Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-21 (2015) 060–065

Optimal Topology for Distributed Fault Detection of Large-scale Systems ⋆

Francesca Boem * **Riccardo M. G. Ferrari** ** **Thomas Parisini** *** **Marios M. Polycarpou** ****

* *Imperial College London, UK,* (f.boem@imperial.ac.uk).
** *Danieli Automation S.p.A., Italy* (r.ferrari@dca.it)
*** *Imperial College London, UK, and University of Trieste, Italy* (t.parisini@gmail.com)
**** *University of Cyprus, Cyprus.* (mpolycar@ucy.ac.cy)

Abstract: The paper deals with the problem of defining the optimal topology for a distributed fault detection architecture for non-linear large-scale systems. A stochastic model-based framework for diagnosis is formulated. The system structural graph is decomposed into subsystems and each subsystem is monitored by one local diagnoser. It is shown that overlapping of subsystems allows to improve the detectability properties of the monitoring architecture. Based on this theoretical result, an optimal decomposition design method is proposed, able to define the minimum number of detection units needed to guarantee the detectability of certain faults while minimizing the communication costs subject to some computation cost constraints. An algorithmic procedure is presented to solve the proposed optimal decomposition problem. Preliminary simulation results show the potential of the proposed approach.

Keywords: Fault detection, Distributed systems, Optimal decomposition, Topology

1. INTRODUCTION

Recently there has been a growing interest towards distributed architectures for fault diagnosis of large-scale and networked systems (see, for instance Boem et al. (2011); Ferrari et al. (2012); Zhang and Zhang (2012); Boem et al. (2013a,b); Keliris et al. (2015); Reppa et al. (2015)). As it is well known, the drawbacks of a centralized fault diagnosis architecture are scalability and robustness. A common solution is to decompose the overall system into subsystems that are monitored by some local agents, which we call Local Fault Diagnosers (LFDs). According to (Šiljak (1978)), the term *system decomposition* refers to the clustering of the states, inputs, and outputs system variables into subsets, which make up the subsystems. Since each LFD is devoted to monitor a subsystem, the decomposition of the overall system defines the topology of the diagnosis architecture. Given a large-scale interconnected system and its structural graph (Šiljak (1978)) whose nodes are the state and the input variables of the system, the goal is to identify: i) the number of local subsystems (and therefore the number of LFDs) needed to monitor the system, and ii) how to assign the system variables of the monitored system to each subsystem. The latter considers also which variables need to be shared among more than one subsystem. In this respect, overlapping decompositions are considered, that is, some state variables may be monitored by more than one LFD. An additional objective is to make the Fault Detection (FD) problem computationally tractable and to guarantee some performances related to given monitoring goals.

The problem of system decomposition is well-known problem in decentralized and distributed control and there are some recent papers presenting algorithms for non-overlapping (Ocampo-Martinez et al. (2011)) and possibly overlapping decompositions (Anderson and Papachristodoulou (2012))). On the other hand, the works proposing distributed monitoring schemes for discrete-time or continuous-time systems, like Boem et al. (2011); Ferrari et al. (2012); Zhang and Zhang (2012), assume that the decomposition of the system into subsystems is given a priori. The aim of this work is to study the decomposition problem specifically for the fault detection task. The goal is to understand how the decomposition and the adoption of distributed approaches can influence the detectability performances. In Bregon et al. (2014), a decentralized fault diagnosis task using structural model decomposition is considered, but an event-based method is implemented in a qualitative approach. In Staroswiecki and Amani (2014), the topology of the information pattern is studied in order to allow fault-tolerant control reconfiguration. In Grbovic et al. (2012), the decomposition is designed using the Sparse Principal Component Analysis algorithm, but the proposed decentralized fault detection architecture is a data-driven approach (see Yin et al. (2014) for a recent survey), while our method is a model-based one (see Venkatasubramanian et al. (2003)).

The main contributions of the paper are: i) a methodology to find the minimum number of LFDs needed to detect a certain set of faults is addressed. Once obtained the minimum required number of LFDs and the variables to be shared, an optimal topology is determined that minimizes the communication costs and satisfies some computational constraints. We show that the decomposition allows to improve detectability; ii) a novel stochastic formulation for the problem of distributed fault detection is proposed, while previous works by the authors presented deterministic approaches [1]; iii) the effectiveness of a consensus approach for diagnosis purposes is demonstrated: the proposed consensus protocol is used as a tool for shared

⋆ This paper has been partially supported by the EPSRC STABLE-NET grant EP/L014343/1.

[1] The proposed approach is based on the model of the system; statistics is introduced to manage the stochastic characterization of the uncertainty and to derive some stochastic thresholds.

10.1016/j.ifacol.2015.09.505

variables estimation and it allows to reduce the uncertainty and to improve detectability.

It is worth noting that, to the best of the authors' knowledge, it is the first time that the system decomposition problem is analyzed specifically for the distributed fault diagnosis purposes. Furthermore, it is the first time that the problem of graph decomposition design is considered in the overlapping case where the nodes to be shared are selected before the decomposition process. In this connection, we remark that in computer science, the problem of graph decomposition has been widely investigated. For instance, a multilevel graph partitioning method is proposed in Karypis and Kumar (1998) and Schloegel et al. (2000), where a non–overlapping decomposition is obtained.

Notation. Given a stochastic variable x, we represent as $\mathbb{E}[x]$ its expected value, and as $\mathrm{Var}[x]$ its variance. Given a vector a, we denote with $a^{(k)}$ its k-th component. Finally, let us denote by present $A^{(k)}$ the k-th row of matrix A.

2. PROBLEM FORMULATION

We introduce in qualitative terms the multi-objective optimization problem designed to obtain an optimal system decomposition for fault detection purposes. The proposed method consists of two phases. Let n^+ be a design parameter, representing the maximum number of state variables that each subsystem may contain due to computational cost limits. Then, we first define the optimization problem $\mathcal{P}_1(n^+)$. Let N denote the number of subsystems, n the number of state variables, and n_I be the number of state variables contained in the I-th subsystem [2], $I = 1, \dots, N$. Let $\mathbb{S}$ be the set of the variables shared among more than one subsystem and d_k be the overlap degree (Ferrari et al. (2012)) of the k-th state variable, $k = 1, \dots, n$, that is, the number of subsystems the variable belongs to. The objective is to find the minimum number of subsystems N^* and the overlap degrees d_k^*, needed to guarantee some detectability conditions $\mathcal{D}(d_k)$ that will be defined in the following, subject to the computational cost constraint. Given n^+, we formulate

$$\begin{aligned} \mathcal{P}_1(n^+) \ : \ N^* = \min_{N, d_k} \ & N \\ \text{s.t.} \ & \\ & \mathcal{D}(d_k) \text{ satisfied} \\ & n_I \leq n^+, \quad \forall I = 1, \dots, N. \end{aligned} \tag{1}$$

The optimal d_k^* characterizes the optimal set $\mathbb{S}^*$ of the variables to be shared. Once the optimal values N^* and d_k^* are obtained from $\mathcal{P}_1(n^+)$, we then formulate a second optimization problem $\mathcal{P}_2(N^*, d_k^*, n^+)$. The objective is to minimize the communication cost C (defined in the following), given the number of subsystems, the variables to be shared and the computational cost constraint:

$$\begin{aligned} \mathcal{P}_2(N^*, d_k^*, n^+) \ : \ \min \ & C \\ \text{s.t.} \ & \\ & N = N^* \\ & \text{satisfy } d_k^*, \ \forall \ k = 1, \dots, n \\ & n_I \leq n^+, \quad \forall I = 1, \dots, N. \end{aligned} \tag{2}$$

The outputs of the second optimization problem are the minimum communication cost C^* and the optimal decomposition Ω^* of the system graph.

Consider a large-scale system described by the possibly non-linear model equations:

$$x(t+1) = f(x(t), u(t)) + \eta(t) + \phi(x(t), u(t), t) \tag{3}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ the control input, $f : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ represent the nominal healthy dynamics, while η describes modeling uncertainties in the state equation. Finally, ϕ is a fault function which is null for $t < T_0$ (T_0 is the time of fault occurrence). Each LFD uses the measurements obtained by some sensors. We assume that the state vector is completely measurable, that is we assume that at least n sensors are available, one for each state component. The possibility of sharing some variables among different subsystems implies the addition of some sensors, so that the LFDs sharing the involved state variable can determine that component value in different ways [3]. In the case that each state variable can be measured by more than one sensor, the output equation is

$$y(t) = Hx(t) + \xi(t),$$

where $y \in \mathbb{R}^p$, with $p \geq n$, represents the measurements vector affected by the measurement noise $\xi \in \mathbb{R}^p$ and H is a $p \times n$ matrix having for each row all the elements null but one, equal to 1. The matrix H describes the relationship between sensors and measured variables: the (i, j)-th element is equal to 1 if the i-th sensor measures the j-th variable. One of the tasks of the design problem is to define the matrix H, that is, to decide how to use the available sensors: we choose the number of sensors measuring each variable and so which variables can take advantage from having redundant measurements. This includes the possibility to add more sensors in order to improve the fault detection performances.

The following assumptions are needed:

Assumption 1. The modeling uncertainty η is an unknown function, modeled as a stochastic process of unknown distribution. We assume to know at each time instant t mean and variance of the stochastic variables $\eta(t)$:

$$\eta(t) \approx (\mu_\eta(t), \sigma_\eta(t)),$$

Assumption 2. The measurement noise ξ is a stochastic process of known distribution. We assume to know at each t the mean and variance of the stochastic variables $\xi(t)$:

$$\xi(t) \approx (\mu_\xi(t), \sigma_\xi(t)).$$

Once the system decomposition is chosen, it is possible to define some local models:

$$\mathscr{S}_I : \begin{cases} x_I(t+1) = f_I(x_I(t), z_I, u_I(t)) + \eta_I(t) \\ \qquad\qquad + \phi_I(x_I(t), u_I(t), t) \\ y_I(t) = x_I(t) + \xi_I(t), \end{cases} \tag{4}$$

where $x_I \in \mathbb{R}^{n_I}$ is the local state vector, $u_I \in \mathbb{R}^{m_I}$ the local input and $y \in \mathbb{R}^{n_I}$ the local output affected by the measurement noise ξ_I, $z_I \in \mathbb{R}^{n_I}$ collects the neighboring state variables affecting local state variables dynamics; f_I is the local nominal function, while η_I describes local uncertainties. Finally, ϕ_I is the local fault function.

Next, we address the model-based FD problem.

3. DISTRIBUTED FAULT DETECTION

We first consider the case in which the k-th state component, with $k = 1, \dots, n$, is monitored by a single LFD. The local estimation model is based on the local system model:

$$\hat{x}_I^{(k)}(t+1) = f_I^{(k)}(y_I(t), v_I(t), u_I(t)) + \lambda(\hat{x}_I^{(k)}(t) - y_I^{(k)}(t)),$$

where v_I is the communicated measurement of z_I, so that $v_I = z_I + \xi_{z,I}$. In the distributed FD architecture, at time $t+1$, the state estimate $\hat{x}_I(t+1)$, computed at time t, is

[2] N and n_I are variables of the optimization problem.

[3] There are many application examples where this fact can be applicable, especially in the more recent times thanks to the low cost of sensor and wireless communication capabilities.