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Model Predictive Control of the Current Density Distribution and Stored Energy in Tokamak Fusion Experiments using Trajectory Linearizations ⋆

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Abstract: Tokamaks are used to confine high temperature plasmas for nuclear fusion research. In this work we apply model predictive control to the transport process in a tokamak plasma that can be described by a set of nonlinear coupled partial differential equations, where the controlled quantities are the current density distribution and stored thermal energy. Applying trajectory linearizations around already commonly predefined feedforward trajectories enables us to use linear MPC techniques that are computationally tractable for implementation on existing tokamaks. Special requirements for the MPC controller are that it should be able to handle real-time-varying references and constraints, whereas the system size, required prediction horizon and available computational time imposes additional challenges. An MPC controller is designed according to the requirements and its performance is analyzed in simulations that approach high performance plasma experiments in the ASDEX Upgrade tokamak. The results show the potential of the controller and encourage its further exploration and use in experiments.

Keywords: Model-based control, Nonlinear systems, Constraints.

1. INTRODUCTION

The tokamak is the furthest developed nuclear fusion device in which a plasma is confined in a torus-shaped device using magnetic fields and heated to achieve fusion reactions. Due to the specific organization of the magnetic field, the plasma transport process can be seen as a 1 dimensional distributed parameter system in the direction of the minor radius, where the plasma quantities as function of the radial coordinate are called profiles (as illustrated in Figure 1). The plasma transport process can be described by a number of nonlinear coupled partial differential equations (fluid transport equations together with the Maxwell equations). A control-oriented introduction to tokamaks and their control is given in (Pironti et al. (2005, 2006)).

Controlling profiles or scalar profile quantities of the nonlinear plasma transport process in (future) tokamaks is a challenging task for a number of reasons. First, the dynamics are described by nonlinear coupled transport equations that are only recently available in the form of control-oriented models (Witrant et al. (2007); Ou et al. (2007); Felici et al. (2011)). Second, measurements in a hot plasma are technologically complicated, limited in quantity and

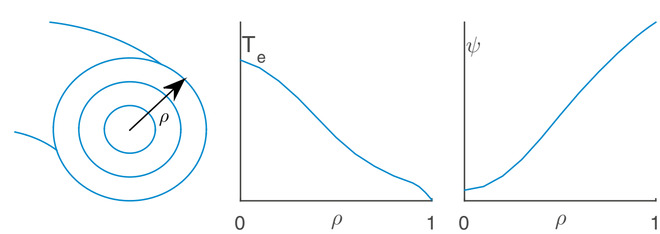

Fig. 1. Illustration of plasma profiles in a tokamak. Radial coordinate ρ (left) and typical profiles of electron temperature $T_e(\rho)$ and poloidal magnetic flux $\psi(\rho)$.

are not routinely available in real-time on many devices, such that feedback control is often limited to a subset of the desired controlled parameters. Third, the plasma profile controller is part of a larger control problem with multiple controllers that may share common actuators that should be governed by a supervisory controller. Depending on the occurrence of actuator events (e.g. actuator failure) or plasma events (e.g. performance and stability degrading plasma instabilities), such a supervisory controller may reallocate actuators to the different control tasks and change control objectives and references in real-time (Winter et al. (2014)). Fourth, actuators are spatially distributed over the radial coordinate and are subjected to strict constraints in amplitude and ramp-rate that thus may change in real-time. Fifth, exceeding certain plasma parameter quantities is known to trigger performance and

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stability degrading plasma instabilities and hence it is desired to keep these plasma parameters within their limits. The energy stored in the plasma increases in larger devices such as ITER (under construction) and DEMO (planned demonstration reactor). Exceeding operational limits with high stored energy may result in unallowable damage to the machine at high cost and loss of expensive operational time. Sixth, the dominant process dynamics exhibit distinct time scales. On currently operational medium-sized tokamaks (e.g. the tokamaks ASDEX Upgrade in Germany and TCV in Switzerland), the fastest time scale is in the order of a few milliseconds and this imposes a strong limit on the available computational time of 2 milliseconds. The slowest time scale is however in the order of seconds, indicating the wide range of time scales present.

Recently, a number of model-based feedback controllers have been proposed for the control of plasma profiles and scalar quantities in tokamaks, where we refer here only to a few of the most recent publications (Barton et al. (2012, 2014); Boyer et al. (2013, 2014); Gaye et al. (2013); Moreau et al. (2013)). Only a few of these controllers have so far been used in dedicated experiments. These contributions impose fixed actuator constraints a posteriori by the use of e.g. an anti-windup loop. Handling of actuator constraints in the controller design itself is done in (Ou et al. (2011); Ouarit et al. (2011); Bribiesca Argomedo et al. (2013)), where actuator constraints are fixed.

