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Control of wheeled mobile robots singularly perturbed by using the slipping and skidding variations: curvilinear coordinates approach (Part II)

C. A. Peña Fernández * J. J. F. Cerqueira * A. M. N. Lima **

* Robotics Laboratory - Department of Electrical Engineering, Federal University of Bahia,Rua Aristides Novis, 02, Federação, 40210-630, Salvador, Bahia, Brasil
** Center of Electrical and Computer Engineering - Department of Electrical Engineering, Federal University of Campina Grande, Rua Agripio Veloso, 882, Universitário, 58429-970, Campina Grande, Paraíba, Brasil

Abstract: In work reported here, it is proposed a method to reduce the effects of slipping and skidding in WMRs not exactly satisfying kinematic constraints. It is studied the case for WMRs whose kinematic constraints are violated owing to deformability or flexibility. To this end, it is considered the static-state linearization whose robustness will be based on singular perturbation methods and an auxiliary control law that is modeled on a suitable transformation for curvilinear coordinates. In results will be seen that the tracking error converges to small ball of the origin whose radius can be adjusted by a known function that depends on the slipping and skidding variations.

Keywords: Singular perturbations method, slipping variation, skidding variation, wheeled mobile robot, curvilinear coordinates.

1. INTRODUCTION

Wheeled mobile robots [WMRs] have two types of kinematic constraints: pure rolling condition and the nonslipping condition, both associated to the contact point between each wheel and the ground [see Campion et al. (1996)]. Often, WMRs are based on assumption that these constraints are satisfied at each instant along motion. However, kinematic constraints are violated when a WMR is moving on a trajectory either accelerating, or decelerating, or cornering at a high speed. In addition, several phenomena contribute with that violation, such as sliding, deformability or flexibility. Thus, the trajectory tracking control of WMRs and its stabilization have given rise to an abundant literature in recent years due to its challenging theoretical nature [see Chwa (2004); Aithal and Janardhanan (2013); Song and Boo (2004); Fernández et al. (2015)]. It is well-known that there does not exist a smooth pure state feedback control law such that the state of a WMR converges to the origin [see Dong (2010); Fernández et al. (2014); Bloch et al. (2000)]. In order to solve this problem, several types of controllers have been proposed, such as time-varying control laws, discontinuous control laws, and hybrid control laws based on linearization local controlling, nonlinear state feedback with singular parameters, or backstepping, see Aithal and Janardhanan (2013); D'Andréa-Novel et al. (1995); Leroquais and D'Andrea-Novel (1996); Motte and Campion (2000); Dong (2010).

In work reported here, we propose a solution for tracking control problems of WMRs which are subject to slipping and skidding effects, i.e., when kinematic constraints of pure rolling and nonslipping are transgressed during the motion. For that purpose, it is considered the classical control law based in static-state feedback linearization whose robustness with respect to the deformability of wheels will be based on singular perturbation methods and an auxiliary control law whose robustness will be based on the slipping and skidding variations. The controller is studied and designed in two parts: in the Part I [first paper], we studied the tracking problem related to WMRs based on singular perturbations method. An auxiliary control law on curvilinear coordinates with regulable convergence radius was established through of the function h_γ. In the Part II [this paper], the regulable convergence radius is improved with inclusion of slipping and skidding variations to describe the function h_γ. Consequently, a hybrid control scheme supported by information of the slipping and skidding variations in the reference trajectory will be synthesized.

This paper is organized as follows: In Section 2, key aspects regarding of Part I are shown. In Section 3 is presented an analysis for the slipping and skidding variations. In Section 4, by using the analysis in Section 3, is found a structure for the function h_γ that includes slipping and skidding variations. Simulation results for trajectory tracking of a WMR [at same used in Part I] are presented in Section 5. Finally, closing remarks are made in Section 6.

2. PRELIMINARIES OF THE PART I

In this section, we briefly recall the singular perturbation formulation cited in the Part I. Thus, let us consider WMRs whose total motion is executed by the action of N wheels such that $N = N_f + N_c + N_o$, where N_f, N_c, N_o represent the number of *fixed* wheels, *centered orientable*

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wheels and *off-centered orientable* wheels, respectively [see Campion et al. (1996)]. Complementarily, like in part I, it was assumed that the configuration of WMRs is fully described by the following vector of generalized coordinates:

$$q = [\, z \;\; \beta_o \;\; \varphi \,]^T \tag{1}$$

where $z = [\, \xi \;\; \beta_c \,]^T \in \mathbb{R}^{3+\delta_d}$, $\xi = [x\,, y\,, \theta\,, \sigma_1 \ldots, \sigma_{N_{\text{ind}}}]^T$, with $3 + N_{\text{ind}}$ DOF, being N_{ind} a value associated with number of wheels that can be oriented independently. Nevertheless, we will consider WMRs whose $N_{\text{ind}} = 0$. Furthermore, $\beta_c \in \mathbb{R}^{N_c}$ contains the guidance of *centered orientable* wheels, $\beta_o \in \mathbb{R}^{N_o}$ contains the guidance of *off-centered orientable* wheels and $\varphi \in \mathbb{R}^N$ contains the pose of all wheels. The variables x, y represent the pose of the local frame $\{L\}$, related to WMR's body, with respect to a global frame $\{W\}$ and θ specifies its orientation. The constant δ_d is called *degree of steeribility* and it is always equal to N_c.

The generalized velocity $\dot{q}$ may be written as:

$$\dot{q} = S(q)\eta + A(q)\varepsilon\mu \quad \left(\begin{bmatrix} \dot{z} \\ \dot{\beta}_o \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \eta + \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \varepsilon\mu \right) \tag{2}$$

where $S(q)) = [S_1 \; S_2]^T \in \mathbb{R}^{(3+N_o+\delta_d+N)\times\delta_u}$, $\eta = [\, u \;\; \zeta \,] \in \mathbb{R}^{\delta_u}$ is the vector that contains the linear and angular velocities of the WMR $[u \in \mathbb{R}^{\delta_u - N_c}]$ and the velocities of centered orientable wheels $[\zeta \in \mathbb{R}^{N_c}]$. The vector $\mu \in \mathbb{R}^{K_r}$ is associated with the violation of kinematic constraints, $\varepsilon > 0$ is a scale factor related to the *flexibility* and $A^T(q) = [A_1^T \;\; A_2^T]^T \in \mathbb{R}^{K_r \times (3+N+N_c+N_o)}$ represents K_r independent constraints, with $K_r < 3 + N + N_c + N_o$. The constant δ_u represents the *degree of maneuverability*.

***Remark** 1.* From Campion et al. (1996), for the type of wheels indicated above $\delta_u \leq 3$ and $\delta_u - \delta_d \leq 2$.

Multiplying both sides of (2) by $A^T(q)$, and due to $A^T(q)S(q) = 0$ we have

$$A^T(q)\dot{q} = A^T(q)A(q)\varepsilon\mu. \tag{3}$$

***Assumption** 1.* $\|A^T(q)A(q)\varepsilon\mu\| \leq \xi^*$, where $\xi^* > 0$, for $\forall t$, is a known function that depends on slipping, skidding and deformation of the wheel.

A singular perturbation model for a WMR can be write as the following space-state [see Fernández et al. (2015, 2014)]:

$$\begin{cases} \dot{x} = Z_0(q)\eta + [\varepsilon Z_1(q) + Z_2(q)]\,\mu + Z_3(q)\tau, \; x(0) = x_0 & (4) \\ \varepsilon\dot{\mu} = G_0(q)\eta + [\varepsilon\, G_1(q) + G_2(q)]\,\mu + G_3(q)\tau, \mu(0) = \mu_0 & (5) \end{cases}$$

where $x = [\, q \; \eta \,]^T \in \mathbb{R}^{3+\delta_d+N_o+N+\delta_u}$ is the state vector with slow variables and $\mu \in \mathbb{R}^{K_r}$ is the vector with the fast variables [see D'Andréa-Novel et al. (1995)]. The vectorial fields $Z_0 \in \mathbb{R}^{(3+\delta_d+N_o+N+\delta_u)\times\delta_u}$, $Z_1, Z_2 \in \mathbb{R}^{(3+\delta_d+N_o+N+\delta_u)\times K_r}$, $Z_3 \in \mathbb{R}^{(3+\delta_d+N_o+N+\delta_u)\times N}$, $G_0 \in \mathbb{R}^{K_r\times\delta_u}$, $G_1, G_2 \in \mathbb{R}^{K_r\times K_r}$, $G_3 \in \mathbb{R}^{K_r\times N}$ are continuously differentiable in the parameters $(x, \mu, \varepsilon, t) \in D_x \times D_\mu \times [0, \varepsilon_0] \times [0, t]$ being $D_x \in \mathbb{R}^{3+\delta_d+N_o+N+\delta_u}$ and $D_\mu \in \mathbb{R}^{K_r}$ open and convex sets. The vector $\tau \in \mathbb{R}^N$ represents the input torques at motors and usually they are expressed as an uniformly bounded function $\tau \triangleq \tau(q, \eta, \mu)$ such that

$$\tau_{\min} \leq \tau \leq \tau_{\max} \tag{6}$$

where $\tau_{\min}, \tau_{\max} \in \mathbb{R}$

The system (4)-(5) is a *standard singular form* if and only if the equation $G_0(q)\eta + G_2(q)\mu + G_3(q)\tau = 0$ has $k \geq 1$ different and isolated roots, denoted by:

$$\bar{\mu}_i = H_i(\bar{x}, t), \; i = 1, \ldots, k. \tag{7}$$

With all functions $\bar{\mu}_i$, it is defined the following *reduced system*:

$$\dot{\bar{x}} = Z_0(q)\eta + Z_2(q)H(\bar{x}, t) + Z_3(q)\tau, \; \bar{x}(0) = x_0 \tag{8}$$

corresponding to the case $\varepsilon = 0$ at system (4)-(5).

***Remark** 2.* In order to ensure the stability around the origin of reduced system (8) we will suppose that the outputs controlled of system (4)-(5) are a subset of q. Like in part I, the outputs controlled for trajectory tracking problems will be associated with the vector z and it will be taken into account a suitable partition on system (4)-(5).

Whenever $\text{rank}\{B(q)\} = \delta_u$, the rank of $S^T(q)B(q)$ is also full and the matrix $S^T(q)M(q)S(q)$ is nonsingular, the generic *static state-feedback* linearization of (4)-(5) (properly partitioned) is defined by

$$\tau = \left[S^T(q)B(q)\right]^{-1} \Big\{ S^T(q) \Big[M(q)S(q)\upsilon + M(q) \left[\frac{\partial S}{\partial q} S(q)\eta\right]\eta - C(q, S(q)\eta) \Big] \Big\}, \tag{9}$$

where $\upsilon \in \mathbb{R}^{\delta_u}$ represents an arbitrary *auxiliary control* law.

2.1 Auxiliary control law: curvilinear coordinates approach used in Part I

Based on the curvilinear approach presented in Part I, the tracking control of any point p on a WMR with desired linear velocity u_1^* can be parameterized by

$$q^* = [\, q_1 \;\; q_2 \;\; q_3 \,]^T = [\, \lambda \;\; d \;\; \alpha \,]^T = f(z) \tag{10}$$

where $f(\cdot) : \mathbb{R}^{3+\delta_d} \to \mathbb{R}^3$ is a known vectorial field, λ is the curvilinear coordinate along any curve with curvature $|\text{curv}(\lambda)| < 1/R$, $\forall\, \lambda$, where $R > 0$ is a constant; d is the coordinate of point p along $N(\lambda)$ [normal vector at λ] and α is the WMR's orientation with respect to $T(\lambda)$ [tangent vector at λ].

Let $e = [\, e_1 \;\; e_2 \;\; e_3 \,]^T$ be the tracking error associated with q^* and $h \in \mathbb{R}^{\delta_u}$ the control inputs of the kinematic controller formally defined by

$$h = \Pi_2^{-1}(z, h_\gamma)\eta, \tag{11}$$

where $\Pi_2(z, h_\gamma) \in \mathbb{R}^{\delta_u \times \delta_u}$ is a nonsingular matrix and $h_\gamma > 0$, for $\forall\, t \geq 0$, is a known function used to regulate the convergence radius. Complementary, the auxiliary control law υ is defined by

$$\upsilon = \left(\frac{\partial \Pi_2}{\partial z}(S_1(q)\eta + A_1(q)\varepsilon\mu) + \frac{\partial \Pi_2}{\partial h_\gamma}\dot{h}_\gamma \right)\rho + \Pi_2(z, h_\gamma)\dot{\rho}, \tag{12}$$

such that $h = \rho \in \mathbb{R}^{\delta_u}$ [from Remark 1, $\rho \leq 3$] with $u_1^* > \delta_v > 0$ and

$$\rho_1 = u_1^*,$$

$$\rho_2 = \begin{cases} -\xi^*\phi_2 \tanh\left(\dfrac{e_3\xi^*\phi_2}{h_\gamma}\right) - (k_3e_3 + e_2)u_1^* + \dfrac{\dot{u}_1^*\phi_1}{(u_1^*)^2}, & \text{if } \delta_u - \delta_d = 2 \\ \zeta_{1_c}^*, & \text{if } \delta_u - \delta_d = 1 \end{cases}$$

$$\rho_3 = \begin{cases} \zeta_{1_c}^*, & \text{if } \delta_u - \delta_d = 2 \\ \zeta_{2_c}^*, & \text{if } \delta_u - \delta_d = 1 \end{cases}$$

where $\zeta_{1_c}^*, \zeta_{2_c}^*$ are desired quantities, $k_3 > 0$ is a constant used like design parameter, $\mathcal{L}$ is the abbreviation of Lie Derivative and