Sensors and Actuators B 192 (2014) 664–672

Contents lists available at ScienceDirect

Sensors and Actuators B: Chemical

journal homepage: www.elsevier.com/locate/snb

Effect of hydrodynamic force on microcantilever vibrations: Applications to liquid-phase chemical sensing

I. Dufour[a,*], E. Lemaire[a], B. Caillard[a], H. Debéda[a], C. Lucat[a], S.M. Heinrich[b], F. Josse[c], O. Brand[d]

[a] Univ. Bordeaux, IMS, UMR 5218, F-33400 Talence, France
[b] Department of Civil, Construction and Environmental Engineering, Marquette University, Milwaukee, WI, USA
[c] Department of Electrical and Computer, Marquette University, Milwaukee, WI, USA
[d] School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA, USA

ARTICLE INFO

Article history:
Received 19 July 2013
Received in revised form 3 October 2013
Accepted 24 October 2013
Available online 12 November 2013

Keywords:
Hydrodynamic force
Microcantilever vibration
Rheology
Chemical detection
In-plane vibration
MEMS

ABSTRACT

At the microscale, cantilever vibrations depend not only on the microstructure's properties and geometry but also on the properties of the surrounding medium. In fact, when a microcantilever vibrates in a fluid, the fluid offers resistance to the motion of the beam. The study of the influence of the hydrodynamic force on the microcantilever's vibrational spectrum can be used to either (1) optimize the use of microcantilevers for chemical detection in liquid media or (2) extract the mechanical properties of the fluid. The classical method for application (1) in gas is to operate the microcantilever in the dynamic transverse bending mode for chemical detection. However, the performance of microcantilevers excited in this standard out-of-plane dynamic mode drastically decreases in viscous liquid media. When immersed in liquids, in order to limit the decrease of both the resonant frequency and the quality factor, and improve sensitivity in sensing applications, alternative vibration modes that primarily shear the fluid (rather than involving motion normal to the fluid/beam interface) have been studied and tested: these include in-plane vibration modes (lateral bending mode and elongation mode). For application (2), the classical method to measure the rheological properties of fluids is to use a rheometer. However, such systems require sampling (no in situ measurements) and a relatively large sample volume (a few milliliters). Moreover, the frequency range is limited to low frequencies (less than 200 Hz). To overcome the limitations of this classical method, an alternative method based on the use of silicon microcantilevers is presented. The method, which is based on the use of analytical equations for the hydrodynamic force, permits the measurement of the complex shear modulus of viscoelastic fluids over a wide frequency range.

1. Introduction

In recent years, interest in microcantilever-based chemical and bio-chemical sensing systems has risen due to their projected high sensitivity [1–5]. The large ratio of surface area to volume makes the microcantilever extremely sensitive to surface processes. For (bio)chemical detection, the microcantilever is usually coated with a (bio)chemically sensitive layer whose purpose is to selectively sorb the analyte or molecule of interest. The sorbed substance can then be detected by monitoring either the resonant frequency shift (dynamic mode) or the quasi-static deflection (static mode). A simplified way to explain the basic principle of such sensors is to say that in the case of dynamic mode operation, the change in mass associated with the sorption of analyte onto/into the sensitive coating causes a shift in resonant frequency, which may be correlated to the ambient concentration of the target substance. For static-mode operation, the sorption of analyte causes a cantilever deflection that is induced by surface stress or by the tendency of the sensitive coating to expand or contract upon analyte sorption or desorption (modification of strain and stress in the coating). In this paper we will focus on the use of the dynamic mode.

At the microscale, cantilever vibrations depend not only on the microstructure's properties and geometry but also on the properties of the surrounding medium (density and viscoelastic properties). In fact, when a microcantilever vibrates in a fluid, the fluid offers resistance to the motion of the beam. The coupling between the structure and the surrounding viscous fluid influences the structure's mechanical resonance; in particular, the fluid causes a decrease of both the resonant frequency and the quality factor and, thus, negatively impacts the device's sensing capabilities.

* Corresponding author.
E-mail address: isabelle.dufour@ims-bordeaux.fr (I. Dufour).

0925-4005/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.snb.2013.10.106

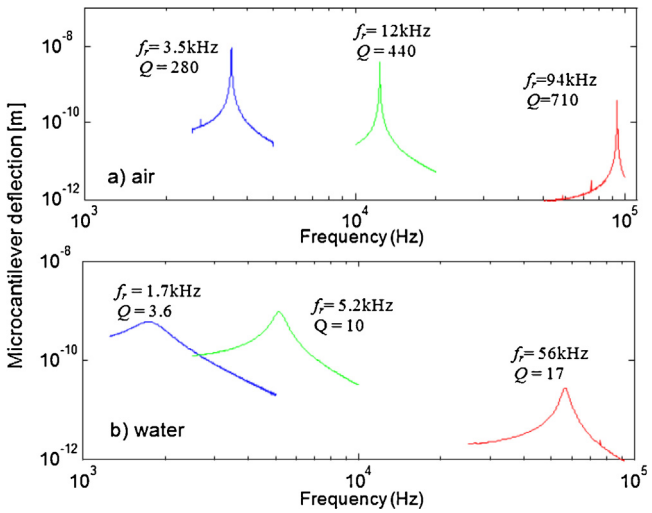

Fig. 1. Measurements of three microcantilevers with different geometries (named in section 4 LL, LH and A respectively): 2810 μm × 100 μm × 20 μm (blue), 1440 μm × 285 μm × 20 μm (green) and 500 μm × 100 μm × 20 μm (red). (a) Spectrum in air. (b) Spectrum in water. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

In gas, resonant frequencies of classical transverse flexural modes may be reduced by a few percent compared with the value in vacuum, whereas the quality factor may exhibit reductions of two orders of magnitude. This performance degradation for conventional out-of-plane flexural resonant modes is further exacerbated in liquids: the resonant frequency and quality factor values are approximately one order of magnitude smaller than their values in the gas phase (Fig. 1).

