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Topology Control for Connectivity Maintenance in Cooperative Mobile Robot Networks

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Abstract: In multi-robot systems, communication is a critical factor to perform cooperative tasks. Maintain communication in a multi-robot systems is a complex task that is not feasible in certain conditions because the environmental interference or failures in the communication devices. This paper presents a topology control algorithm for maintaining connectivity in a network with dynamic number of mobile robots, using concepts of graphs theory and consensus. The algorithm and the theoretical basis involved are detailed along the paper and simulations evaluating the effectiveness of this algorithm are proposed at the end.

Keywords: connectivity maintenance, distributed algorithm, multi-robot networks, consensus, optimization, predictive control.

1. INTRODUCTION

The use of mobile robots in task automation has become practicable with technology advances, however, the high cost of these robots compromises the use of this technology. Various strategies are used to reduce the implementation costs of tasks resolutions performed by robots, and one of these strategies is the use of distribution of complex tasks among several simpler and cheaper robots. Some of these tasks can be highlighted, such as: forest monitoring, search and rescue, and military applications (Murray (2007); Lucas et al. (2012); Chung et al. (2011)).

The communication topology of a multi-robot system imposes influence on the quality of the information exchanged between the robots. It happens because the robots are mobile and the topology is dynamic, this way it changes along time. Control this topology can add characteristics that improve the communication in the multi-robot network, making the execution of the cooperative task more robust.

Several works with topology control to optimize the communication process among the robots in a multi-robot system are presented in the literature. Most of them uses graph properties that make redundancy of the communication channels among the robots, resulting in networks topologically fault tolerant. The works of Li and Hou (2004) and Ahmadi and Stone (2006) use the exchange of special messages among the robots of a multi-robot system to detect the redundancy of communication channels, then use synchronization parameters to maintain this state of the network even in case of disconnection. The solution proposed by Wagenpfeil et al. (2009) uses the repulsion and attraction forces for model a decentralized control law that moves some robots to build a redundant network. Other similar work is proposed by Casteigts et al. (2010), where virtual forces are used to attract neighbors and make the fundamental triangular structure, ensuring the redundancy of communication channels in a decentralized way. Finally, Fiacchini and Morarescu (2014) present some essential conditions for preserve certain topology types in unstable environments. Based on these and other works, this paper introduces a robust topology control algorithm for maintenance of connectivity in cooperative multi-robot systems with variable number of robots using some graph properties and consensus theory.

This document is organized as follows: in section 2 a short introduction of graph and consensus theory is presented; in section 3 the proposed topology control algorithm is detailed; in section 4 some simulations are made for show the effectiveness of the proposed algorithm; and finally in section 5 a concluding analysis and directions for future works are given.

2. COOPERATION AND CONSENSUS

In cooperative robotics, consensus can be defined as a compromise between the robots in a multi-robot system to reach a target or common task, consolidating the joint action of these robots. Mathematically the consensus is formed by the modeled variables that describes a cooperative behavior between the robots of a multi-robot system. These variables are known as *information state* and it can assume several values and dimensions.

Is common the use of graphs and its algebraic forms, as adjacency matrices, to represent the state of the network in a multi-robot system. A graph can be defined as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{r_1, \ldots, r_n\}$ is the set of nodes or robots in this network and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges or

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links. The algebraic form of the graph can be represented by an adjacency matrix defined as $A = [a_{ij}] \in \mathbb{R}^{n\times n}$, where n is the number of nodes in the graph $\mathcal{G}$ and a_{ij} represent the neighborhood relationship between i and j, with $a_{ij} > 0 \leftrightarrow (i, j) \in \mathcal{E}$ and $a_{ij} = 0$ in another case.

To represent the dynamic consensus, first order dynamics equations are used, like defined by Olfati-Saber et al. (2007):

$$\dot{x}_i(t) = \sum_{j=1;j\neq i}^{n} a_{ij}(x_i(t) - x_j(t)), \quad i = 1, \dots, n, \qquad (1)$$

where a_{ij} is the neighborhood relationship between i and j, t is the time instant and x is the consensus variable.

When the consensus on x is reached, the information state for all robots in the network is the same, in other words:

$$\lim_{t\to\infty} |x_i(t) - x_j(t)| = 0 \quad \forall i, j \in \{1, \dots, n\}.$$

However, to reach this global consensus, some conditions must be ensured. According to Moallemi and Roy (2006) is strictly necessary that exist at least one oriented spanning tree in the network for the correct propagation of the information. This implies that in an application involving a multi-robot system, exists at least one robot from where all other robots receive information, directly or indirectly.

2.1 Consensus Based in Predictive Control

Extending the classical consensus algorithm (1), Ordoñez et al. (2012) presents a formulation of the consensus algorithm based on decentralized predictive control for the problem of *rendezvous*, where a multi-robot system must reach a common point known only by some robots. The algorithm is written as a quadratic programming function where the p occurrences of the consensus variable – in this case, the position – are estimated to not break the constraints of the problem.

Basically (1) is rewritten as a quadratic optimization problem with the functional J_i representing the consensus actions for the velocity of robot i, as follows:

$$\begin{aligned} J_i[k] = \sum_{j=1}^{n} \|x_i[k] - x_j[k]\|^2_{a_{ij}} + \|x_i[k] - x_r[k]\|^2_{a_{ir}} \\ + \|\Delta v_i[k]\|^2_{\gamma_i}, \quad i = 1, \dots, n, \qquad (2) \end{aligned}$$

where the notation $\|x\|^2_Q \equiv x^\top Q x$, and $a_{ir} > 0$ if the robot i knows the reference and γ_i is a penalization factor to the velocity variation of the robot i (Δv_i).

The first term of (2) penalizes the distance between robots, this imply when $t \to \infty$, the distance between i and j tends to zero. The second term is used to minimize the distance between the robot i and the reference point of k prediction instant, and this term performs the *rendezvous* of all robots with the reference. The third term ensure the velocity normalization to a smooth behavior without discontinuities.

The complete algorithm is presented as a distributed optimization problem where each robot must minimize the functional J_i respecting some constraints, as follows:

$$\begin{aligned} \min_{v_i} \ & \sum_{k=1}^{p} J_i[k] \qquad (3) \\ \text{s.t. } & x_i[k] = x_i[k-1] + \Delta k v_i[k] \\ & x_j[k] = x_j[0] \qquad j \neq i \\ & \Delta v_i[k] = v_i[k] - v_i[k-1] \\ & v_i^{min} \leq v_i[k] \leq v_i^{max}, \end{aligned}$$

where k is a discrete time variable, p is the prediction horizon, Δk is the update step, x is the position, v is the velocity, v_i^{max} and v_i^{min} are the velocity saturation limits for the robot i.

3. COMMUNICATION TOPOLOGY CONTROL

In this section is proposed a topology control algorithm to build a network tolerant to communication faults using a graph property called bi-connectivity. This property is a result of Menger's theorem, where the connectivity is characterized through the distinct paths between any two nodes of the network. More specifically, the theorem 1 define this idea.

Theorem 1. The *vertex connectivity* or κ-connectivity is defined when there are at least κ distinct paths between any two different nodes in a graph. As consequence of this, $\kappa - 1$ vertex can be removed from the network without disconnection.

To transform an arbitrary network in a bi-connected network ($\kappa = 2$), it requires that the "critical nodes" – which can disconnect the original graph when excluded from it – are corrected.

The following subsections presents two parts of the proposal: in 3.1 is presented the method to fix the criticality of nodes based on Traveling Salesman Problem, and in 3.2 is proposed the connectivity control algorithm based on work of Ordoñez et al. (2012) to ensure the bi-connectivity of the network.

3.1 Fixing the Criticality of Nodes Using TSP

The Traveling Salesman Problem (TSP) is a classical *NP-Hard* problem of combinatorial optimization where a salesman must travel the shortest path between all cities of a certain place, passing only once in each city and returning to the begin after reach the last city.

The path covered by the salesman is a Hamiltonian bi-connected cycle, and it can be used to generate the bi-connectivity structure in an arbitrary graph. In the literature, several algorithms to solve TSP can be found, but most of them have exponential order of complexity.

In this work the solution used to solve the TSP was presented by Miller et al. (1960), which have a quadratic order of complexity $\mathcal{O}(n^2)$. This solution is known as MTZ formulation, and is showed below: