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# Robust Fault Detection and Diagnosis for Multiple-Model Systems with Uncertainties ⋆

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**Abstract:** In this paper, a robust fault detection and diagnosis (FDD) method is proposed for multiple-model systems with modeling uncertainties. A compensation step is introduced to modify the mixed states and their variances obtained through the interacting multiple model (IMM) approximation and to solve the uncertainty problem. The degree of compensation is governed by a modification parameter determined by the orthogonality principle, which means that the estimation error calculated in the sub-filter using the true system models should be orthogonal to the residual error vector. To avoid over compensation in the unmatched models, a minimization procedure is used to derive the overall modification parameter. When the modification parameter is equal to one, the proposed method reduces to the IMM algorithm. An experiment is conducted through the ball and tube system to demonstrate the effectiveness of the proposed method.



*Keywords:* Fault detection and diagnosis, multiple model system, uncertainties, compensation

## 1. INTRODUCTION

Due to its importance in industry, the fault detection and diagnosis (FDD) problem has drawn considerable attention during the last three decades. The main task of FDD is to detect and identify possible faults as early as possible so as to prevent disaster event and severe performance degradation (Hwang et al (2010)). As the modern engineering systems are becoming increasingly complex, systems may suffer from different types of potential failures, such as single actuator failure, single sensor failure, and simultaneous sensor or actuator failure, etc, which cannot be modeled appropriately by a single mathematical model (Zhang et al (1991)). To overcome this problem, the stochastic hybrid systems which are driven both by continuous and discrete mechanisms are introduced. For such systems, one of the most effective FDD approaches is based on the interacting multiple model (IMM) method, where a bank of model-matched Kalman filters (KFs) are operated to provide the overall estimate (Blom et al (1988); Johnston et al (2001)). The FDD declaration is generally to choose the most possible model at each time step. A distinct advantage of this framework is that it can deal with different types of failures in an uniform way. Compared with other multiple-model (MM) methods (Willsky (1976)), the model-matched filters in IMM interact with each other, leading to more suitable FDD structure and improved estimation performance.

In the IMM-based FDD approach, the model set is assumed to be known apriori. In practical applications, however, model set design is one of the most challenging tasks, and a reasonable model set is critical to the algorithm performance. As the systems always operate in an uncertain environment, it is near impossible to cover all the possible behaviour patterns of the system investigated. On the other hand, increasing the number of modes does not necessarily lead to improved model set. On the contrary, it has been proved that the performance will deteriorate if too many models are used in the IMM method (Li (2001)), as extra competitions will be introduced. Therefore, one cannot solve the uncertainties inherent in the modeling process by increasing the number of system modes. In view of this, one possible method is to use the so-called variable structure filtering algorithms (Li (2001); Li et al (1996); Ru et al (2008)), and another possibility is to apply the guaranteed-cost technique to ensure that the variance of estimation error is upper bounded for all admissible uncertainties (Souza et al (2002); Du et al (2012)). For more details, one can refer to Tudoroiu et al (2007); Kim et al (2008); Orguner et al (2008).

In this paper, we propose a new robust IMM method for multiple-model systems with uncertainties. Different from the methods mentioned above, it introduces a compensation step to ensure that the estimation errors in the true

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modes are orthogonal to the residual errors as much as possible. In this way, the recursive structure of IMM method is kept, and more importantly, no additional information is to be assumed. The remainder of this paper is organized as follows. In section 2, we describe the mathematical model of the system, and illustrate the traditional IMM-based FDD method. The main results are given in section 3, where the robust IMM method is developed. In section 4, we test the proposed algorithm with an experiment setup to demonstrate the effectiveness of the proposed method. Section 5 concludes this paper.

## 2. PROBLEM FORMULATION AND PRELIMINARIES

In the IMM-based FDD algorithm, a linear Markov jump system is used to model the dynamics of the investigated systems, which is represented in state space by

$$x_k = A(r_k)x_{k-1} + B(r_k)u_k + G(r_k)w_k \,, \tag{1}$$
$$y_k = C(r_k)x_k + v_k \,, \tag{2}$$

where $x_k$ is the state vector, $y_k$ is the measurement vector, $u_k$ is the input variable, $w_k$ and $v_k$ are the uncorrelated system and measurement white noises with known variances $R_k$ and $Q_k$ respectively, and $A(r_k)$, $B(r_k)$, $C(r_k)$, and $G(r_k)$ are the system matrices, which are dependent on the system mode $r_k$. As this paper considers the uncertainty problem, all the system matrices belong to the nominal models. Here, $\{r_k\}$ is a sequence following a Markov chain, taking values in a finite state set $M = \{1, 2, \cdots, N\}$. The transition probabilities from mode at time $k-1$ to mode at time $k$ are defined as

$$\pi_{ij} = P_r(r_k^j | r_{k-1}^i) \,,$$
$$0 \leq \pi_{ij} \leq 1, \quad \sum_{i=1}^{N} \pi_{ij} = 1 \,,$$

where $i, j = 1, 2, \cdots, N$ are mode indexes, and $r_k^j \triangleq \{r_k = j\}$. For notational simplicity, the system matrices under mode $j$ at time $k$ are denoted by $A_j$, $B_j$, $C_j$, and $G_j$.

