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Variational Analytic Programming for Synthesis of Optimal Control for Flying Robot

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Abstract: The paper presents a novel numerical approach for automatic feedback control system synthesis. The controller is supposed to be described by the control function that depends on objects state. The developed variational analytic programming method allows to construct automatically the mathematical expression of the control function and also to adjust the parameters. The method organizes search of the optimal control function over the set of the small variations of the given basic solution. Search efficiency depends on the quality of the chosen basic solution. An example of automatic control system synthesis for flying robot with state constraints is shown.

Keywords: robust robot control, learning robot control, mobile robots and vehicles.

1. INTRODUCTION

The problem of control system synthesis arises when we need to perform an automatic mode of robot's control for example to make the robot autonomously follow a certain spatial trajectory. To solve the control synthesis problem is to find a multidimensional control function. The arguments of this control function are the components of the state vector of the plant. If we place this control function into the right parts of differential equations describing robot's motion we obtain the system of differential equations independent of the components of the control vector. Solution of the obtained autonomous control system ensures meeting the goal objectives from different initial conditions with optimal value of the quality criterion.

To solve the problem of synthesis means to construct the control function that depends on object's state and also tune the parameters. Analytical methods apply to a small class of the systems like linear system with quadratic functional (Letov, Lee and Marcus). Currently the control synthesis problem is solved in most cases manually basing on the developer's experience when an engineer construct the mathematical expression and then adjust only the parameters.

We are working on research and development of numerical methods to solve the problem of control system synthesis. Our method of the network operator solves the problem of control synthesis rather successfully (Diveev 2012, 2013). Based on the experience and analysis of the problem a researcher gives a basic solution. The evolutionary algorithm adjusts the mathematical expression and the parameters. The algorithm finds the optimal solution on the set of small variations of the basic solution. When solving complex control problems for effective work of algorithm the researcher must determine the proper basic solution. The restriction of the network operator consists in the fact that it searches solution on the set of functions with one or two arguments, therefore one has to use two network operators with arithmetic and logic functions for complicated control (Atiencia Vilagomes et al.). The method of analytical programming involves functions with any number of arguments (Zelinka 2002, 2005). But without using the principle of small variations of basic solution it is difficult to solve problems of control synthesis, since besides the optimal value of goal function the solution also should provide the achievement of control objective. We have used the principle of small variations of basic solution for the method of analytical programming in order to solve the problems of control synthesis. We call the new method as a variational analytic programming.

We have considered the problem of control synthesis for flying robot as an example. The unmanned flying robot must follow the given trajectory in a space. Through the application of the method of the variational analytic programming we have solved the problem of synthesis and determined the function of control which enables to achieve the aim of control for different initial values.

2. VARIATIONAL ANALYTIC PROGRAMMING

As a method of symbolic regression the variational genetic programming allows to present mathematical expressions as codes. These codes are compositions of elementary functions. Let us define the ordered set of functions with certain number of arguments

$$\mathrm{F}_i = \left(f_{i,1}(z_1,\dots,z_i),\dots,f_{i,m_i}(z_1,\dots,z_i)\right) , \tag{1}$$

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where $f_{i,j}(z_1,\ldots,z_i)$ is a function numbered j with the number of arguments i, $j=1,\ldots m_i$, $i=0,\ldots,n$.

So the joint set is

$$\mathrm{F}=\bigcup_{i=0}^{n}\mathrm{F}_i\ . \tag{2}$$

Let us number all elements of the joint set as follows

$$\mathrm{F}=(f_1,\ldots,f_D), \tag{3}$$

where

$$D=\sum_{i=0}^{n}m_i\ , \tag{4}$$

$$f_1=f_{0,1},\ f_1=f_{0,1},\ f_2=f_{0,2},\ldots,f_{m_0}=f_{0,m_0},$$
$$f_{m_0+1}(z)=f_{1,1}(z),\ldots,f_{m_0+m_1}(z)=f_{1,m_1}(z),$$
$$f_{m_0+m_1+1}(z_1,z_2)=f_{2,1}(z_1,z_2),\ldots,$$
$$f_{m_0+\ldots+m_n}(z_1,\ldots,z_n)=f_{n,m_n}(z_1,\ldots,z_n).$$

Set of functions without the arguments or with zero number of arguments is to be considered apart. For any mathematical expression this is the set of parameters and variables

$$\mathrm{F}_0=(f_{0,1},\ldots,f_{0,m_0})\ =(x_1,\ldots,x_N,q_1,\ldots,q_p). \tag{5}$$

According to the method of analytical programming the code of a mathematical expression is performed in the form of ordered set of integers

$$\mathrm{C}=(c_1,\ldots,c_K)\ , \tag{6}$$

where $c_i\in\{1,\ldots,D\}$, $i=\overline{1,K}$.

Each integer value is a number of the element from the joint set (2). The notation has prefix syntax when the code of function is placed before the codes of arguments. The length of the code is limited. Additional set of functions (5) without arguments is used to represent coded functions correctly.

To obtain the mathematical expression from the code it is necessary to know the amount of elements in each set of functions: $m_0,\ldots,m_n$, and the amount of variables and parameters N, p used.

The amount of arguments of function i and the number of function j are defined through the value of the element c_k

$$i=\begin{cases}0,\ \text{if}\ 0\le c_k\le m_0\\ \alpha,\ \text{if}\ \sum_{r=0}^{\alpha-1}m_r\le c_k\le\sum_{r=0}^{\alpha}m_r\ ,\ \alpha=\overline{1,m_n}\end{cases}, \tag{7}$$

$$j=c_k-\sum_{r=0}^{i-1}m_r\ ,\ 1<i\le n. \tag{8}$$

If c_k corresponds to a function without arguments ($i=0$) then the variable x_j or parameter q_j is defined using the amount of variables N and parameters p

$$j=\begin{cases}c_k,\ \text{if}\ c_k\le N\\ c_k-N\ \text{otherwise}\end{cases}\quad i=0, \tag{9}$$

where c_k corresponds to variable x_j if $c_k\le N$, or to parameter q_j if $N<c_k\le m_0$.

To describe a vector of mathematical expressions one set of integers with the certain number of elements for each component of the vector is used. Let the vector of mathematical expression has M components. Allocate L positions of the code for each component. Code of each component i of the mathematical expression includes $k_i\le L$ elements, $i=\overline{1,M}$. Set zero value to those elements that are not used to form the code

$$\mathrm{C}=(\underbrace{c_1,\ldots,c_{k_1},0,\ldots,0}_{L},\ldots$$
$$\ldots,\underbrace{c_{L(M-1)+1},\ldots,c_{L(M-1)+k_M},0,\ldots,0}_{L}), \tag{10}$$

where L is the number of positions in the code for each component of the vector of mathematical expressions, k_i is the length of the code of the component i, $i=\overline{1,M}$.

In the code of the vector of mathematical expressions (10) the following condition is applied

$$c_j=0,\ \text{if}\ L(i-1)+k_i<j\le Li\ ,\ i=\overline{1,M}\ . \tag{11}$$

Index of the element is used to define the correctness of the code of mathematical expression. Let the element $c_j\ne0$ in the code (6). Define the number of the components i for the vector of mathematical expressions from the ratio

$$i=\left\lfloor\frac{j-1}{L}\right\rfloor+1. \tag{12}$$

In order to write the index of the element $c_j\ne0$ correctly the following conditions must be fulfilled

$$T(j)>0\ ,\ j\ne k_i\ , \tag{13}$$
$$T(k_i+L(i-1))=0\ , \tag{14}$$

where $T(j)$ is the index of the element j in the code of mathematical expression. To calculate the index $T(j)$ of the element j if $c_j\ne0$ we use the ratio

$$T(j)=1-(j-L\beta)+\sum_{k=L\beta+1}^{j}i_k\ , \tag{15}$$

where

$$i_k=\begin{cases}0,\ \text{if}\ 0\le c_k\le m_0\\ \alpha,\ \text{if}\ \sum_{r=0}^{\alpha-1}m_r\le c_k\le\sum_{r=0}^{\alpha}m_r\ ,\ \alpha=\overline{1,m_n}\end{cases}, \tag{16}$$

$$\beta=\left\lfloor\frac{j-1}{L}\right\rfloor. \tag{17}$$