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Robust receding horizon control for linear systems with state and input dependent disturbances

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Abstract: A robust model predictive controller is proposed for constrained linear systems with unknown disturbances that are subject to state and input dependent piecewise affine bounds. The formulation encompasses problems involving model uncertainty arising from unknown system parameters and linearisation errors. We propose a computationally efficient method for the online optimization of a receding horizon min-max cost over the class of feedback strategies subject robust constraint satisfaction and provide an illustrative numerical example.

Keywords: Robust optimal control, min-max control, multiparametric programming

1. INTRODUCTION

Robust model predictive control (RMPC) algorithms compute a control law that optimises a predicted performance criterion subject to state and control constraints for all possible realisations of model uncertainty. Witsenhausen (1968) proposes to formulate the RMPC problem as a game in which the adversary tries to maximise the effect of model uncertainty on the cost and the controller chooses its strategy to minimise the cost for the worst case system parameters and disturbances. This formulation is widely accepted in the RMPC literature (e.g. Bemporad and Morari, 1999; Bemporad et al., 2003; Lee and Yu, 1997). The predicted control law may be an open-loop policy, in which case the controller chooses its strategy for the whole prediction horizon before the adversary decides its strategy, or a closed-loop policy in which the horizon is split into stages and the actions of controller and adversary at each stage depend on the outcome of the previous stage). Theoretical properties of these controllers are well understood (e.g. Mayne et al., 2000). However, the numerical computation of robust model predictive controllers still poses a challenge and only few tractable solutions have been proposed.

Various strategies have been proposed to solve RMPC problems for linear systems subject to linear state and input constraints, with model uncertainty in the form of bounded additive disturbances and/or multiplicative parameters. For the case of unknown additive disturbances, Scokaert and Mayne (1998) propose a scenario-based approach to solve a sequence of min-max problems online, however the computational complexity of this approach grows exponentially with the length of the horizon. A parametrised algorithm was proposed by Rakovic et al. (2012) which reduces the RMPC problem to a single linear program to be solved online, but although this is a closed loop strategy the parametrisation is in general suboptimal. Explicit RMPC algorithms optimise the worst case predicted performance for each admissible model state offline and use results from multi-parametric programming to parametrise the optimal control law as a piecewise affine function of the state (e.g. Bemporad et al., 2003; Diehl and Björnberg, 2004). However, this requires extensive offline computation and becomes inefficient for a high-dimensional state space. An alternative approach that similarly exploits the piecewise affine structure of the solution, but with less redundancy, was proposed by Buerger et al. (2011) and extended in Buerger et al. (2013). In this work an active set solver is designed to solve a sequence of min-max problems explicitly online.

In almost all existing methods for RMPC, the model disturbances are assumed to be confined to sets that are constant, i.e. independent of the system state and control inputs. Disturbances that depend on model states (or control inputs) are considered in Kuntsevich and Pshenichnyi (1996); Rakovic et al. (2006). However general formulations of state and input dependence can result in non-convex constraints (Rakovic et al., 2006). To avoid this we make use of recent algorithmic advances (Schaich and Cannon, 2015) that enable a class of polytopic disturbance sets that depend affinely on the system state and control input to be handled using convex constraints. This provides an algorithmic and analytical framework which is used in the current paper to embed affine state and input dependent disturbance constraints into the framework of the active set RMPC solver framework proposed by Buerger et al. (2011). Disturbances that belong to sets which depend on model states and inputs can account for multiplicative model uncertainty as well as linearisation errors, and the approach of this paper is therefore able to handle linearisation errors in RMPC non-conservatively.

This paper is structured as follows: After describing the control problem in Section 2 we summarise preliminary

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results which are necessary to formulate the considered problem explicitly in Section 3. Section 4 discusses the recursive solution for equality constrained min-max recursions, one of the essential parts of the active set solver, and this is applied to the min-max RMPC problem in Section 5 for the case that the active set of constraints is known. A line search to determine the active set of constraints is described in Section 6 and bounds on the computational complexity of the approach are discussed in Section 7. An application of the proposed algorithm to an example problem is presented in Section 8 and conclusions are drawn in Section 9.

2. PROBLEM STATEMENT

We consider a computationally efficient method for solving online min-max receding horizon control problems that exploits the structure of multi-parametric programming solutions. Our approach determines the optimal feedback law without approximating the cost or the constraints of the problem, and without making restrictive assumptions on the parametrisation of control inputs. To make it possible to obtain a closed form solution of the min-max optimal control problem we assume a linear-quadratic problem formulation. Thus we assume linear dynamics

$$x^+ = Ax + Bu + Dw, \tag{1}$$

where A, B, D are real matrices of appropriate dimension, $x, x^+ \in \mathcal{X} \subseteq \mathbb{R}^n$ denote the state and successor state respectively, $u \in \mathcal{U} = \{u : F_i u \leq 1 \,\forall i \in \mathcal{I}^{\mathcal{U}}\} \subset \mathbb{R}^p$ is the control input and $w \in \mathcal{W}(x,u) \subset \mathbb{R}^q$ is an unknown disturbance input, $\mathcal{I}^{\mathcal{U}}$ denotes the index set of the constraint set $\mathcal{U}$. The disturbance set $\mathcal{W}(x,u)$ is assumed to have a known dependence on the values of x and u, according to

$$\mathcal{W}(x,u) = \{w : Gw \leq H(x,u)\}, \tag{2}$$

where each element of $H(x,u)$ is a convex, piecewise affine function for all $(x,u) \in \mathcal{X} \times \mathcal{U}$. The control objective is to minimize the worst case value, over all future model uncertainty, of the cost

$$\sum_{k=0}^{\infty} \left(\|x_k\|_Q^2 + \|u_k\|_R^2 - \gamma^2 \|w_k\|^2 \right), \tag{3}$$

where the subscript k denotes the realisation of a variable at time k, $\|x\|_Q^2$ denotes the quadratic form $x^T Q x$, Q and R are given positive definite matrices and γ is a given scalar weight that controls the l^2-gain of the closed loop system from the disturbance sequence to the state and control sequences.

Given a prediction horizon of N steps over which the predicted control law is to be optimized, we define the min-max optimal control problem with m stages to go as

$$J_m^*(x) = \min_u \max_{w,x^+} \tfrac{1}{2}\left(\|x\|_Q^2 + \|u\|_R^2 - \gamma^2 \|w\|^2 \right) + J_{m-1}^*(x^+) \tag{4a}$$

subject to

$$x^+ = Ax + Bu + Dw \tag{4b}$$
$$u \in \mathcal{U} \tag{4c}$$
$$w \in \mathcal{W}(x,u) \tag{4d}$$
$$(x,u) \in \mathcal{Z}_m \tag{4e}$$

for $m = 1, \ldots, N$, with $J_0^*(x) = \|x\|_{P_0}^2$. The stagewise state and input constraints $\mathcal{Z}_m$ are defined recursively by

$$\begin{aligned} \mathcal{Z}_m = \{(x,u) : u \in \mathcal{U} & \\ \wedge\ Ax + Bu + Dw \in \mathcal{X}_{m-1} \,\forall\, w \in \mathcal{W}(x,u)\} & \quad (5a) \\ \mathcal{X}_m = \{x : \exists\, u \in \mathcal{U}, (x,u) \in \mathcal{Z}_m\} & \quad (5b) \\ & \quad (5c) \end{aligned}$$

with

$$\mathcal{X}_0 = \mathcal{X}^\infty. \tag{5d}$$

Thus the constraints $(x,u) \in \mathcal{Z}_m$ ensure that the terminal state, namely x^+ at $m = 1$, is contained in $\mathcal{X}^\infty$. We define $\mathcal{X}^\infty$ as the maximal robust positive invariant (MRPI) set (Blanchini and Miani, 2007)), for the closed-loop system governed by a given state feedback controller $u = Kx$:

$$\mathcal{X}^\infty = \{x : (A+BK)x + Dw \in \mathcal{X}^\infty \;\forall w \in \mathcal{W}(x, Kx)\}, \tag{6}$$

and we define P_0 so that $J_0^*(x)$ is the worst case value of the cost (3) under $u = Kx$ in the absence of constraints. Recently Schaich and Cannon (2015) proposed an algorithm to compute MRPI sets for linear systems subject to state and input dependent disturbance constraints. In this paper we combine this approach with the multistage multi-parametric programming method of Buerger et al. (2013) to derive an efficient algorithmic solution to the online MPC optimization of (4a-e).

3. CONTROLLABLE SETS

In this section we summarise results on controllable sets that allow the constraints of the problem (4) to be formulated in a convenient way. The following result shows that the MRPI set (6) associated with the pointwise polytopic set (2) is polytopic if H is elementwise convex and piecewise affine. For a proof and detailed discussion of this result we refer the reader to Schaich and Cannon (2015).

Theorem 1. (Schaich and Cannon (2015)). Let $\mathcal{X} \subseteq \{x : \Gamma x \leq \mathbf{1} \wedge -\Gamma x \leq \mathbf{1}\}$ be a polyhedral set, let the pair $(A+BK, \Gamma)$ be observable, and let $\mathcal{W}(x, Kx)$ be defined as in (2) with an elementwise convex piecewise affine function $H(x, Kx)$ which is bounded for all finite x. Then the MRPI set $\mathcal{X}^\infty$ is polytopic, i.e. it has the representation

$$\mathcal{X}^\infty = \{x : \Lambda_i x \leq \lambda_i, \forall\, i \in \mathcal{I}^\infty\}. \tag{7}$$

Remark 2. The conditions of Theorem 1 require that $\mathcal{W}(x,u)$ is polytopic for all finite (x,u), i.e. it must have a convex hull representation $\mathcal{W}(x,u) = \text{conv}_k\{w_k(x,u)\}, k \in \{1, \ldots, r\}$ where each vertex, $w_k(x,u)$, is a piecewise affine function of (x,u), i.e.

$$\begin{aligned} w_k(x,u) = \max\{ & W_{k,1}^x x + W_{k,1}^u u + w_{k,1}, \\ & W_{k,2}^x x + W_{k,2}^u u + w_{k,2}, \ldots\} \end{aligned} \tag{8}$$

where the maximisation is performed elementwise.

We now derive the representation of $\mathcal{Z}_m$ and $\mathcal{X}_m$ as defined in (5). According to Theorem 1 the MRPI set $\mathcal{X}^\infty$ is polytopic and has the representation (7). Therefore the recursion (5a) defines $\mathcal{Z}_1$ as