Chemical detection using microcantilever-based sensors is based on the measurement of the resonant frequency shift Δf_r which is induced by the mass increase of the vibrating microstructure during sorption of the target species into the sensitive coating [6]:

$$\Delta f_r = -f_r \frac{\Delta m}{2m} \tag{1}$$

with f_r and m being the resonant frequency and mass of the microstructure, respectively. According to the partition coefficient (K) of the analyte/sensitive coating pair [6], which is defined as the ratio of the concentration of the analyte in the coating and the ambient concentration, C_A, of the analyte in the surrounding medium, the mass variation Δm is proportional to the concentration C_A. Thus, the sensitivity of the chemical sensor, S, is proportional to the resonant frequency of the microcantilever in the surrounding medium (gas or liquid) [6]:

$$S = \frac{\Delta f_r}{C_A} = -\frac{f_r K V_L}{2m} \propto f_r \tag{2}$$

with V_L defined as the volume of the sensitive layer.

The accuracy of the resonant frequency measurement depends on the sharpness of the resonant peak which is characterized by the quality factor, Q, associated with a particular resonant mode, while the noise N corresponding to the resonant frequency measurement can be considered to be inversely proportional to the quality factor [6]:

$$N \propto \frac{1}{Q} \tag{3}$$

According to Eqs. (2) and (3), reduced values of the resonance characteristics (resonant frequency and quality factor) in liquid media (Fig. 1) adversely affect the sensitivity and the detection limit of these types of sensors. Therefore, for liquid-phase chemical sensing applications it is imperative that the effect of the liquid on the resonant characteristics of the sensing device be understood and considered in the sensor design in order to achieve desired levels of sensing performance (sensitivity and limit of detection).

In the present paper, we propose alternative uses of microcantilevers, including the implementation of unconventional vibrational modes to overcome the aforementioned obstacles for liquid-phase operation. By understanding in detail the influence of the hydrodynamic force on the microcantilever's vibrational spectrum, the following two objectives may be achieved: (1) optimization of microcantilevers for chemical detection in liquid media and (2) extraction of the mechanical properties of complex fluids which is of particular importance for microfluidic systems and for chemical detection in these types of environments.

The paper is organized such that a review of hydrodynamic force expressions and their impact on the resonant frequency and quality factor is given in Section 2 for the cases of out-of-plane and in-plane vibrations. In Section 3 we present how one may use the knowledge of the hydrodynamic force to choose appropriate vibrational modes and microcantilever geometries for chemical detection in liquid media. A second example of how a firm grasp of fluid-structure interaction may be used to advantage is the implementation of a microcantilever to extract the rheological properties of a fluid, which is described in Section 4.

Notation (Fig. 2): The geometry of the cantilever is defined by the width b, thickness h, and length L. Coordinate x is measured along the beam length. The properties of the cantilever material are the Young's modulus E and the mass density ρ. The fluid (gas or liquid) properties are defined as ρ_f, the fluid's mass density, η the fluid's dynamic viscosity and $G^* = G' + jG''$, the complex shear modulus of the fluid.

2. Influence of hydrodynamic force on microstructure resonant characteristics

In this paper, we focus on three distinct vibration modes (Fig. 2): the transverse (out-of-plane) bending mode, the lateral (in-plane) bending mode and the elongation mode (also in-plane). A fourth mode that may also have interesting potential for both chemical detection and rheological sensing is the torsional mode, whose hydrodynamic force and resonant characteristics have been studied in [7,8]; however, that mode is not included in the present study.

2.1. General modeling of the hydrodynamic force

When a microcantilever vibrates in a viscous fluid (gas or liquid), the fluid offers resistance to the motion. Depending on the vibration mode and on the cantilever surface there are hydrodynamic pressure forces (normal to the surface) and hydrodynamic viscous forces (tangential to the surface) (Fig. 3).

The force per unit length, F_{fluid}, which is the consequence of all normal and tangential stresses exerted by the fluid on all the surfaces of the cantilever, can be written in the frequency domain as [9]:

$$F_{fluid} = -[j\omega g_1(x,\omega) - \omega^2 g_2(x,\omega)]w(x,\omega) \tag{4}$$

where x is the longitudinal coordinate, ω the radial frequency of vibration, w the microcantilever deflection in the case of bending modes or the microcantilever axial displacement in the case of the elongation mode, and g_1 and g_2 are functions depending on the fluid properties and the microcantilever cross-sectional geometry. They may be interpreted, respectively, as the distributed damping coefficient of the fluid and the distributed effective fluid mass (per unit length of beam). In general, these quantities are both frequency-dependent and x-dependent; however, in what follows g_1 and g_2