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Convex Synthesis of Multivariable Static Discrete-time Anti-windup via the Jury-Lee Criterion

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Abstract
Due to its ease of application, the circle criterion has been widely used to guarantee the stability of many anti-windup schemes. While the Popov criterion gives less conservative results, it has been conjectured in the literature that it cannot be used for convex anti-windup synthesis. This paper shows that the conjecture does not necessarily apply in the discrete-time setting. We show how the search for optimal parameters corresponding to the Jury-Lee criterion (a discrete counterpart of the Popov criterion) can be formulated as a convex search via a linear matrix inequality (LMI). The result is then extended to two existing multivariable static anti-windup schemes with stable open-loop plants. Two numerical examples of multivariable anti-windup controller synthesis are provided, and it is shown that in both cases the synthesis using the Jury-Lee criterion can allow better performance than existing methods which use the circle criterion alone.

1. INTRODUCTION

Most practical control systems which are designed based on linear theory have to deal with physical constraints such as saturations on the actuators. When the outputs of the controllers reach their limitations, so-called windup effects can take place. These might, in turn, cause performance degradation, large overshoots in the output and sometimes instability (Campo and Morari [1990], Kothare et al. [1994]). These phenomena have been observed since 1950's in both analog (Lozier [1956]) and digital (Fertik and Ross [1967]) control loops.

Techniques for addressing the windup effects have been widely studied in the continuous-time domain and numerous anti-windup schemes have been developed to improve the stability and performance of the controllers. Most of the traditional design techniques developed are either based on static (zero order) (Hanus et al. [1987], Wada and Saeki [1999], Saeki and Wada [2002], Mulder et al. [2001], Marcopoli and Phillips [1996]), or dynamic (low-order and full-order) (Turner and Postlethwaite [2004] Grimm et al. [2003] Zheng et al. [1994]) anti-windup compensators. Furthermore, as the extension to discrete-time setting appears straightforward, almost all existing continuous-time anti-windup schemes have their own digital versions (for example Syaichu-Rohman and Middleton [2004], Massimetti et al. [2009], Hermann et al. [2006]). It has been argued (e.g. Saeki and Wada [2002],Turner and Postlethwaite [2004]) that static anti-windup may be the most desirable structure from a practical point of view. Moreover, most practical controllers nowadays are implemented digitally using computers, which increases the importance in designing the anti-windup in discrete-time.

In early studies, a great deal of stability analysis was done for closed-loop systems having sector-bounded nonlinearities, both in continuous- and discrete-time domains. This has led to the derivation of various stability tests such as the circle, off-axis circle, and Popov criteria and the use of Zames-Falb multipliers. The extension of the theorems to stability analysis of existing anti-windup schemes has also been widely considered (see, for examples, Pittet et al. [1997], Feron et al. [1996], Kothare and Morari [1999] in continuous-time, and Cao and Lin [2003] in discrete-time). In Kothare and Morari [1999], the stability analysis of multivariable anti-windup designs is presented in a unified multiplier framework with the circle, off-axis circle, and Popov criteria and the use of Zames-Falb multipliers as special cases. It is also shown in their paper that the Zames-Falb multiplier gives the best stability margin even though the search for an optimal solution via the approach may be computationally intractable (i.e. it is non-convex).

As for the anti-windup design problems, most of the continuous-time anti-windup schemes base their synthesis on the circle criterion and/or use a quadratic Lyapunov function (Cao and Lin [2003], Weston and Postlethwaite [1998], Marcopoli and Phillips [1996], Mulder et al. [2001]). Similar techniques are also incorporated into digital anti-windup schemes (e.g. Massimetti et al. [2009], Grimm et al. [2008], Pan and Kapila [2002], Hermann et al. [2006]

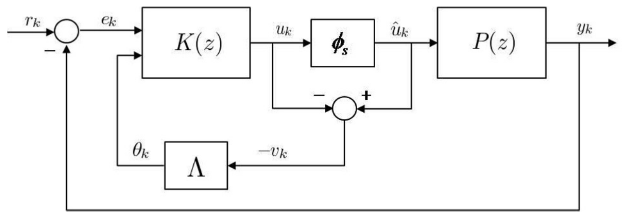

Figure 1. General static anti-windup scheme.

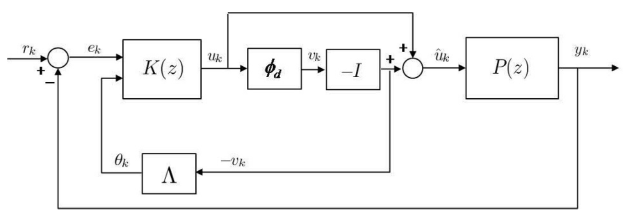

Figure 2. Equivalent representation of Figure 1.

