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Scenario Based Implicit Dual Model Predictive Control

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Abstract: Model Predictive Control (MPC) has become a popular control strategy, especially within process control, where the handling of constraints and multiple inputs and outputs are essential. As the performance of the controller is crucially linked to the models predictive capabilities, model uncertainty can reduce the performance significantly. Robust MPC has been proposed to handle model uncertainty, but often leads to overly conservative solutions. In this paper, we propose a new stochastic scenario based formulation for robust MPC, where feedback is explicitly introduced in the optimization problem, to allow both state and parameter updates. The updates are conducted based on measurements from the different scenarios, and we use an Ensemble Kalman Filter (EnKF) for state and parameter updating. The resulting controller is an implicit dual MPC, and as shown in an example, applies perturbations for identification only if it will return itself over the prediction horizon.

Keywords: model predictive control, optimal control, dual control, stochastic control, system identification

1. INTRODUCTION

MPC solves an optimization problem at every control step. The plant behaviour is predicted by a model, obtaining a trajectory over the prediction horizon. This allows formulating constraints on both the decision variables and the future predicted states. Only the first control step is used, because at the next control step the optimization problem is solved over again. The prediction horizon is then shifted forward in a receding manner, and because of this, MPC is also known as *receding horizon control*.

The model is crucial for the performance of an MPC. If the model fails to capture the dynamics of the plant, the calculated control will be suboptimal for the real plant. An MPC will never be better than the model used in the optimization. Even when great care is taken in the modelling phase, there can be uncertainty in the model. Some parameters can be difficult to estimate precisely, while quantities such as future raw material cost are stochastic in nature. Also the estimate of the current state can be uncertain. Although both states and parameters are known to be uncertain, they are usually treated as if they were precisely known. This results in the expected value problem, that is, the optimization problem is solved assuming the expected value of the states and parameters are the actual ones. The expected value problem basically ignores the uncertainty involved. This often yields very good performance, and when the actual quantities are close to the expected value of the estimates, the solution is nearly optimal for the real plant. However, the performance can deteriorate quickly when the actual parameters deviate from the expected value.

A recent extensive review of MPC is provided in Mayne (2014), where a substantial part is on MPC for uncertain systems. When dealing with uncertain systems, there are several challenges emerging. Because of the uncertainty, it is not possible to perfectly predict the behaviour of the plant. However, we assume that we will obtain measurements in the future, and we can apply feedback to compensate for the model uncertainty. Depending on whether this feedback possibility is included in the optimization problem, we arrive at what is known as open loop or closed loop predictions. In the case of open loop predictions, the control input is calculated as a sequence over the control horizon, with no concept of feedback. The control is, however, applied in a feedback manner, since the problem is solved at every control step. This control strategy is also denoted as *open loop feedback*. Closed loop predictions on the other hand, foresee that measurements will become available, and is denoted as *closed loop feedback*. It is often posed as an optimization problem optimizing over feedback policies, such as $u_k = K_k x_k$, where x_k and u_k are the states and control input at time t_k. If we do not parametrize the control in terms of feedback policies, the closed loop predictions naturally lead to a multi-stage stochastic programming problem, as in Lucia et al. (2013). Exact full state information at every stage is assumed, but relaxed in later work. In Subramanian et al. (2014), an Extended Kalman Filter (EKF) is used in a multi-stage output NMPC, while an Unscented Kalman Filter (UKF) is used in the recently published work Subramanian et al. (2015).

This partitioning in open and closed loop predictions is irrelevant for the deterministic case, as there is no new information to be obtained, and the two are equivalent (Rawlings and Mayne, 2009). Another aspect when controlling uncertain systems, is the possibility of gaining new knowledge of the uncertain parameters in the future. System identification is often conducted as a separate

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task, independent of control. The two tasks are, however, closely coupled, as the result of the system identification is directly linked to the control input applied.

Control of uncertain or time varying systems have been studied within adaptive control, and the branch on dual control is especially relevant from an MPC perspective. In dual control, the controller has the two goals of controlling the system and obtaining more information about the uncertain system. These goals are often viewed as conflicting, as perturbations for better identification will decrease the performance of the control. However, better identification might yield a reward in the future because it allows for better control, and the perturbation should only be conducted if it will return itself. The dual goal of the controller was first considered in the seminal work of Feldbaum (1960). The dual problem is however computationally intractable for all except the simplest problems, and approximations must be applied. Dual control is often divided into explicit and implicit dual control. In the case of *explicit dual control*, there is an explicit trade-off between controlling the plant and reducing the uncertainty. *Implicit dual control* on the other hand, rewards reduction in uncertainty only if it can be exploited at later stages. The formulation suggested in this paper is an implicit dual controller based on scenarios.

