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Separable Nonlinear Model Predictive Control via Sequential Quadratic Programming for Large-scale Systems ⋆

Liang Lu *,**

* *Department of Chemical and Biomolecular Engineering (BK21 Plus Program), Korea Advanced Institute of Science & Technology, Daejeon, Korea.*
** *Laboratory for Automatic Control and System Dynamics, Technische Universität Chemnitz, Chemnitz, Germany.*
(e-mail: liangup@gmail.com)

Abstract: In this paper, a sequential quadratic programming method is presented for large-scale nonlinear and possibly non-convex model predictive control (MPC) optimization problem which is often set up with a separable objective function. By introducing the so-call consensus constraints to separate the couplings among the subsystems. The resulting QP subproblem is formulated in a separable form, which makes it possible to use the existing alternating direction methods, like ADMM, to efficiently compute Newton steps for the overall system in a distributed way. In order to enforce the convergence rate of the distributed computation, a distributed line search with local merit functions is also proposed.

Keywords: Sequential quadratic programming, separable model predictive control, alternating direction method of multipliers, nonlinear systems, non-convex optimization, large-scale optimization, distributed line search.

1. INTRODUCTION

Model predictive control (MPC) is by far the most successful control technology for constrained systems, which require to solve an optimization problem at each sampling time, based on a given system model to plan the future control moves. Three essential components of MPC are moving or shifted horizon, online optimization and simulation model. During the last two decades, a lot of well-established theoretical results were developed on how to guarantee feasibility and stability for constrained linear or piecewise affine systems (Mayne et al. (2000)). On the other hand, considerable efforts have been put on developing fast real-time MPC optimization algorithms to solve the formulated MPC optimization problem efficiently for such systems. Specifically, Nesterov's fast gradient methods Richter et al. (2012), interior-point methods Wang and Boyd (2010); Rao et al. (1998), active-set methods Ferreau et al. (2013) and alternating direction method of multipliers (ADMM) Annergren et al. (2012) have been adopted for the banded sparse QP structure of MPC problems with quadratic costs. In particular, if the dimension of such a system is small, an offline explicit MPC control law can be obtained multi-parametric (mixed-integer) linear or quadratic programming (Bemporad et al. (2002)).

Nowadays, in the era of "Big Data", the challenges for MPC are how to solve the optimization problem online for large-scale systems and how to handle the nonlinearities in the systems' dynamics and constraints (Mayne (2014)).

For large scale systems, various decomposable convex optimization techniques have been tailored for distributed MPC to solve optimization problem with separable and parallelizable structure. The idea is to break the problem into a bunch of subproblems that can be managed, solve the subproblems locally and communicate cooperatively (see, for instance, Camponogara et al. (2002); Christofides et al. (2013); Stewart et al. (2010); Scattolini (2009) and the references therein). Existing distributed optimization algorithms can be classified into two categories: one class is dual decomposition with proximal gradient descent methods (Bertsekas (1999); Boyd et al. (2010)) and the other is to apply second order Newton type methods (Wei et al. (2013)). For the first category, dual decomposition with first order gradient based methods and alternating direction methods are the most commonly used optimization algorithms. In Giselsson et al. (2013), the distributed MPC controller is designed based on dual decomposition and its dual problem can be solved in parallel with accelerated gradient method, which guarantees a $\mathcal{O}(\frac{1}{k^2})$ convergence rate, and the stopping condition was proposed in Giselsson and Rantzer (2013) with some suboptimal performance. The result of distributed MPC via ADMM algorithm was presented in Summers and Lygeros (2012), Farokhi et al. (2013) and Lu (2014). In contrast to the first two papers which uses CVX solver to solve the ADMM update steps, Lu (2014) employs Riccati recursion for the main steps of ADMM iterations. In addition, Lu (2014) also shows linear convergence rate satisfaction for the linear quadratic

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MPC setup with polyhedra constraints. In Pu et al. (2014), the authors propose an inexact fast alternating minimization algorithm (FAMA) for distributed model predictive control. For the second order Newton type methods, some distributed Newton algorithms can be found in Wei et al. (2013) for network utility optimization problem and in Liu et al. (2013) for cross-layer network optimization. These results need a block diagonal structure of the Hessian matrix in order to solve the Newton systems for each subsystem in a distributed way.