In the IMM-based FDD method, a set of models are used to represent all the possible system structures. Compared with the single-model-based methods, the multiple failures scenarios can be considered and handled more conveniently under the framework of the multiple-model system (1) and (2). This is one of noticeable advantages of the IMM method. For example, denote the normal case by mode 1, and the single sensor failure by mode 2. Then, the only difference between the measurement matrices $C_1$ and $C_2$ is that a column of $C_2$ is zero. In this way, the element of the measurement that corresponds to the zeroed row will only consist of additive white noise $v_k$, and does not contain any information about state. Another common failure is the actuator failure. Similarly, let mode 3 denote a single actuator failure, then a column of input matrix $B_3$ will be zeroed. The result is that the element of control input $u_k$ that corresponds to the zero column will have no effect on the system dynamics. Following the same line, a total sensor failure and a total actuator failure can also be modeled conveniently.

### 2.1 IMM-based FDD

In this section, we will review the well-known IMM-based FDD method using the nominal model (1) and (2). The main purpose is to introduce some notations and variables which will be used in the following section. In the IMM-based FDD method, $N$ KFs are run in parallel to provide the overall estimation and FDD results. Let $x^i_{k-1|k-1}$, $\Delta^i_{k-1|k-1}$, and $\upsilon^i_{k-1}$ denote the estimate, variance, and mode probability of the $i^{\text{th}}$ KF respectively, which are provided by the last cycle of computation. At time $k$, the input to the $j^{\text{th}}$ KF can be computed by the IMM approximation,

$$\bar{x}^j_{k-1|k-1} = \sum_{i=1}^{N} \upsilon^{ij}_{k-1|k} \, x^i_{k-1|k-1} \,, \tag{3}$$

$$\bar{\Delta}^j_{k-1|k-1} = \sum_{i=1}^{N} \upsilon^{ij}_{k-1|k} \Big[ \Delta^i_{k-1|k-1} + \tilde{x}^j_{k-1|k-1} (\tilde{x}^j_{k-1|k-1})^T \Big] \,, \tag{4}$$

where $\tilde{x}^j_{k-1|k-1} = x^i_{k-1|k-1} - \bar{x}^j_{k-1|k-1}$, and

$$\upsilon^{ij}_{k-1|k} = \frac{\pi_{ij} \upsilon^i_{k-1}}{\sum\limits_{i=1}^{N} \pi_{ij} \upsilon^i_{k-1}} \,, \tag{5}$$

where the superscript $ij$ indicates the transition from mode $i$ to mode $j$. Then, the posterior estimates $x^j_{k|k}$ and variances $\Delta^j_{k|k}$ are obtained by all $N$ KFs, where $j = 1, 2, \cdots, N$. As the posterior mode probabilities $\upsilon^j_k$ play an important role in the FDD process, the likelihood is used to update the predicted mode probabilities $\upsilon^j_{k|k-1}$ as

$$\upsilon^j_k = \frac{L^j_k \upsilon^j_{k|k-1}}{\sum\limits_{j=1}^{N} L^j_k \upsilon^j_{k|k-1}} \,, \tag{6}$$

where predicted mode probabilities $\upsilon^j_{k|k-1}$ and likelihoods $L^j_k$ can be computed by, respectively,

$$\upsilon^j_{k|k-1} = \sum_{i=1}^{N} \pi_{ij} \upsilon^i_{k-1} \,,$$
$$L^j_k = \frac{1}{\sqrt{\left|2\pi S^j_k\right|}} \exp\left( -\frac{1}{2} e^j_k \left(S^j_k\right)^{-1} \left(e^j_k\right)^T \right) ,$$

with

$$e^j_k = y_k - C_j (A_j \bar{x}^j_{k-1|k-1} + B_j u_k) \,, \tag{7}$$
$$S^j_k = C_j (A_j \bar{\Delta}^j_{k-1|k-1} A^T_j + G_j R_k G^T_j) C^T_j + Q_k \,. \tag{8}$$

Once the posterior mode probabilities $\upsilon^j_k$ are available, the fault declaration can be made according to the following logic.