Systems & Control Letters 119 (2018) 1–7

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Robustness to noise of distributed averaging integral controllers in power networks ☆

Erik Weitenberg *, Claudio De Persis

Engineering and Technology Institute Groningen and Jan Willems Center for Systems and Control, University of Groningen, 9747 AG Groningen, The Netherlands

ARTICLE INFO

Article history:
Received 9 January 2018
Received in revised form 2 May 2018
Accepted 17 June 2018

Keywords:
Lyapunov methods
Networked systems
Power networks
Robustness analysis
Cyber–physical systems

ABSTRACT

We investigate the robustness of distributed averaging integral controllers for optimal frequency regulation of power networks to noise in measurements, communication and actuation. Specifically, using Lyapunov techniques, we show a property related to input-to-state stability of the closed loop system with respect to this noise. Using this result, a tuning trade-off between controller performance and noise rejection is highlighted.

1. Introduction

The modern AC power system balances supply and demand in real time despite faults and fluctuations in demand, supply and transport. Adequate control techniques on the supply side ensure all units on the network enjoy a stable voltage amplitude and frequency, which is critical for safety and performance. Traditionally, these challenges have been addressed using centralized control on multiple time scales, exploiting the large inertia in generation units to compensate for the relatively small effect of fluctuations and faults.

Recently, increasing prevalence of renewable low-inertia generation units has increased volatility of supply on small and large time scales. Additionally, the emergence of so-called microgrids has introduced the compelling case of a small-scale network that can operate independently of the larger power grid, relying on small local generators. Inspired by this, an active research area has emerged to deal with this volatility in a decentralized and flexible way.

This work focuses on the secondary control layer. Various approaches for secondary control have been taken in recent years, for example primal–dual methods [1–3], internal-model control [4,5] and distributed averaging integral (DAI) control [6–8,5]. We investigate the latter approach.

Previously the performance of the DAI controller has been addressed e.g. by Flamme et al. [9], who derived a $\mathcal{H}_2$-optimum for the controller parameters under measurement noise. Similarly, Wu et al. [10] use $\mathcal{H}_2$ techniques to find the optimal communication topology for the DAI controller. Additionally, Andreasson et al. [11] performed an analysis of the linearized system. In the present work however, we additionally consider frequency noise, and provide a stability certificate for the non-linear system instead of a linearized one. This has the additional advantage of making the work applicable to other systems with similar strongly convex dynamics.

1.1. Main contribution

To our knowledge, while the DAI controller offers stability [5] and exponential convergence [12], its robustness to noise in frequency measurements, actuation and communication has not been formally established. Recently, it was shown that the so-called *leaky integral controller* offers attractive robustness properties and tuning opportunities, though it lacks exact frequency regulation [13]. In this work, we show that the DAI controller in fact satisfies an input-to-state stability with restrictions property and robustness with respect to measurement noise, and for completeness also to actuation and communication noise. The analysis builds on results from Weitenberg et al. [12], but the ISS-with restrictions result pursued in this paper, as opposed to the exponential stability

☆ This work was partially supported by the NWO-URSES project ENBARK (Grant number is 408-13-037), the DST-NWO IndoDutch Cooperation on "Smart Grids - Energy management strategies for interconnected smart microgrids" and the STW Perspectief program "Robust Design of Cyber–physical Systems" –"Energy Autonomous Smart Microgrids".

* Corresponding author.
E-mail addresses: e.r.a.weitenberg@rug.nl (E. Weitenberg), c.de.persis@rug.nl (C. De Persis).

https://doi.org/10.1016/j.sysconle.2018.06.003
0167-6911/

result from Weitenberg et al. [12], requires to extend these results to the presence of disturbances and eventually show that the Lyapunov function proposed in Weitenberg et al. [12] is indeed an ISS-Lyapunov function. The result obtained is analogous to the result obtained in Weitenberg et al. [13] for the leaky integral controller, but the use of the distributed averaging controllers considered in this paper calls for a Lyapunov function different from Weitenberg et al. [13], which requires some adjustments in the analysis. Moreover, we show how this result can be exploited in the choice of tuning parameters for the controllers, highlighting a trade-off between robustness to noise and speedy response to fluctuations in demand. This makes the DAI controller a well-performing and comparably robust alternative to the leaky integral controller, *if a communication network is available.*

The remainder of this paper is organized as follows. In Section 2, the power network model is introduced, as well as the control objectives and the DAI controller. The energy function used to analyze the closed-loop system is introduced, along with its various properties, in Section 3. In Section 4, we exploit this energy function to derive a robustness property of the closed-loop system. This leads to an interesting trade-off between robustness and performance, which is highlighted in Section 5.

