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On the role of dissipativity in economic model predictive control ⋆

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Abstract: In this paper, we examine and discuss the role of dissipativity in economic model predictive control. We review some recent results relating dissipativity with the concept of optimal steady-state operation, and we show that *strict* dissipativity is necessary and sufficient for a slightly stronger property than optimal steady-state operation. We discuss the importance of this result for giving closed-loop performance guarantees in economic MPC. Furthermore, we present extensions for the case of optimal periodic operation.

Keywords: Economic MPC, Dissipativity, Optimal steady-state operation, Optimal periodic operation

1. INTRODUCTION

Economic model predictive control (MPC) is a variant of MPC where, in contrast to standard tracking MPC, the control objective is not necessarily the stabilization of an a priori given setpoint (or trajectory), but the optimization of some general performance criterion, possibly related to the economics of the considered system. In recent years, different economic MPC schemes have been proposed and studied in the literature, using different assumptions and/or additional terminal constraints or cost terms, see, e.g., (Angeli et al., 2012; Amrit et al., 2011; Heidarinejad et al., 2012; Müller et al., 2013b; Grüne, 2013; Ferramosca et al., 2014) and the recent survey article by Ellis et al. (2014).

Due to the use of a general performance criterion, the optimal operating regime for the considered system might not be stationary, but can be some periodic orbit or even more complex. Hence an interesting question is to classify what the optimal operating regime is for a given system and a given cost function. Furthermore, it is desirable to guarantee that the closed-loop system, resulting from application of an economic MPC scheme, "finds" the optimal operating behavior, i.e., converges to the optimal trajectory. To this end, a certain dissipativity condition has turned out to play a crucial role. Namely, dissipativity with respect to a supply rate involving the employed stage cost function is both necessary and sufficient such that the optimal operating regime is stationary, i.e., at some steady-state (Angeli et al., 2012; Müller et al., 2013a, 2015; Faulwasser et al., 2014). Furthermore, the same dissipativity property (strengthened to strict dissipativity) can be used to conclude that the optimal steady-state is an asymptotically stable equilibrium point for the resulting closed-loop system, see, e.g., Angeli et al. (2012); Amrit et al. (2011); Grüne (2013); Zanon et al. (2014). For the case where periodic operation is optimal, some first generalizations of these results have recently been studied by Grüne and Zanon (2014) and Müller and Grüne (2015a,b).

The contribution of this paper is to provide a comprehensive treatment of the role played by dissipativity in the context of economic MPC. To this end, we first review some of the results mentioned above concerning the relation between dissipativity and optimal steady-state operation. After that, we show that *strict* dissipativity is both necessary and sufficient for a slightly stronger property than optimal steady-state operation (see Section 3). The implications and importance of this result, also for establishing desired convergence properties for the closed-loop system, are then discussed in Section 4. Section 5 provides extensions of the previous results to the case where periodic operation in contrast to steady-state operation is optimal.

2. PRELIMINARIES AND SETUP

Denote by $\mathbb{I}$ the set of integer numbers, by $\mathbb{I}_{[a,b]}$ the set of integers in the interval $[a, b] \subseteq \mathbb{R}$, and by $\mathbb{I}_{\geq a}$ ($\mathbb{I}_{\leq a}$) the set of integers greater (less) than or equal to a. We consider discrete-time nonlinear systems of the form

$$x(t+1) = f(x(t), u(t)), \qquad x(0) = x_0, \tag{1}$$

where $f : \mathbb{X} \times \mathbb{U} \to \mathbb{R}^n$, $x(t) \in \mathbb{X} \subseteq \mathbb{R}^n$ and $u(t) \in \mathbb{U} \subseteq \mathbb{R}^m$ are the system state and the control input, respectively, at time $t \in \mathbb{I}_{\geq 0}$, and $x_0 \in \mathbb{X}$ is the initial condition. The system is subject to pointwise-in-time state and input constraints

$$(x(t), u(t)) \in \mathbb{Z} \subseteq \mathbb{X} \times \mathbb{U} \tag{2}$$

for all $t \in \mathbb{I}_{\geq 0}$. For a given control sequence $u = (u(0), \dots, u(K)) \in \mathbb{U}^{K+1}$ (or $u = (u(0), \dots) \in \mathbb{U}^\infty$), denote by $x_u(t, x_0)$ the corresponding solution of system (1)

