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Networked control of uncertain systems via the coarsest quantization and lossy communication[☆]

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ABSTRACT

We study a networked control problem of uncertain systems based on quantized signals sent over unreliable, lossy communication channels. The coarsest quantization is characterized for attaining the objective of quadratic stability of the overall closed-loop system in a stochastic sense. Our result indicates that the coarsest quantization is given by the logarithmic type and that more information is required through finer quantization for more uncertain systems and more unreliable channels. The characterization provides an analytic upper bound for the coarseness, which is tight for some special cases and generalizes known results for systems without uncertainties and for channels without losses.

1. Introduction

Discretization of signals through quantization has long been recognized as an important part of digital control systems, potentially introducing interesting nonlinear effects. However, more recently, studies of networked control systems have strongly motivated the consideration of quantization from the viewpoint of information constraints. Quantized signals can be measured in terms of bit rates, and a fundamental problem in this setting is to find the lowest possible amount of information that a control signal should possess to achieve control objectives. For stabilization of linear systems, various limitations in quantized signals have been found including the celebrated minimum data rate theorem (see, e.g., [1,2]).

One line of research initiated by the work of [3] is to characterize the coarseness in quantizers to guarantee quadratic stability of networked control systems. The coarsest quantizer is known to take the logarithmic form with the special structure that quantization is fine around the origin, but becomes coarser for larger inputs. This structure is intuitively reasonable for the purpose of controlling the states towards the origin. Moreover, coarseness in quantization has a limitation depending on the level of instability in the plant dynamics, that is, more unstable systems require finer quantization. Related works include [4] which studies control performance with quadratic costs and the H^∞ norm, and [5] which takes account of random data packet losses in the characterization. Further, a sampled-data control approach is developed in [6] to guarantee quadratic stability in the continuous-time domain.

The focus of this paper is on dealing with uncertainties in control systems. Several approaches can be found in the literature of quantized control dealing with this aspect. The first approach is based more on the perspective of robust control. For data-rate limited control, the work [7] provides an upper bound on the data rate to stabilize quantized feedback systems. For control with coarse quantization, the work [8] considers robust control problems for both static and dynamic logarithmic quantizers. Stabilization of stochastic systems with time-varying uncertainties can be found in [9]. More recently, in [10,11], the minimum data rate was derived for parametrically uncertain systems controlled over unreliable channels. An approach based on switching control is developed in [12], where a suitable controller is chosen by a switching controller which estimates plant parameters on-line. On the other hand, adaptive control strategies can handle uncertainties with fewer constraints and have been studied in [13,14] for linear plants and [15,16] for nonlinear plants.

In this paper, following the formulation of [3], we study the stabilization problem for a class of uncertain linear systems with parametric uncertainties [17]. It is a generalization of our recent work [18] to the case where the network is unreliable in that data packets transmitted may become lost stochastically; the non-uncertain case was dealt with in [5]. Our goal is to characterize the coarsest quantization scheme for such systems to achieve stochastic quadratic stability under a given quadratic Lyapunov function.

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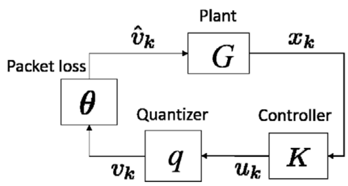

Fig. 1. Networked control system.

It will be demonstrated that more uncertain systems or more unreliable channels mandate finer quantization for stabilization. In our development, we show that the consideration of packet losses in this context complicates the problem, making it difficult to obtain closed form solutions that hold in general. However, our study leads us to an analytic bound on the coarseness for the special cases when the level of loss probability or uncertainties is small. This bound generalizes those of [3,5,18]. The relations among them will be discussed in detail.

This paper is organized as follows. In Section 2, we formulate the problem of quantized control for uncertain networked systems over lossy channels. In Section 3, we provide bounds on the coarsest quantizer for a given Lyapunov function. The proofs of the results are given in Section 4. In Section 5, we illustrate the proposed approach via a numerical example. Finally, concluding remarks are given in Section 6. This paper is based on [19], but contains the full proofs of the results and more discussions.

2. Problem formulation

Consider the networked control system depicted in Fig. 1. We first describe the system setup briefly. The plant G is a single-input single-output discrete-time linear system and has uncertain parameters. The control signal $u_k \in \mathcal{R}$ is generated by the controller K and then quantized by the quantizer q before being sent over the network. In the channel, we assume that there is no network latency and the data rate is high enough to transfer all the data within the sampling period. To simplify the problem, we assume that the word length is infinite. That is, the quantized signal takes discrete values which form an infinite set.

