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Power law fluid viscometry through capillary filling in a closed microchannel

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ABSTRACT

In this work we analyze the capillary filling dynamics of non-Newtonian fluids that can be modeled with a power law constitutive equation. We solve the Poiseuille equations for an hydrophilic closed channel where capillary pressures drives the fluid in until a rest position given by the barometric pressure is reached. We show that this dynamics can be used to measure both the coefficient k and exponent n, that describes the power law fluid viscosity and we ran tests on Soda Lime Glass microchannels. Using a simple experimental setup with a USB Microscope and a custom image processing software we were able to measure the power law parameters of whole blood, wall varnish and DI water.

The exponents were also obtained from the velocity profiles inside the microchannel using a custom μPIV setup matching both results with those measured with a standard Brookfield Rotational Microviscometer.

1. Introduction

One of the ongoing challenges in fluid rheometry is to provide fast and accurate viscosity measurements using very low sample volumes. Since the 1980s this problem was approached by miniaturizing internal flow sensors like capillary viscometers, or external flow sensors like Stokes and Rotational Viscometers. The latter has become one of today's industry standards, as rotational devices with cone-plate geometry can be used to directly measure the shear stress τ by setting different shear rates γ, thus calculating the viscosity μ as the ratio between these two variables ($\mu = \tau/\gamma$).

The miniaturization of fluid containers has dropped the minimum volume sample required down to ~500 μl [1] and in order to scale further down, novel microfluidic approaches have raised over in the recent years [2]. Standard photolithography microfabrication techniques widely used in microelectronics are now being used to manufacture external flow sensors, oscillators with a viscosity dependant resonance frequency [3], or capillary driven flow inside microchannels [4–6]. While these devices may use 10 μl of sample volume, they rely on a fluid dynamics model with the viscosity embedded in one or more parameters. Not any fluid can be measured in every viscometer and special care must be taken when analyzing non-Newtonian fluids.

In their 2005 work, Strivastava et al. [7], proposed a capillary flow viscometer using a microfluidic chip and a video recorder setup. The fluid dynamics of a fluid drop driven by capillarity is described by Poiseuille equation where viscosity, capillary pressure and channel geometry are binded constants. Srivastava's chip uses two independent channels to measure the dynamics and the capillary pressure, but the main drawback is the need of a third and a fourth channel where a test fluid has to be inserted to calibrate the geometrical dimensions. In 2006 [8] they showed how the same chip could be used to measure non-Newtonian fluids whose viscosity followed a power law model, a.k.a. a power law fluid. They used a known solution for a power law Poiseuille equation and showed that they could measure them with the same experimental setup, although the need of a test calibration fluid persisted.

Following the concept of capillary viscometer on a microfluidic chip, we have previously shown [9] that the analysis of the capillary filling in a closed channel provides enough information to compute both capillary pressure and viscosity in a single channel device. Moreover, we show that controlling the accuracy of the chip fabrication process [10] avoids the need of a test fluid, an attractive feature for applications in point of care devices. In this work, we analyze the capillary filling flow of a power law fluid and solve the modified Poiseuille equations for the velocity profile and the mean velocity across the channel. Then, we verify the model by testing fluids with a known non-Newtonian behavior. First, we show the power law behavior by μPIV measurements, contrasting with a

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commercial Rotational Microviscometer, and finally we obtain the power law viscosity parameters with our single channel design.

2. Theory: capillary filling of a power law fluid in a closed microchannel

The Ostwald de Waele power law constitutive equation is a simple model to describe the changes of viscosity with the flow shear rate. In this model the viscosity is given by [11]

$$\mu = k\gamma^{n-1} \tag{1}$$

where the constant k is the viscosity at $\gamma = 1\,s^{-1}$, and n the exponent of the power law. In this model three kinds of fluids are distinguished: a pseudoplastic or shear thinning fluid for $n < 1$, a dilatant fluid for $n > 1$ and a Newtonian fluid with constant viscosity for $n = 1$.

The capillary filling inside a microchannel, described by a power law Poiseuille equation, can be studied to explicitly determine both viscosity parameters k and n. Here we present the solution of a fully developed Poiseuille flow inside a closed microchannel.

2.1. Pipe flow of a power law fluid

When a power law fluid inside a circular pipe of radius R is set to a pressure difference dP/dx, the velocity profile $u(r)$ is given by [12]

$$u(r) = \frac{1}{1+(1/n)}\left(\frac{1}{2k}\frac{dP}{dx}\right)^{1/n}\left(R^{1+(1/n)} - r^{1+(1/n)}\right) \tag{2}$$

where it can be seen that for $n = 1$ the Newtonian parabolic profile is obtained. Fig. 1a shows the profile for pseudoplastic and dilatant fluids, respectively a flattened or stretched parabola. Thus, a fit over a velocity profile can be used to measure the power law index n of a fluid in a pipe flow.