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with initial condition $x_u(0, x_0) = x_0$. For a given $x \in \mathbb{X}$, by $\mathbb{U}^N(x)$ we denote the set of all feasible control sequences of length N, i.e., $\mathbb{U}^N(x) := \{u \in \mathbb{U}^N : (x_u(k, x), u(k)) \in \mathbb{Z}\ \forall k \in \mathbb{I}_{[0,N-1]}\}$. Similarly, the set of all feasible control sequences of infinite length is denoted by $\mathbb{U}^\infty(x)$. Define the set $\mathbb{Z}^0$ as the largest "forward invariant" set contained in $\mathbb{Z}$, i.e., the set which contains all elements in $\mathbb{Z}$ which are part of a feasible state/input sequence pair:

$$\mathbb{Z}^0 := \{(x, u) \in \mathbb{Z} : \exists v \in \mathbb{U}^\infty(x) \text{ s.t. } v(0) = u\} \subseteq \mathbb{Z}. \quad (3)$$

Denote by $\mathbb{X}^0$ the projection of $\mathbb{Z}^0$ on $\mathbb{X}$, i.e., $\mathbb{X}^0 := \{x \in \mathbb{X} : \mathbb{U}^\infty(x) \neq \emptyset\}$.

System (1) is equipped with a stage cost function $\ell : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$ specifying the performance criterion to be minimized. In the context of economic MPC, ℓ can be some general function, and need not be positive definite with respect to a setpoint (or more general, set) to be stabilized as in standard tracking MPC. In economic MPC, the control input to system (1) is now computed at each time instant $t \in \mathbb{I}_{\geq 0}$ with current system state $x = x(t)$ by minimizing, with respect to $u \in \mathbb{U}^N(x)$, the following finite-horizon cost function:

$$J_N(x, u) := \sum_{t=0}^{N-1} \ell(x_u(t, x), u(t)) \quad (4)$$

Then, the first element of the optimal input sequence [1] $u^*_{N,x}$ is applied to system (1) and the procedure is repeated again at time $t + 1$. As discussed in the introduction, an additional terminal cost term and/or suitable terminal constraints are added to the above optimization problem in various economic MPC schemes available in the literature.

Let S be defined as the set of all feasible state/input equilibrium pairs of system (1), i.e.,

$$S := \{(x, u) \in \mathbb{Z} : x = f(x, u)\}, \quad (5)$$

which is assumed to be non-empty. In the following, we assume that a (possibly non-unique) optimal state/input equilibrium pair (x^*, u^*) exists, i.e., (x^*, u^*) satisfies

$$\ell(x^*, u^*) = \inf_{(x,u)\in S} \ell(x, u). \quad (6)$$

For a given $M \in \mathbb{I}_{\geq 1}$, denote by $\mathcal{C}_M$ the set of states which can be steered to x^* in M steps in a feasible way, i.e.,

$$\mathcal{X}_M := \{x \in \mathbb{X} : \exists u \in \mathbb{U}^M(x) \text{ s.t. } x_u(M, x) = x^*\}. \quad (7)$$

Next, let $\mathcal{R}_M$ be the set of states which can be reached from x^* in M steps in a feasible way, i.e.,

$$\mathcal{R}_M := \{x \in \mathbb{X} : \exists u \in \mathbb{U}^M(x^*) \text{ s.t. } x_u(M, x^*) = x\}. \quad (8)$$

Note that $\mathcal{C}_M \cap \mathcal{R}_M \neq \emptyset$, as by definition x^* is contained in both $\mathcal{C}_M$ and $\mathcal{R}_M$. Now define the set $\mathcal{Z}_M$ as the set of state/input pairs which are part of a feasible state/input sequence pair staying in $\mathcal{C}_M \cap \mathcal{R}_M$ for all times:

$$\mathcal{Z}_M := \{(x, u) \in \mathbb{Z} : \exists v \in \mathbb{U}^\infty(x) \text{ s.t. } v(0) = u, \\ x_v(t, x) \in \mathcal{C}_M \cap \mathcal{R}_M\ \forall t \in \mathbb{I}_{\geq 0}\} \subseteq \mathbb{Z}^0. \quad (9)$$

As already discussed in the introduction, in this paper we study and discuss the role of dissipativity in economic MPC. The concept of dissipativity dates back to Willems (1972) (see also (Byrnes and Lin, 1994) for a discrete time version) and is as follows.