2. Setting

The power network is viewed as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The systems at the nodes are partitioned into a set of $n_{\mathbf{G}}$ generators and a set of $n_{\mathbf{L}}$ loads, with $n = n_{\mathbf{G}} + n_{\mathbf{L}}$. As such, $\mathcal{V} = \mathcal{V}_{\mathbf{G}} \cup \mathcal{V}_{\mathbf{L}}$. The graph's edges represent the m physical power lines between the various power systems.

We denote the $n \times m$ incidence matrix of $\mathcal{G}$ by $\mathcal{B}$. Without loss of generality, we assume the first $n_{\mathbf{G}}$ rows of $\mathcal{B}$ correspond to the generator nodes and the others to the loads. Accordingly, we write $\mathcal{B}^\top = [\mathcal{B}_{\mathbf{G}}^\top, \mathcal{B}_{\mathbf{L}}^\top]$.

We model the power network using the Bergen–Hill equations [14,15].

$$\dot{\theta}_{\mathbf{G}} = \omega_{\mathbf{G}} \tag{1a}$$

$$M_{\mathbf{G}}\dot{\omega}_{\mathbf{G}} = -D_{\mathbf{G}}\omega_{\mathbf{G}} - \mathcal{B}_{\mathbf{G}}\Gamma \sin(\mathcal{B}^\top\theta) + u \tag{1b}$$

$$D_{\mathbf{L}}\dot{\theta}_{\mathbf{L}} = -\mathcal{B}_{\mathbf{L}}\Gamma \sin(\mathcal{B}^\top\theta) - P. \tag{1c}$$

Here, $\theta \in \mathbb{R}^n$ denotes the vector of voltage angles of the synchronous machines and loads at the nodes, relative to a frame of reference rotating at a nominal frequency ω^*, usually 50 or 60 Hz. Likewise, $\omega \in \mathbb{R}^n$ denotes a machine's frequency deviation from ω^*. D and M are diagonal $n \times n$ matrices encoding the droop gain and inertia at each node respectively, with the understanding that inertia at the load nodes is zero. As with $\mathcal{B}$, the subscript $_{\mathbf{G}}$ and $_{\mathbf{L}}$ denote partition of vectors and (diagonal) matrices into source and load nodes, i.e. $\theta = [\theta_{\mathbf{G}}^\top, \theta_{\mathbf{L}}^\top]^\top$, $\omega = [\omega_{\mathbf{G}}^\top, \omega_{\mathbf{L}}^\top]^\top$, $M = \text{block diag}(M_{\mathbf{G}}, M_{\mathbf{L}})$ et cetera. Γ is a diagonal $m \times m$ matrix encoding the susceptance B_k of the power lines and the voltage amplitudes V_i and V_j at each edge as $\Gamma_{kk} = B_k V_i V_j$, for each edge $k = (i, j) \in \mathcal{E}$. Finally, $u \in \mathbb{R}^{n_{\mathbf{G}}}$ is the control input and $P \in \mathbb{R}^{n_{\mathbf{L}}}$ is the demand at the load nodes. In the Bergen–Hill model, these load nodes are assumed to be dynamic as opposed to static impedance loads, which are subsequently absorbed into the line susceptances in a reduced network [14].

For ease of analysis, we will use the following equivalent form of (1), in which we introduce the potential function $U(\theta) = -\mathbb{1}^\top \Gamma \cos(\mathcal{B}^\top\theta)$:

$$\dot{\theta} = \omega \tag{2a}$$

$$M_{\mathbf{G}}\dot{\omega}_{\mathbf{G}} = -D_{\mathbf{G}}\omega_{\mathbf{G}} - \nabla U(\theta)_{\mathbf{G}} + u \tag{2b}$$

$$\mathbb{0} = -D_{\mathbf{L}}\omega_{\mathbf{L}} - \nabla U(\theta)_{\mathbf{L}} - P. \tag{2c}$$

Remark 1. The analysis in this paper of the behavior of the DAI controller is not limited to the swing equations seen in power networks, but to a large class of nonlinear passive networks [16]. In fact, as long as the potential function U is strongly convex and the diagonal matrices $M_{\mathbf{G}}$ and D are positive definite, the results hold.