In general, most of the work on MPC for uncertain systems is on *robust MPC*. An overview of robust MPC can be found in Rawlings and Mayne (2009, Ch. 3). For robust MPC, the problem is formulated to minimize the worst case behaviour over all possible realizations of the uncertainty. Constraints must also be satisfied for all possible realizations. In order to make the problems tractable, the uncertainty is required to be restricted to some compact set. When model uncertainties and disturbances are unbounded, robust feasibility can never be guaranteed, and an alternative formulation is necessary. Examples of this is to require constraints to be satisfied in terms of expected value (Primbs and Sung, 2009), or by probabilistic constraints (Schwarm and Nikolaou, 1999).

There is little published work on dual MPC. In Genceli and Nikolaou (1996), additional constraints to the MPC formulation is introduced to assure persistent excitation in the input signal, allowing for efficient system identification. A similar approach is also found in Marafioti et al. (2014). In Bayard and Schumitzky (2010), an implicit dual controller using particle filtering and forward dynamic programming is proposed. They use a policy-iteration method, but is limited to a finite number of control values at each control step. An explicit dual MPC formulation is found in Heirung et al. (2013), where there is a trade-off between minimizing uncertainty or maximizing information content, and controlling the nominal trajectory.

The formulation in Subramanian et al. (2015) has a clear similarity to our approach, both employs a stochastic multi-stage formulation incorporating future measurements. Our approach, however, also update the uncertain parameter estimates. In this way, we obtain an implicit dual formulation. Furthermore, Subramanian et al. (2015) employ an UKF for model updating, while we propose to use an EnKF. The formulation of Subramanian et al. (2015) could, however, be extended to also update parameter estimates, making it an implicit dual formulation as well.

The remainder of the paper is organized as follows: In section 2, the basic idea of the proposed controller is presented, including the mathematical formulation and the details on the EnKF. In section 3, we perform a case study illustrating the effect of the formulation, and provide a discussion of the results in section 4. At the end, some concluding remarks are given.

2. SCENARIO BASED IMPLICIT DUAL MPC

Including future measurements in the optimization problem is not trivial. However, we want to include the fact that at later control steps, we know more than we currently do. This new knowledge is obtained through measurements, which allows us to update both state and parameter estimates. In this paper, we formulate the problem as a multi-stage stochastic programming problem, where future measurements are explicitly included in the optimization problem. The idea is that at the current time step, we have a set of N scenarios, also denoted as an ensemble, describing what we know about the model and state estimates. The states are propagated forward in time by the simulation model. At the next control step, we know that a measurement will become available, but we do not know what it will be. Our best estimate is, however, that it will be based on one of the N scenarios. If we use one of the scenarios as a "virtual" measurement, we can update all the scenarios. However, we do not know what the measurement will be, and all scenarios are just as likely as the others. We can first update the ensemble using the first scenario, to obtain N new model scenarios. This can be done for all the other scenarios, resulting in N^2 scenarios. Doing this recursively for n^r time steps, we will have N^{n^r+1} scenarios. As this grows exponentially, the number of steps has to be kept low. After the n^r steps, we assume no model updates are available, propagating the scenarios over the rest of the prediction horizon without further "branching". A simple schematics of the control structure is seen in Figure 1. Model propagation is illustrated by solid lines, while dotted lines are model-updating using virtual measurements. We start with 3 scenarios, and after 1 model update we have 9 scenarios. The control input has to be the same for the first step, and all scenarios coming from the same state/parameter update, at a total of 4 decision variables for the figure shown.

Although the problem size grows quickly, the hope is that including just a few steps will be enough to include the effect of feedback. For an MPC, the optimization problem will be solved at every control step in a receding horizon manner. In this work, we focus on the optimization problem at a single time step, although it can easily be extended in a receding horizon manner.

Using scenarios, also denoted as realizations, suggest that some sort of particle filter could be used for the state and parameter updates. In this work, we use the Ensemble Kalman Filter (EnKF) to update the estimates for the virtual measurements. This naturally leads to a set of N new scenarios for each new measurement, and no re-sampling is needed. Furthermore, by treating the parameters as state