Definition 1. The system (1) is dissipative on a set $\mathbb{W} \subseteq \mathbb{Z}$ with respect to the supply rate $s : \mathbb{W} \to \mathbb{R}$ if there exists a storage function [2] $\lambda : \mathbb{W}_\mathbb{X} \to \mathbb{R}_{\geq 0}$ such that the following inequality is satisfied for all $(x, u) \in \mathbb{W}$:

$$\lambda(f(x, u)) - \lambda(x) \leq s(x, u). \quad (10)$$

If there exists $\rho \in \mathcal{K}_\infty$ such that for all $(x, u) \in \mathbb{W}$

$$\lambda(f(x, u)) - \lambda(x) \leq -\rho(|x - x^*|) + s(x, u), \quad (11)$$

then system (1) is strictly dissipative on $\mathbb{W}$.

An equivalent characterization of dissipativity can be obtained via the so-called *available storage*, defined as

$$S_a(x) := \sup_{T\geq 0, u\in\mathbb{U}^\infty(x)} \sum_{t=0}^{T-1} -s(x_u(t, x), u(t)). \quad (12)$$

Namely, it was shown by [3] Willems (1972) that system (1) is dissipative on $\mathbb{Z}^0$ with respect to the supply rate s if and only if $S_a(x) < \infty$ for all $x \in \mathbb{X}^0$. Furthermore, in an analogous fashion one can show that that system (1) is dissipative on $\mathbb{Z}^0$ with respect to the supply rate s and with a storage function λ which is bounded on $\mathbb{X}^0$ if and only if S_a is bounded on $\mathbb{X}^0$, i.e., $S_a(x) \leq c < \infty$ for all $x \in \mathbb{X}^0$ and some $c \geq 0$.

3. DISSIPATIVITY AND OPTIMAL STEADY-STATE OPERATION

Given the system dynamics (1), the constraint set $\mathbb{Z}$ and the cost function ℓ, an interesting question is to determine what the optimal operating regime looks like, i.e., what system behavior results in an optimal performance. To this end, the following definition of optimal steady-state operation was considered in Angeli et al. (2012).

Definition 2. System (1) is *optimally operated at steady-state*, if for each $x_0 \in \mathbb{X}^0$ and each $u \in \mathbb{U}^\infty(x)$ the following holds for all $t \in \mathbb{I}_{\geq 0}$:

$$\liminf_{T\to\infty} \frac{\sum_{t=0}^{T-1} \ell(x_u(t, x), u(t))}{T} \geq \ell(x^*, u^*). \quad (13)$$

System (1) is *suboptimally operated off steady-state*, if in addition for each $x_0 \in \mathbb{X}^0$ and each $u \in \mathbb{U}^\infty(x)$ at least one of the following two conditions holds:

$$\liminf_{T\to\infty} \frac{\sum_{t=0}^{T-1} \ell(x_u(t, x), u(t))}{T} > \ell(x^*, u^*) \quad (14a)$$

$$\liminf_{t\to\infty} |x_u(t, x) - x^*| = 0 \quad (14b)$$

The definition of optimal steady-state operation means that no feasible solution can have an (asymptotic) average performance which is better than the performance of the best steady-state, while suboptimal operation off steady-state means that each solution has an (asymptotic) average performance which is strictly worse than the performance of the best steady-state, or "passes by" the optimal steady-state infinitely often. The following theorem from Angeli et al. (2012) shows that a certain dissipativity property is sufficient for optimal steady-state operation of system (1).

[1] In the following, we assume that for all $x \in \mathbb{X}^0$, a minimizing control sequence $u^*_{N,x} \in \mathbb{U}^N(x)$ exists, i.e., such that $J_N(x, u^*_{N,x}) = \inf_{u\in\mathbb{U}^N(x)} J_N(x, u)$.

[2] Here, $\mathbb{W}_\mathbb{X}$ denotes the projection of $\mathbb{W}$ on $\mathbb{X}$.

[3] We note that while this was established by Willems (1972) for continuous-time systems without constraints, the same result can be obtained in an analogous fashion for our setting of discrete-time systems with state and input constraints.