Is is usually easier to determine the mean velocity with an experimental setup that follows the free surface meniscus across the channel. The cross section of microchannels in microfluidic devices is seldom circular and an equivalent hydraulic diameter D is calculated. Rewriting R as a function of the hydraulic diameter D, the mean velocity u is given by [13]

$$u = \frac{D}{2((1/n)+3)}\left(\frac{D}{4k}\frac{dP}{dx}\right)^{1/n} \tag{3}$$

Eq. (3) can also be used to calculate both viscosity parameters k and n, although the parameters remain binded to the pressure gradient dP/dx which has to be independently determined.

2.2. Capillary flow in a closed channel

If a power law fluid sample is placed at the entrance of an hydrophilic microchannel the fluid will enter driven by a capillary pressure P_c. In a channel with an inlet and an outlet, *an open channel*, the fluid slows down continuously with the pressure gradient $\frac{\Delta P}{x}$, where ΔP is just P_c and x the fluid column length highlighted by the meniscus position. In a *closed channel* (Fig. 1-b) with an inlet and no outlet, the fluid compresses the trapped air and slows down until a rest position L_f, where the compressed air pressure P_x compensates both atmospheric pressure P_0 and capillary pressure. In this case $\Delta P = P_0 + P_c - P_x$. Replacing ΔP in Eq. (3) describes the transient flow, but instead of explicitly writing P_x and P_c it is convenient to rewrite these as a function of geometrical parameters.

Considering an isothermal compression of the air inside the microchannel, P_x can be approximated by

$$P_x(x) = P_0\frac{L}{L-x} \tag{4}$$

where L is the total length of the channel. Notice that Eq. (4) shows that P_x can be determined without any dedicated pressure sensor by measuring the atmospheric pressure and the position of the fluid column. Furthermore, at the rest position $P_x(L_f) = P_0 + P_c$, and then the capillary pressure P_c can also be calculated as

$$P_c = P_0\frac{L_f}{L-L_f} \tag{5}$$

Using Eqs. (4 and 5) and replacing $\frac{\Delta P}{x}$ in Eq. (3) the mean velocity $u(x)$ is

$$u = \frac{D}{2((1/n)+3)\left(\frac{D}{4k}\frac{P_0}{(1-(L_f/L))}\frac{L_f-x}{x(L_t-x)}\right)}^{1/n} \tag{6}$$

and this equation can be used to calculate n and k, by independently measuring P_0 and the geometrical parameters D, L and L_f.

3. Experimental details

To analyze the transient capillary flow of a power law fluid, we have used a microfluidic device featuring a custom design with a long serpentine packaged in a 10 mm × 10 mm × 1 mm glass chip. The microchannels with a 40 μm width and a 15 μm depth were built using photolithography and wet etching techniques on soda lime glass wafers. After bonding to a seconds glass wafer, individual chips were diced opening a side inlet where the testing fluids drops are then placed. The channels hydraulic diameter D_h was calculated after [15] for an isotropically etched cross sections, obtaining $D_h = 20.7$ μm.

Whole blood and a diluted water based wall varnish *(1:10 – Varnish:DI water)* were selected as test power law fluids, and DI water as a reference Newtonian fluid. The fluids rheology were determined using a commercial Brookfield Rotational Microviscometer with a sample volume of 250 μl for shear rates between 10 s^{-1} and 250 s^{-1}.

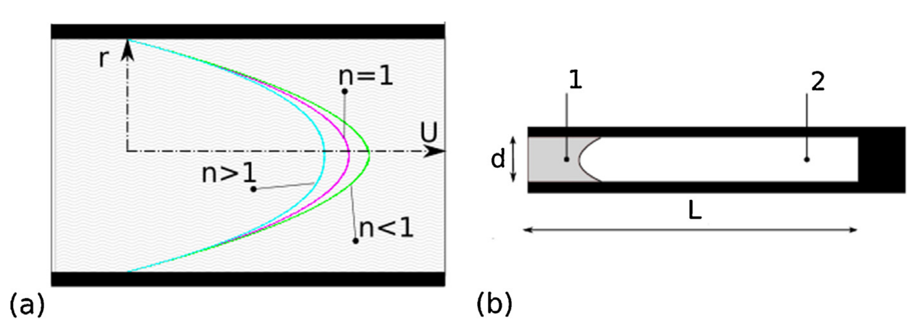

Fig. 1. Schematic of the power law fluid flow inside a microchannel. (a) A power law constitutive equation is defined featuring the viscosity constant k and the exponent n. (a) The parabolic velocity profile is stretched or flattened depending on the exponent n. (b) Schematic of a fluid entering closed channel by capillary pressure. Region 1: fluid, region 2: trapped gas.