Model predictive control (Camacho and Bordons (2004); Rossiter (2013); Mayne (2014)) appears, due to its constraint handling capabilities and straightforward handling of MIMO systems, as a promising control method to increase the performance and stability of tokamaks plasmas, while ensuring the satisfaction of important machine and physical limits. So far MPC has not been researched for profile control, except for the initial work in (Ou et al. (2011); Ouarit et al. (2011)). The controllers in these works handle only fixed actuator constraints, contain a model with a single PDE and yield nonlinear optimization problems that cannot be solved in real-time on existing devices.

In our recent work (Maljaars et al. (2015)) we proposed an MPC approach for the control of one particular magnetic profile important for the stability and performance of a plasma. We used trajectory linearizations to formulate an MPC controller that can track a predefined reference trajectory in the presence of time-varying actuator and state constraints. Simulation results of the future ITER tokamak showed the performance of the controller in reducing tracking error and handling time-varying constraints with computational times that enable implementation on currently operational tokamaks.

This work extends our previous work in various ways. In this paper we not only control a magnetic profile, but also a kinetic quantity being the plasma stored energy. In addition, the controller can handle real-time-varying references that differ from the nominal trajectory used for the trajectory linearizations. Constraints are added that limit a weighted combination of inputs and outputs. To improve tracking and constraint handling during the stationary phase of a tokamak plasma, state disturbance estimation is incorporated. The performance of the proposed MPC controller is shown in simulation using the nonlinear plasma transport simulator RAPTOR (Felici and Sauter (2012)) with parameters that are chosen to approximately represent high performance plasma experiments in the ASDEX Upgrade tokamak (Schweinzer et al. (2012)).

This paper is organized as follows. In section 2 we introduce the process model, the phases in a tokamak experiment and formulate the controller requirements. Based on these controller requirements, in section 3 we present the MPC controller design. The performance of the designed controller is analyzed in similations in section 4. Finally, conclusions and an outlook are given in section 5.

2. PROCESS MODELING AND CONTROLLER REQUIREMENTS

In this section we describe briefly the process model and the phases of a typical tokamak experiment after which we define the controller requirements.

2.1 Process model in nonlinear state space format

In this work we are interested in the spatial and temporal evolution of the current density distribution and the temporal evolution of the stored energy in the plasma in response to the various actuators. The underlying dynamics of both quantities are mainly the nonlinear coupled evolution of the poloidal magnetic flux $\psi(\rho, t)$ and the electron temperature $T_e(\rho, t)$ as a function of a radial coordinate ρ, represented by partial differential equations (PDEs) (Hinton and Hazeltine (1976)) that can be written as:

$$k_1 \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial \rho}\left(k_2 \frac{\partial \psi}{\partial \rho}\right) + k_3 + k_u u_{\text{int}}, \quad (1)$$

$$c_1 \frac{\partial T_e}{\partial t} = \frac{\partial}{\partial \rho}\left(c_2 \frac{\partial T_e}{\partial \rho}\right) + c_3 + c_u u_{\text{int}}. \quad (2)$$

These are two non-linear coupled diffusion equations, where the quantities k_1, k_2, k_3, k_u, c_1, c_2, c_3 and c_u are functions of the radial coordinate ρ and nonlinearly dependent on the profiles ψ and T_e and other prescribed profiles including the geometry of the plasma. The radial coordinate $\rho \in \mathbb{R}$ equals 0 in the plasma core and 1 at the plasma edge.

The actuators act either on the boundary (ρ=1) or the interior of the plasma ($\rho \leq 1$). The plasma current I_p[MA] acts on the boundary of ψ. The interior actuators are given in the vector $u_{\text{int}} \in \mathbb{R}^{n_{u_{\text{int}}}}$ and the quantities $k_u(\rho)$ and $c_u(\rho)$ describe their spatial localization. We choose as interior actuators the auxiliary spatially distributed actuators that heat the plasma and can drive current in the plasma. These auxiliary heating and current drives are comprised of a power request to two electron cyclotron beams ($P_{ec,1}$[MW] and $P_{ec,2}$[MW]) at different locations in ρ and a power request to the Neutral Beam Injection (NBI) system (P_{nbi}[MW]). We assume that low level controllers ensure that the requested power is delivered.

The current density distribution and the stored energy are a function of ψ and T_e respectively. The so-called inverse safety factor profile is a direct measure of the current density distribution in the plasma and an often used measure for stability and performance of the plasma.