The generator nodes are controlled by distributed averaging integral controllers [17,5,18]. These controllers are equipped with a communication network $\mathcal{G}_u = (\mathcal{V}_{\mathbf{G}}, \mathcal{E}_u)$, consisting of all generator nodes and an edge set possibly different from that of $\mathcal{G}$. Under mild assumptions (detailed later) and noise-free circumstances, these controllers minimize a quadratic cost function $C(u) = \frac{1}{2}\sum_{i\in\mathcal{V}_{\mathbf{G}}} Q_i u_i^2$ while ensuring that $\sum_{i\in\mathbf{G}} u_i = \sum_{i\in\mathbf{L}} P_i$ [18]. This allows the user to guarantee economically optimal operation, in addition to frequency regulation.

We apply the DAI controller with measurement noise ν_1. Additionally, we allow for communication noise ν_2 to occur before transmission.

$$\dot{u}_i = -\sum_{j\in\mathcal{N}i} Q_i u_i - Q_j(u_j + \nu_{2,j}) - Q_i^{-1}(\omega_i + \nu_{1,i}). \tag{3}$$

We define the noise ν_ω so that both the measurement noise and the communication noise are encapsulated in it. That is, $\nu_{\omega,i} := \nu_{1,i} - \sum_{j\in\mathcal{N}i} Q_i Q_j \nu_{2,j}$. As a result, we write the controller in vector form as

$$\dot{u} = -\mathcal{L}_u Q u - Q^{-1}(\omega_{\mathbf{G}} + \nu_\omega). \tag{4}$$

The noise $\nu_\omega = \nu_\omega(t)$ is assumed to be an infinity-norm-bounded function of time. Likewise, and for the sake of completeness, we assume the control input contains noise, replacing (2b) by

$$M_{\mathbf{G}}\dot{\omega}_{\mathbf{G}} = -D_{\mathbf{G}}\omega_{\mathbf{G}} - \nabla U(\theta)_{\mathbf{G}} + u + \nu_u, \tag{5}$$

where again, $\nu_u = \nu_u(t)$ is an infinity-norm-bounded function of time.

For ease of analysis, we now apply a coordinate transformation on the rotor angles θ. Following [12,13], instead of these, we use the offset from the average of the angles, setting $\delta := \Pi\theta := (I - \frac{1}{n}\mathbb{1}\mathbb{1}^\top)\theta$. Note that $\mathcal{B}^\top\Pi = \mathcal{B}^\top$, as $\mathcal{B}^\top\mathbb{1} = \mathbb{0}$. We will commit a slight abuse of notation by using the symbol U to also refer to the potential as a function of δ.

2.1. Steady state analysis

The system (2) in closed loop with distributed averaging integral controllers is well studied [8,18,12]. In the noise-free case, the system converges exponentially to a synchronous solution $\bar{\delta}$, $\bar{\omega} = \mathbb{0}$, $\bar{u}$ satisfying

$$\mathbb{0} = -\nabla U(\bar{\delta}) + \text{col}(\bar{u}, -P) \tag{6}$$

$$\bar{u} = Q^{-1}\mathbb{1}_{\mathbf{G}} \frac{\mathbb{1}_{\mathbf{L}}^\top P}{\mathbb{1}_{\mathbf{G}}^\top Q^{-1}\mathbb{1}_{\mathbf{G}}}, \tag{7}$$

provided the following assumption holds:

Assumption 1 (*Feasibility*)**.** There exists a vector $\bar{\delta} \in \operatorname{Im}\Pi$ such that (6)–(7) is satisfied. Moreover, there exists a $\rho \in \left(0, \frac{\pi}{2}\right)$ such that $\mathcal{B}^\top\bar{\delta}$ is in the *interior* of $\Theta := [\rho - \frac{\pi}{2}, \frac{\pi}{2} - \rho]^n$.

It will be convenient for later analysis to write the closed-loop system in incremental form [see e.g. 5], recalling that the notation $v_{\mathbf{G}}$, $v_{\mathbf{L}}$ is used to partition a vector v into subvectors for the sources and loads:

$$\dot{\delta} = \Pi\omega \tag{8a}$$