The plant G is an n-dimensional autoregressive system with uncertain parameters, which may be time varying, as

$$y_{k+1} = a_{1,k}y_k + a_{2,k}y_{k-1} + \cdots + a_{n,k}y_{k-n+1} + \hat{v}_k, \tag{1}$$

where $\hat{v}_k \in \mathcal{R}$ is the input and $y_k \in \mathcal{R}$ is the output. The parameters $a_{i,k}$ are uncertain and take the form $a_{i,k} = a_i^* + \Delta_{i,k}$, $i = 1, \ldots, n$, $k \in \mathcal{Z}_+$, where a_i^* is the nominal part and $\Delta_{i,k}$ is the uncertain part. Let $\Delta_k \in \mathcal{R}^{1\times n}$ be the uncertainty vector given by $\Delta_k := \begin{bmatrix}\Delta_{n,k} & \Delta_{n-1,k} & \cdots & \Delta_{1,k}\end{bmatrix}$. We assume that the uncertainty is contained in the bounded set $\mathcal{D} \subset \mathcal{R}^n$ as

$$\Delta_k \in \mathcal{D}, \ \forall k \in \mathcal{Z}_+. \tag{2}$$

The uncertain plant (1) can be expressed in the controllable canonical form as

$$x_{k+1} = A(\Delta_k)x_k + b\hat{v}_k, \ \ y_k = cx_k,$$

where $x_k := [y_{k-n+1} \ y_{k-n+2} \ \cdots \ y_k]^T \in \mathcal{R}^n$ is the state, and the system matrices $A(\Delta_k) \in \mathcal{R}^{n\times n}$, $b \in \mathcal{R}^{n\times 1}$, and $c \in \mathcal{R}^{1\times n}$ are given by

$$A(\Delta_k) := \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ a_{n,k} & a_{n-1,k} & \cdots & a_{1,k} \end{bmatrix}, \ b := \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \ c := b^T. \tag{3}$$

We make the assumption that the matrix $A(\Delta_k)$ has at least one unstable eigenvalue for any Δ_k. For later use, denote the nominal A-matrix by $A^* := A(0)$.

The control signal is generated over the network. First, the controller K provides the control input u_k by

$$u_k = \sum_{i=1}^{n} f_i y_{k-i+1} = fx_k, \tag{4}$$

where $f \in \mathcal{R}^{1\times n}$ is the feedback gain to be designed.

For this control signal to be sent over a network channel, it is quantized by the memoryless quantizer $q(\cdot)$ as

$$v_k = q(u_k). \tag{5}$$

Here, the quantizer is a piecewise constant function to be designed so as to minimize the necessary communication under the measure of its *coarseness* [3] given by

$$d_q := \lim_{\epsilon\to 0} \sup \frac{\sharp u[\epsilon]}{-\ln \epsilon}, \tag{6}$$

where $\sharp u[\epsilon]$ denotes the number of quantized levels of q in the interval $[\epsilon, 1/\epsilon]$. Hence, it measures the density of the discrete values according to a certain logarithmic weight.

The channel is unreliable as the data packets transmitted may get lost with a certain loss probability [20]. When the packet is lost, the control input applied at the actuator becomes zero. The loss process denoted by $\theta_k \in \{0, 1\}$ for $k \in \mathbb{Z}_+$ is independent and identically distributed (i.i.d.) and is specified as $\text{Prob}\{\theta_k = 0\} = p$ and $\text{Prob}\{\theta_k = 1\} = 1 - p$, where $p \in (0, 1)$ is the loss probability. Based on this process, the control input is given by

$$\hat{v}_k = \theta_k v_k.$$

In our networked control problem, the objective is to achieve stabilization of the closed-loop system under the notion of stochastic quadratic stability as defined next.

Consider the nonlinear discrete-time time-varying system given by

$$x_{k+1} = g(x_k, \theta_k, k), \tag{7}$$

where $x_k \in \mathbb{R}^n$ is the state, and $\theta_k \in \{0, \ldots, N-1\}$ is the mode of the system at time $k \in \mathbb{Z}_+$. The mode is a stochastic process and is i.i.d. as $p_i = \text{Prob}\{\theta_k = i\}$. The function $g(x, \theta, k)$ satisfies $g(0, \theta, k) = 0$ for any θ and k, and thus the origin $x = 0$ of the system is an equilibrium.

Definition 1. For the system in (7), the origin is said to be stochastically quadratically stable if there exist a positive-definite function $V(x) := x^T Px$ and a positive-definite matrix R such that

$$\text{E}\left[V(x_{k+1})|\, x_k\right] - V(x_k) \le -x_k^T R x_k, \quad \forall x_k \in \mathbb{R}^n. \tag{8}$$

The condition (8) is a sufficient condition for the origin of the system in (7) to be mean square stable, i.e., for every initial state x_0, it holds $\lim_{k\to\infty} E[\|x_k\|^2|\, x_0] = 0$, where $\|\cdot\|$ is the Euclidean norm. For stochastic systems, it is essential that in (8), the absolute averaged decreasing rate of a Lyapunov function V is bounded by a quadratic function of x. Unlike in the deterministic case, taking $R = 0$ in (8) is not enough to guarantee stability [5].

The quantized control problem of the paper can be stated as follows: For the networked uncertain control system in Fig. 1, given the loss probability p, find the quantizer $q(\cdot)$ which is the coarsest in the sense of (6) and guarantees stochastic quadratic stability of the origin.

The interesting aspect of the approach based on the quadratic stability notion above is that it leads us to analytic limitations on the coarseness of quantizers. The work of [5] considered the case