Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-23 (2015) 352–359

Scenario and Adaptive Model Predictive Control of Uncertain Systems

Lorenzo Fagiano * **Georg Schildbach** ** **Marko Tanaskovic** *** **Manfred Morari** ***

* *ABB Switzerland Ltd., Corporate Research (lorenzo.fagiano@ch.abb.com).*
** *University of California at Berkeley (schildbach@berkeley.edu)*
*** *ETH Zürich, Automatic Control Laboratory, (tmarko@control.ee.ethz.ch, morari@control.ee.ethz.ch)*

Abstract: Two recent predictive control approaches for constrained systems subject to uncertainty are reviewed. The first one, named *scenario MPC*, is best suited for stochastic systems where a certain share of constraint violations is tolerated and rewarded. The approach is able to control precisely the share of violations that occur during closed loop operation, under quite general assumptions on the involved stochastic variables. The second technique, named *adaptive MPC*, is cast in a different framework, where the aim is to enforce robustly the system constraints and a stochastic characterization of the uncertainty is not required. The algorithm embeds a real-time set membership identification strategy that yields a refined set of unfalsified models at each time step, hence reducing the size of the model uncertainty and improving the closed loop performance over time. After recalling the main results pertaining to each approach, their applicability, strengths and weaknesses are discussed, as well as open issues that can be subject of future research.

Keywords: Model Predictive Control, Scenario MPC, Stochastic MPC, Adaptive MPC, Robust MPC, Set Membership Identification, Soft Constraints, Stochastic Systems, Scenario Optimization, Chance Constraints

1. INTRODUCTION

Among the research topics concerned with Model Predictive Control, the study of its robustness properties and the development of approaches that can cope with uncertainty and disturbances have been active subjects for more than two decades, during which theoretical results and design techniques have been delivered for both linear and nonlinear systems, in presence of different descriptions of the uncertainty. A conspicuous number of contributions considered non-stochastic models for the uncertainty affecting the system, where the uncertain parameters and/or external disturbances are assumed to be of the "unknown but bounded" (ubb) type, typically confined in polytopic sets, see for example the works of Polak and Yang (1993); Michalska and Mayne (1993); Kothare et al. (1996); Lee and Kouvaritakis (2000); Primbs and Nevistic (2000); Diehl and Björnberg (2004); Grimm et al. (2004); Langson et al. (2004); Mayne et al. (2005); Bravo et al. (2006); Goulart et al. (2006); Pin et al. (2009). In a non-stochastic framework, the aim is usually to absolve a reference tracking task while enforcing constraints and retaining closed loop stability for all possible realizations of the uncertainty within the assumed bounds. This is often referred to as worst-case approach, since the control input is determined by those values of the uncertain parameters or disturbances which determine the worst situation possible for the system in its current state.

In relatively more recent times, research efforts have been increasingly focusing also on stochastic uncertainty models, see e.g. the work of Li et al. (2002); Cannon et al. (2011); Chatterjee et al. (2011); Cinquemani et al. (2011); Schwarm and Nikolaou (1999); de la Peña et al. (2005); Cannon et al. (2009); Primbs and Sung (2012); Arellano-Garcia and Wozny (2009); Bemporad and Cairano (2011); Bernardini and Bemporad (2012); Cairano et al. (2014). A stochastic characterization of the uncertainty naturally leads to the concept of probability of constraint violation, hence opening a framework where violating constraints is tolerated up to a certain desired probability. Such a concept is of little interest if the optimization of the performance is by itself driving the system's trajectory inside the constraints. A more intriguing framework for stochastic uncertainty models is one where a better performance is achieved when constraints are violated, i.e. when enforcing the constraints and optimizing performance are "opposite" tasks, so that it is of interest to attain exactly the prescribed probability of constraint violations, which reflects a tradeoff between cost optimization and safe system operation. This framework is interestingly connected to economic MPC problems (see e.g. the special issue edited by Christofides and El-Farra (2014)), when the economic cost criterion drives the state trajectory on the constraints.

Notwithstanding the relatively large number of contributions in the literature, research on MPC approaches that take into account either stochastic or non-stochastic uncertainty models is still very active, tackling more and more complex problems or aiming at devising more efficient or practical implementations. In recent years, we developed two new techniques to deal with stochastic and non-stochastic uncertainty, respectively.

The first one, named *scenario MPC* (Schildbach et al. (2014)), allows the control designer to control precisely the share of violations that occur during closed loop operation. The approach can be applied to any uncertain linear system, i.e. any probability distribution on any support set can be treated, as well as any kind of functional dependence of the system matrices or the exogenous disturbances from the uncertain variables. Moreover, the optimization program to be solved at each time step is

10.1016/j.ifacol.2015.11.305

convex and the number of constraints scales linearly with the number of control inputs, hence it is independent of the prediction horizon and from the number of uncertain variables. The approach builds up on the theory of scenario optimization and its application to multi-stage uncertain decision problems and MPC, see the works by Campi and Garatti (2008, 2011); Calafiore and Fagiano (2013a,b); Matuško and Borrelli (2012); Prandini et al. (2012); Schildbach et al. (2012); Vayanos et al. (2012); Schildbach et al. (2013).

The second technique, named *adaptive MPC* (Tanaskovic et al. (2014a)), is cast in a non-stochastic framework where the only prior knowledge on the system is given by some (eventually very loose) bounds on its impulse response coefficients and a bound on the additive output disturbances. The approach combines a real-time set membership identification algorithm and a robust predictive control one: the former is used to refine, at each time step, the set of models that are consistent with the available prior information and the collected input-output measurements, while the latter is used to compute a control input that enforces robustly the system constraints and ensures recursive feasibility. The use of model adaptation in MPC has been proposed several times in the literature, see e.g. the works by Kim et al. (2008); Kim and Sugie (2008); Aswani et al. (2013); Adetola et al. (2011); Veres and Norton (1993); however among these previous contributions, those that lead to a convex optimization problem are cast in a different framework with respect to the one we propose, and none of them consider at the same time the presence of non-zero measurement noise on the feedback variables, hard output constraints, and multiple-input, multiple-output (MIMO) plants.

In this paper, we provide an overview of these two approaches and of the main related theoretical results, and, in light of the experience we accumulated on their use, we discuss their applicability, strengths and weaknesses, as well as open issues that can be subject of future research.

2. SCENARIO MODEL PREDICTIVE CONTROL

2.1 Problem settings

We consider a discrete-time system model with a linear stochastic transition map

$$x(t+1) = A(\delta(t))x(t) + B(\delta(t))u(t) + w(\delta(t)),\ x(0) = \bar{x}, \tag{1}$$

for some fixed initial condition $\bar{x} \in \mathbb{R}^{n_x}$. The matrices $A(\delta(t)) \in \mathbb{R}^{n_x \times n_x}$ and $B(\delta(t)) \in \mathbb{R}^{n_x \times n_u}$ as well as the additive disturbance $w(\delta(t)) \in \mathbb{R}^{n_x}$ are random, as they are known functions of a primal uncertainty $\delta(t)$. For notational simplicity, $\delta(t)$ comprises all uncertain influences on the system at time $t \in \mathbb{N}$.

Assumption 1. (a) $\{\delta(0), \delta(1), ...\}$, are independent and identically distributed (i.i.d.) random variables on a probability space $(\Delta, \mathbf{P})$; (b) i.i.d. samples of $\delta(t)$ can be obtained, either empirically or by a random number generator.

The support set Δ of $\delta(t)$ and the probability measure $\mathbf{P}$ on Δ are entirely generic. In fact, Δ and $\mathbf{P}$ do not need to be known explicitly. The exact number of samples which we require at point (b) of the Assumption will become concrete in the main results recalled in the next section.

The system (1) can be controlled by inputs $\{u(0), u(1), ...\}$, belonging to a set of feasible inputs, $\mathbb{U} \subset \mathbb{R}^{n_u}$. In particular, $u(t)$ should be determined by a static feedback law

$$\psi : \mathbb{R}^{n_x} \to \mathbb{U} \qquad \text{with} \qquad u(t) = \psi(x(t)) \ ,$$

based only on the current state of the system.

The cost function is given by the time-average of stage costs $\ell : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}_{0+}$,

$$\frac{1}{T} \sum_{t=0}^{T-1} \ell\big(x(t), u(t)\big) \ . \tag{2}$$

The stage cost ℓ is chosen by the designer according to the control objective (e.g. consumed energy, tracking error, etc.); typical choices are to consider the behavior of a nominal model (i.e. with some fixed value of the system matrices and of the exogenous disturbance) or the expected behavior of the uncertain system. Value-at-risk formulations can be also treated, see Schildbach et al. (2014) for the full details. The time horizon T is considered to be very large and generally not precisely known, as it represents the total period of time during which the control system operates.

The minimization of the cost is subject to keeping the state inside a set $\mathbb{X}$ for a given fraction of all time steps. More precisely, we denote with $M(t) := \mathbf{1}_{\mathbb{X}^{\mathrm{C}}}(x(t+1))$ the random variable indicating that $x(t+1) \notin \mathbb{X}$. Here $\mathbf{1}_{\mathbb{X}^{\mathrm{C}}} : \mathbb{R}^{n_x} \to \{0, 1\}$ is the indicator function on the complement $\mathbb{X}^{\mathrm{C}}$ of $\mathbb{X}$. The goal is to limit, in closed loop operation, the expected time-average of constraint violations below a given violation level $\varepsilon \in (0, 1)$:

$$\mathbf{E}\Big[\frac{1}{T} \sum_{t=0}^{T-1} M(t)\Big] \leq \varepsilon \ . \tag{3}$$

We assume that the following conditions hold for the described control problem:

Assumption 2. (a) The state of the system can be measured at each time step t. (b) The set of feasible inputs $\mathbb{U}$ is bounded and convex. (c) The state constraint set $\mathbb{X}$ is convex. (d) The stage cost $\ell(\cdot, \cdot)$ is a convex function.

Assumption 2(b) holds for most practical applications, moreover very large artificial bounds can always be introduced for input channels without natural bounds. Uncertain state constraint sets, i.e. $\mathbb{X}(\delta)$, can be straightforwardly included as long as for any fixed value of δ the set $\mathbb{X}(\delta)$ is convex.

Combining what we introduced so far, the *optimal control problem (OCP)* can be stated as follows:

$$\min_{\psi(\cdot)} \quad \frac{1}{T} \sum_{t=0}^{T-1} \ell\big(x(t), u(t)\big) \ , \tag{4a}$$

$$\text{s.t.}\ \ x(t+1) = A(\delta(t))x(t) + B(\delta(t))u(t) + w(\delta(t)),\ \ ,$$
$$x(0) = \bar{x}, \quad \forall\ t = 0, ..., T-1 \ , \tag{4b}$$

$$\mathbf{E}\Big[\frac{1}{T} \sum_{t=0}^{T-1} \mathbf{1}_{\mathbb{X}^{\mathrm{C}}}(x(t))\Big] \leq \varepsilon \ , \tag{4c}$$

$$u(t) = \psi(x(t)) \quad \forall\ t = 0, ..., T-1 \ . \tag{4d}$$

Since the initial state $\bar{x}$ is given, only the state feedback law $\psi(\cdot)$ is a free variable in (4).

The OCP is generally intractable, as it involves an infinite-dimensional decision variable $\psi(\cdot)$ and a large number of constraints (growing with T). A possible way to solve the OCP in an approximate way is to rely on an MPC strategy with a shorter horizon $N \ll T$. In particular, a Stochastic MPC (SMPC) approach often considered in the literature