For nonlinear programming (NLP), sequential quadratic programming (SQP) methods and interior point methods are two effective and promising approaches (Nocedal and Wright (2006)). SQP methods solve a sequence of convex quadratic approximation of the original nonlinear, possibly non-convex problem iteratively. Interior point methods approximate a path that approaches the solution. For distributed nonlinear optimization, these two types of methods have also been studied for many years. In Necoara et al. (2009), the authors combine sequential convex programming (SCP) and some smoothing techniques to solve the nonlinear optimal control problem for large-scale networked control systems. In Necoara and J. Suykens (2009), the authors develope an interior point Lagrangian decomposition method for separable convex optimization problem with application to distributed MPC. Especially, in Annergren et al. (2015), the authors propose a distributed primal-dual interior-point method for loosely coupled convex optimization problems which is similar to Necoara and J. Suykens (2009) and employ ADMM to solve the distributed Newton Equations. Motivated by these results, this paper presents a sequential quadratic programming method combined with ADMM for separable nonlinear model predictive control problem, and the problem setup maybe non-convex. By introducing the so-called consensus constraints, we can decouple the couplings among the subsystems. In order to enforce the convergence, this paper also proposes a distributed line search method with local merit functions.

The organization of the paper is as follows. In Section 2, we provide the notations and the separable nonlinear MPC setup that will be dealt with in this paper. In Section 3, sequential quadratic programming method is reviewed for the nonlinear optimization problem with equality and inequality constraints. Section 4 describes ADMM update procedures for separable convex optimization problem. Section 5 presents the combined SQP and ADMM algorithm with distributed line search strategy for nonlinear large-scale MPC problems. Section 6 concludes the paper.

2. PROBLEM FORMULATION

Consider a discrete-time nonlinear system with a non-overlapping partition of M subsystems $\mathcal{S}_i$, $i = 1, \ldots, M$. The subsystems are connected with a fixed undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, M\}$ denotes the subsystem nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set that specifies pairs of connected nodes, and $(i, j) \in \mathcal{E}$ means subsystems $\mathcal{S}_i$ and $\mathcal{S}_j$ are neighbors. Denote $\mathcal{N}_i = \{i\} \cup \{j \mid (i, j) \in \mathcal{E}\}$ the index of subsystem $\mathcal{S}_i$ and all its neighbors.

Let $x_k^i \in \mathbb{R}^{n_i}$ denote the state of subsystem $\mathcal{S}_i$, i.e.

$$x_k = [(x_k^1)^{\mathrm{T}}, (x_k^2)^{\mathrm{T}}, \cdots, (x_k^M)^{\mathrm{T}}]^{\mathrm{T}},$$

with $\sum_{i=1}^M n_i = n$. Each subsystem $\mathcal{S}_i$ has its own input vector $u_k^i \in \mathbb{R}^{m_i}$,

$$u_k = [(u_k^1)^{\mathrm{T}}, (u_k^2)^{\mathrm{T}}, \cdots, (u_k^M)^{\mathrm{T}}]^{\mathrm{T}},$$

with $\sum_{i=1}^M m_i = m$. The dynamics for subsystem $\mathcal{S}_i$ can be expressed as

$$x_{k+1}^i = f^i(x_k^j, u_k^j; j \in \mathcal{N}_i), \quad x_0^i = \bar{x}_i, \quad i = 1, \ldots, M. \tag{1}$$

The state and control vector of subsystems $\mathcal{S}_i$'s, $i = 1, \ldots, M$ satisfy local, possibly non-convex constraints

$$x_k^i \in \mathbb{X}^i, u_k^i \in \mathbb{U}^i, \quad i = 1, \ldots, M.$$

The global constraint sets for the state and input are defined as

$$\mathbb{X} = \mathbb{X}^1 \times \cdots \times \mathbb{X}^M, \quad \mathbb{U} = \mathbb{U}^1 \times \cdots \times \mathbb{U}^M.$$

For each subsystem $\mathcal{S}_i$, the local objective function is

$$\sum_{k=0}^{N-1} \ell^i(x_k^i, u_k^i) + \ell_f^i(x_N^i).$$

Consider the nonlinear distributed model predictive control (MPC) setup,

$$\min \sum_{i=1}^M \left(\sum_{k=0}^{N-1} \ell^i(x_k^i, u_k^i) + \ell_f^i(x_N^i) \right), \tag{2}$$

$$\begin{aligned}
\text{s.t.} \quad & x_{k+1}^i = f^i(x_k^j, u_k^j; j \in \mathcal{N}_i), \quad x_0^i = \bar{x}_i, && (2.1)\\
& x_k^i \in \mathbb{X}^i, \quad k = 0, 1, \ldots, N-1, && (2.2)\\
& u_k^i \in \mathbb{U}^i, \quad k = 0, 1, \ldots, N-1, && (2.3)\\
& x_N^i \in \mathbb{X}_f^i. && (2.4)
\end{aligned}$$

2.1 Separable Nonlinear MPC Formulation for Distributed Systems

For the distributed MPC problem (2), the vector variables of subsystem $\mathcal{S}_i$ are $\mathbf{x}^i = [(x_0^i)^{\mathrm{T}}, \ldots, (x_N^i)^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{u}^i = [(u_0^i)^{\mathrm{T}}, \ldots, (u_{N-1}^i)^{\mathrm{T}}]^{\mathrm{T}}$. We denote the stacked vectors of state and control vector variables of subsystem $\mathcal{S}_i$ and its neighbors by $\mathbf{x}_{\mathcal{N}_i}$ and $\mathbf{u}_{\mathcal{N}_i}$, which satisfy constraints $\mathbf{x}_{\mathcal{N}_i} \in \mathbb{X}^{\mathcal{N}_i}$ and $\mathbf{u}_{\mathcal{N}_i} \in \mathbb{U}^{\mathcal{N}_i}$. Set $\mathbf{v} = [(\mathbf{x}^1)^{\mathrm{T}}, \ldots, (\mathbf{x}^M), (\mathbf{u}^1)^{\mathrm{T}}, \ldots, (\mathbf{u}^M)]^{\mathrm{T}}$ as the collection of global variable, and $\mathbf{z}^i = [\mathbf{x}_{\mathcal{N}_i}, \mathbf{u}_{\mathcal{N}_i}]$ as the local variables which collect the variables that the dynamics of subsystem $\mathcal{S}_i$ in (1) is involved with. The corresponding constraints on $\mathbf{z}^i$ is denoted as $\mathbb{Z}^{\mathcal{N}_i} = \mathbb{X}^{\mathcal{N}_i} \times \mathbb{U}^{\mathcal{N}_i}$.

The nonlinear MPC setup (2) can be equivalently formulated as a separable optimization problem

$$\min \sum_{i=1}^M \phi^i(\mathbf{z}^i), \tag{3}$$

$$\begin{aligned}
\text{s.t.} \quad & h^i(\mathbf{z}^i) = 0, \quad i = 1, \ldots, M, && (3.1)\\
& c^i(\mathbf{z}^i) \le 0, \quad i = 1, \ldots, M, && (3.2)\\
& \mathbf{z}^i = E_i \mathbf{v}, \quad i = 1, \ldots, M, && (3.3)
\end{aligned}$$

where ϕ^i is the local objective function for the local variable $\mathbf{z}^i$. The constraints (3.1) and (3.2) include the dynamical constraint (2.1) and the constraint $\mathbb{Z}^{\mathcal{N}_i}$, the constraint (3.3) is the so-called consensus constraints, and E_i's decouple the local variable sets from the global variable $\mathbf{v}$.