Gomes da Silva and Tarbouriech [2006]) to achieve stability. This is mainly due to their ease of applications and the convexity obtained during the synthesis. Hence, the focus of the anti-windup design in both time domains has been mostly devoted to satisfying certain $l_2(\mathcal{L}_2)$ or H_∞ performance requirements (Wada and Saeki [1999], Marcopoli and Phillips [1996], Mulder et al. [2001], Hermann et al. [2006]). On the other hand, it has been observed that other less conservative criteria such as the Popov criterion do not lead to convex formulation when used for synthesis (see Gomes da Silva and Tarbouriech [2006], Kapila et al. [2001], Weston and Postlethwaite [1998] and Feron et al. [1996] for discussion on this). This has also been stressed in Grimm et al. [2003] if an attempt is made to use the quadratic-plus-integral Lyapunov function (which is often associated with Popov criterion) for synthesis purposes. In Kapila et al. [2001], however, the Popov criterion is used in the anti-windup schemes but only sub-optimal solutions can be found. In summary, to the best of the authors knowledge, there is not much attention being directed towards applying criteria other than the circle for anti-windup synthesis due to the difficulties in achieving the optimal solutions.

In this paper, the focus is on the synthesis of static discrete-time anti-windup schemes with stable open-loop plants. The novelty of this paper is that the static anti-windup controller synthesis problem using the Jury-Lee criterion (Jury and Lee [1964a], Jury and Lee [1964b]), which is a discrete-time counterpart of the Popov criterion, is formulated into a convex search over an LMI where an optimal solution can be found. This directly shows that the conjecture of the Popov criterion leading to nonconvex solution does not necessarily apply in the discrete-time setting. The new stability criterion is then extended to existing anti-windup schemes in the literature which follow the conventional two-step paradigm: the linear controller is designed first ignoring the saturation and the anti-windup compensation is added to attenuate the performance degradation resulting from the saturation (Wada and Saeki [1999], Marcopoli and Phillips [1996]).

This paper is structured as follows: Section 2 presents the problem formulation of a standard static anti-windup scheme where the static gain is fed back into the controller's input. Section 3 formulates the anti-windup stability conditions via the Jury-Lee criterion into an LMI. The result of Section 3 is extended to two existing static anti-windup schemes (Wada and Saeki [1999], Marcopoli and Phillips [1996]) in Section 4. In Section 5, we provide some numerical examples of multivariable anti-windup controller synthesis to compare the performance of the Jury-Lee criterion and the circle criterion under given performance requirements. The conclusion is given in the last section.

The notation used in this paper is standard throughout. We denote x_k for $x(k)$ and $G^*(z)$ as the complex conjugate of $G(z)$. If $M \in \mathbb{C}^{p\times p}$, we write $\mathrm{He}(M) = M + M^*$. We omit the upper triangle half of the Hermitian matrix M as it is always the complex conjugate of the lower triangle half.

2. PROBLEM FORMULATION

Figure 1 shows a standard static anti-windup scheme (Wada and Saeki [1999], Hermann et al. [2006]) with a stable, strictly proper plant $P(z)$

$$\begin{aligned} x^p_{k+1} &= A_p x^p_k + B_p \hat{u}_k \\ y^p_k &= C_p x^p_k \end{aligned} \tag{1}$$

and a controller $K(z)$

$$x^c_{k+1} = A_c x^c_k + B_c e_k + B_c \theta_k \tag{2}$$

$$u_k = C_c x^c_k + D_c e_k. \tag{3}$$

where $x^p_k \in \mathbb{R}^{np}$, $x^c_k \in \mathbb{R}^{nc}$,$u_k \in \mathbb{R}^{nu}$ and $y_k \in \mathbb{R}^{ny}$. The saturation nonlinearity is described as $\hat{u}_k = \phi_s(u_k)$ where

$$(\phi_s(u_k))_i = \begin{cases} -1 & \text{for } u^i_k < -1 \\ u^i_k & \text{for } -1 \le u^i_k \le 1 \\ 1 & \text{for } u^i_k > 1. \end{cases} \tag{4}$$

When there is no saturation, the system will act linearly since the controller output u_k will be the same as the plant input $\hat{u}_k$. However, when the controller output reaches the saturation levels, the difference between u_k and $\hat{u}_k$ will be fed back into the input of the controller via a static gain $\Lambda \in \mathbb{R}^{nc\times nu}$ as $\theta_k = -\Lambda v_k = \Lambda(\hat{u}_k - u_k)$. It is standard to represent the loop around the saturation ϕ_s as the deadzone nonlinearity ϕ_d as shown in Figure 2 (Wada and Saeki [1999] Mulder et al. [2001] Marcopoli and Phillips [1996]). The deadzone function ϕ_d can be expressed as $\phi_d(u_k) = u_k - \phi_s(u_k)$. Hence $\phi_d(u_k) = \left[(\phi_d(y_k))_1, \cdots, (\phi_d(y_k))_p\right]^T$.

Since we assume the plant is given and the controller has already been designed first to achieve acceptable performance in the unsaturated region, the only design parameter is the static gain Λ. Therefore, the problem formulation is to optimize the static gain which can minimize the effect of the nonlinearity (in some sense) while preserving the stability. In the next section, we will show how the Jury-Lee criterion can be formulated into a convex search in the anti-windup synthesis problem.

3. ANTI-WINDUP STABILITY CONDITIONS

The closed-loop system as shown in Figure 3 consists of a stable, strictly proper LTI plant $\tilde{G}(z)$ in negative feedback with a static nonlinearity ϕ. To guarantee the stability of the system, we begin with the set of nonlinearities described as follows: