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Efficient Implementation of Step Response Prediction Models for Embedded Model Predictive Control ⋆

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Abstract: Different formulations of the step response prediction model used in Model Predictive Control (MPC) are examined in this paper. The advantages and disadvantages of the models are uncovered, and a class of model errors that lead to the deterioration of the prediction capability of step response models is identified. Particular attention is paid to the effect of *small* truncation errors, and the results show that such errors might lead to poor predictions, if the last element in the step response sequence is used to extend the prediction model. Several implementation aspects that are crucial for embedded targets with limited resources are also discussed. The results of the performance and computational enhancements proposed in this paper are analyzed in a simulation study.

Keywords: Model predictive control, step response models, unmatched model error, embedded systems.

1. INTRODUCTION

Model Predictive Control (MPC) is an advanced control method for constrained multivariable control problems. MPC relies on an *internal* dynamic model of the plant and available plant measurements (or estimates) in order to predict the future behavior of the plant. The model describes the dynamic and static interactions between inputs (also known as *manipulated variables* or MVs), outputs (*controlled variables* or CVs), and *disturbance variables* (DVs).

In a model-based control approach the accuracy of the model used has a significant effect on the performance of the controller, and in the case of MPC, inaccurate predictions can lead to undesirable control performance. The type of model used also incorporates specific characteristics and capabilities to the MPC scheme. For instance, the use of step response models is limited to asymptotically stable and integrating plants, whereas a state-space or ARX model can also describe unstable systems. A *nonlinear* model will introduce different properties (including significant advantages and disadvantages) to an MPC scheme compared to the use of a *linear* model, which is the main focus of this paper.

Dynamic Matrix Control (DMC), Cutler and Ramaker (1979), which is among the first MPC schemes developed in the 1970s, uses step response models, and the use of step response models is still common in industrial MPC implementations (Strand and Sagli, 2003; Maciejowski, 2002; Camacho and Bordons, 2007; Qin and Badgwell, 2000; Garcia et al., 1989). This is mainly because step response models are easy to build, understand, and maintain (Strand and Sagli, 2003; Lee et al., 1994). The limitations of step response models are also well covered in the MPC literature (see for example Lundström et al. (1995); Lee et al. (1994); Qin and Badgwell (2000); Maciejowski (2002)). However, many practical applications exist where the limitations are not considered crucial for control performance.

A known issue of step response models is the large amount of data usually required in order to produce accurate enough predictions in MPC. The amount of data tends to be very large for applications where fast sampling rates are necessary for desired control performance targets (Lundström et al., 1995; Hovd et al., 1993; Maciejowski, 2002). Due to practical limits on the amount of step response data that can be used in an application, truncated models may be considered. Nevertheless, the extent of truncation is limited. In fact, large truncation errors may not only lead to poor performance, but also instability (Lundström et al., 1995). Different techniques have been introduced to capture the residual neglected when the step response sequence is truncated (see for example Hovd et al. (1993); Lee et al. (1994)). However, such techniques tend to incorporate other types of model representations, leading to "hybrid" formulations, and may introduce complications that reduce the main attractiveness of step response models. It is therefore the goal of this work to enhance the efficiency of prediction models that rely only on step response data.

In this paper, different formulations of the step response prediction model are examined, and implementation aspects that are crucial for embedded targets with limited resources are discussed. The properties of the models are analyzed in a simulation study. It is common practice to use the assumption $s(N+i) \approx s(N)$, for $i \geq 1$ to extend a sequence of N step response coefficients in order to (for example) achieve appropriate dimensions in a matrix-vector formulation for multiple-input-multiple-output (MIMO) systems. Nevertheless, it is shown

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in this paper that a more cautious use of $s(N+i) \approx s(N)$ is important in order to avoid the violation of the constant *unknown* disturbance assumption commonly used in step response prediction models.

2. STEP RESPONSE PREDICTION MODELS

When a step response model of a single-input single-output (SISO) system is used for prediction in MPC, an estimate of the future plant output trajectory, $\{y(k+j), j=1,\ldots,H_p\}$, can be obtained based on the knowledge of the past control moves, $\{\Delta u(k-i), i=1,\ldots,N\}$, and the measured plant output $y_m(k)$. The prediction horizon is specified by H_p, and N is the number of step response coefficients, $s(i)$, used in the step response model:

$$y(k) = \sum_{i=1}^{N-1} s(i)\Delta u(k-i) + s(N)u(k-N) \quad (1)$$

Model (1) assumes that the step response coefficients are obtained from a process that is initially at steady-state, with all inputs and outputs at zero. The model (1) is valid only if the process is asymptotically stable, implying that the coefficients in $s(i)$ reach constant values after N sampling periods, i.e. $s(N+1) \approx s(N)$.

2.1 Standard formulation

Based on (1), prediction of the future output trajectory can be computed using

$$\tilde{y}(k+j|k) = \sum_{i=j+1}^{N-1} s(i)\Delta\tilde{u}(k+j-i) + s(N)\tilde{u}(k+j-N) + \sum_{i=1}^{j} s(i)\Delta u(k+j-i), \quad (2)$$

where $\Delta u(k) = u(k) - u(k-1)$. The *unknown* terms consist of the present and future input moves $\Delta u(\cdot)$, while the *known* terms contain the past input $\tilde{u}(\cdot)$ and past input moves $\Delta\tilde{u}(\cdot)$. Model (2) is referred to as prediction model Variant A.

2.2 Disturbances

Without output feedback, the cumulative effects of unmeasured disturbances and model errors will lead to inaccurate predictions. A disturbance model $v(k+j|k)$ is therefore used:

$$\hat{y}(k+j|k) = \tilde{y}(k+j|k) + v(k+j|k), \quad (3)$$

$$v(k+j|k) = v(k|k) = y_m(k) - \tilde{y}(k|k-1). \quad (4)$$

The model (4) is a usual choice known as a *bias* term used in correcting $\tilde{y}(k+j|k)$, and it provides integral action in MPC. Throughout this paper, $\tilde{\cdot}$ is used on output vectors to indicate that the predictions are not corrected, and the use of $\hat{\cdot}$ implies that a bias correction is applied. The simple bias term (4) assumes that an additive (step) disturbance acts on the plant output, and the disturbance remains constant for $j=1,\ldots,H_p$. If this assumption does not hold in a given situation, the output prediction will be incorrect, and it may lead to poor control performance.

If a disturbance variable d can be measured, a *disturbance term* that contains the step response model relating the measured disturbance to each controlled variable can be added to the prediction model. Since the future changes in disturbance are not always known at the current time k, a usual assumption is that $\Delta d(k+j) = 0,\ j \geq 1$.

2.3 Alternative formulations of the standard prediction model

An alternative formulation to (2) is derived in Maciejowski (2002):

$$\tilde{y}(k+j|k) = \sum_{i=j+1}^{N} s(i)\Delta\tilde{u}(k+j-i) + s(j)\tilde{u}(k-1) + \sum_{i=1}^{j} s(i)\Delta u(k+j-i), \quad (5)$$

where only the past control input $\tilde{u}(k-1)$ is used, instead of $\{\tilde{u}(k+j-N),\ j=1,\ldots,H_p\}$, as stated in (2). Model (5) is referred to as Variant B.

In (3), the *known* terms of $\hat{y}(k+j|k)$ define the predicted unforced response of the plant $\hat{y}_f(k+j|k)$, also known as the *free response*. From (3) and (4),

$$\hat{y}(k+j|k) = \sum_{i=1}^{j} s(i)\Delta u(k+j-i) + \hat{y}_f(k+j|k), \text{ where}$$

$$\hat{y}_f(k+j|k) = \sum_{i=j+1}^{N-1} s(i)\Delta\tilde{u}(k+j-i) + s(N)\tilde{u}(k+j-N) + y_m(k) - \sum_{i=1}^{N-1} s(i)\Delta\tilde{u}(k-i) - s(N)\tilde{u}(k-N), \quad (6)$$

and $\hat{y}_f(k+j|k)$ the response at each point along the prediction horizon, if the future inputs remains the same as $\tilde{u}(k-1)$.

Using (6), another formulation, Variant C, is derived in Camacho and Bordons (2007), where the state space realization of step response models are emphasized. Considering $N > H_p$, the free response can be written in a more compact form (Camacho and Bordons, 2007):

$$\hat{y}_f(k+j|k) = y_m(k) + \sum_{i=1}^{N} \big(s(i+j) - s(i)\big)\Delta\tilde{u}(k-i), \quad (7)$$

where $s(i+j) - s(i) \approx 0$, for $i > N$, has been used. The prediction model can be written in a matrix-vector form by considering the following definitions:

$$\hat{Y}(k+1) = [\hat{y}(k+1|k) \ \ldots \ \hat{y}(k+H_p|k)]^T \quad (8a)$$

$$\hat{Y}_f(k+1) = [\hat{y}_f(k+1|k) \ \ldots \ \hat{y}_f(k+H_p|k)]^T \quad (8b)$$

$$\Delta U(k) = [\Delta u(k) \ \ \Delta u(k+1) \ \ldots \ \Delta u(k+H_u-1)]^T \quad (8c)$$

The prediction model variant C will then take the form:

$$\hat{Y}(k+1) = \bar{\Theta}\Delta U(k) + \hat{Y}_f(k+1), \quad (9)$$

$$\text{where} \quad \bar{\Theta} = \begin{bmatrix} s(1) & 0 & \cdots \\ s(2) & s(1) & \ddots \\ \vdots & \vdots & \ddots \\ s(H_p) & \ldots & s(H_p-H_u+1) \end{bmatrix} \quad (10)$$

Since $\hat{Y}_f(k+1)$ is the response when no future control moves are applied, it depends on the state of the plant, defined as

$$\hat{X}(k) = [y_m(k) \ \ \Delta u(k-1) \ \ldots \ \Delta u(k-N+1)]^T \quad (11)$$

$$\Longrightarrow \hat{Y}_f(k+1) = \mathscr{A}_f\hat{X}(k), \quad (12)$$

where $\mathscr{A}_f$ can be extracted from (7) by direct inspection.

$$\mathscr{A}_f = \begin{bmatrix} 1 & s(2)-s(1) & s(3)-s(2) & \ldots & s(N)-s(N-1) \\ 1 & s(3)-s(1) & s(4)-s(2) & \ldots & s(N)-s(N-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & s(1+H_p)-s(1) & s(2+H_p)-s(2) & \ldots & s(N)-s(N-1) \end{bmatrix} \quad (13)$$

The prediction model (9) can therefore be written as

$$\hat{Y}(k+1) = \bar{\Theta}\Delta U(k) + \mathscr{A}_f\hat{X}(k), \quad (14)$$