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Robust sensitive fault detection and estimation for single-rate and multirate nonlinear sampled-data systems

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ABSTRACT

This paper investigates the problem of fault detection and estimation for nonlinear sampled-data systems in the presence of unknown exogenous inputs. Both cases of single-rate and multirate sampling are treated and general observer-based frameworks are provided using discrete-time approximation to estimate fault signals of any sources. We show that these frameworks are input-to-error stable with respect to the estimation error and can simultaneously enhance robustness against unknown inputs and sensitivity to faults in a mixed H_-/H_∞ sense for the unknown exact discrete-time model of the plant. Our results are then applied to a class of Lipschitz nonlinear systems with a refined Euler approximate model to derive sampled-data fault estimation techniques where stability and H_-/H_∞ optimization are ensured using linear matrix inequalities (LMIs). Simulation results of a flexible joint robot illustrate the effectiveness of the proposed methodologies.

1. Introduction

Due to the inevitable occurrence of malfunctions in practical control systems and the significance of promptly detecting and identifying the faulty signals to meet performance and safety requirements and to avoid catastrophic incidents, extensive research has been devoted to fault detection and estimation (FDE) design in a variety of industries such as aerospace, robotics, power plants, and process control (see e.g., [1,2] for a survey of recent results). One critical and well-recognized aspect of any fault detection technique is to be *sensitive* to faults and *robust* against unknown inputs such as disturbances, and measurement noise.

In a typical fault detection application a continuous-time plant is controlled using a digital computer interfaced using sample and hold devices. The FDE algorithm is implemented in discrete-time at the controller side. We will refer to this configuration as a sample-data system [3]. Sampled-data fault detection is therefore of paramount importance and has been extensively studied during the past decade, specially for LTI systems ([4,5] to name a few). For nonlinear systems, on the other hand, the fault detection literature has been primarily focussed on either continuous-time systems, where both plant and controller are assumed to work in continuous-time, or discrete-time systems, where both plant and controller are represented in discrete-time. See for example [6–8]. Nonlinear sampled-data fault detection research has remained relatively unexplored. Note that, unlike the linear case, the exact discrete-time model of a nonlinear plant is typically not available due to the non-existence of closed-form solutions of the initial value problem. Thus, a fault detection scheme for a nonlinear system must account for the fact that the predictions of the approximate discrete-time models will deviate from the true plant output or state, even during normal operation. Sampled-data control of nonlinear plants using approximate models has been studied by Nesic and Teel in a series of papers (see [9–12]). See also [13–17] for related results for multirate systems. Some results on sampled-data fault-detection for nonlinear plants were recently reported in [18]. This reference proposes a sampled-data fault estimation algorithm for a class of Lipschitz systems using Euler approximate models and relying on the sampled-data observer proposed by [12]. However, the effect of unknown inputs, sensor faults and fault sensitivity are not considered.

In this paper our goal is to present a prescriptive FDE framework for nonlinear sampled-data systems in the presence of exogenous disturbances and actuator, sensor and system faults, which simultaneously take the issues of sensitivity and robustness into account. We first study the single-rate case where all the inputs and outputs are sampled at the same sampling rate and propose a general observer-based structure for FDE design based on an

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approximate discrete-time plant model. We derive sufficient Lyapunov and *consistency* conditions which guarantee input-to-state stability (ISS) from disturbances to the fault estimation error in spite of unknown exact discrete-time plant model [14,19]. The robustness and sensitivity of the sampled-data FDE scheme is evaluated by applying H_-/H_∞ optimization (see [8,20] for different techniques), where H_∞-norm and H_--index measure the minimum influence of the fault and maximum influence of the unknown inputs on the residuals, respectively. Then, as a special case we employ our general FDE setup to a class of Lipschitz nonlinear systems to establish a constructive design algorithm via Euler approximation and linear matrix inequalities (LMIs). This LMI formulation, which minimizes the H_--norm and maximizes the H_--norm, can be solved using efficient numerical algorithms and are much simpler to solve than the Hamilton–Jacobi inequalities in [8], do not depend on the exact discrete-time model of the plant assumed in [8], and covers [18] as a spacial case with zero disturbance and no sensor faults.

The second part of the paper deals with multirate nonlinear systems where input and output signals are sampled at different frequencies [13–17]. One crucial property that is more challenging to satisfy in the multirate case is to detect a fault in a timely manner as close as possible to its occurrence in real time. There are several studies on the FDE problem for linear multirate systems [4,21–23]. However, to the best of our knowledge, no formal study of multirate fault detection for nonlinear systems has ever been reported in the literature. Inspired by the general framework for multirate observers design in [14], we extend our results in the first half of the article to construct a multirate sampled-data FDE setup that consists of a fast-rate fault estimator, modified residual error and two periodic switches to predict the missing inter-sample inputs and outputs. It is proved that if the given fast-rate FDE is input-to-error stable and satisfies a certain H_-/H_∞ index, then under standard Lyapunov-ISS, continuity and consistency assumptions, the proposed multirate FDE preserves stability and the H_-/H_∞ performance criterion for the unknown exact discrete-time model of the multirate plant. Finally, considering a Lipschitz structure, we formulate a systematic multirate FDE scheme using LMIs.

Our main contributions compared with the previous results on this topic are as follows:

- Laying out general guidelines for designing fault detectors or estimators for nonlinear sampled-data systems that take the unknown exact discrete-time plant model into account (ignored in the current literature). These guidelines are then applied to Lipschitz nonlinear systems to obtain some algorithmic mechanisms.
- Covering both single-rate and multirate sampling scenarios. The multirate case has not been studied in the literature so far.
- Exploiting a new notion for stability analysis of fault estimation schemes with guaranteed robustness against disturbances and sensitivity to faults.

2. Problem setting and preliminaries

Consider a general nonlinear system described by

$$G: \begin{cases} \dot{x}(t) = f(x(t), u(t), d(t), v(t)) & \text{(a)} \\ y(t) = g(x(t), u(t), d(t), v(t)) & \text{(b)} \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the measured output, $d \in \mathbb{R}^q$ stands for the unknown exogenous inputs, and $v \in \mathbb{R}^l$ represents the fault vector to be detected which can be sensor, actuator, or component faults. Also, f and g are continuously differentiable functions zero at zero, and d and v are assumed to be $\mathcal{L}_2$-norm bounded. The dynamical plant G is controlled via the standard sampled-data configuration of Fig. 1 with a zero-order hold device (D/A converter) $\mathcal{H}$ and a sampler (A/D converter) $\mathcal{S}$ and the FDE block to be designed.

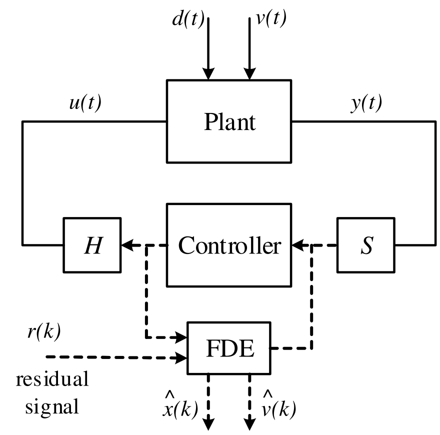

Fig. 1. Structure of the single-rate sampled-data FDE.

We first study the single-rate case where all the sample and hold devices operate at the same sampling period $T > 0$. The multirate case will be addressed in Section 5. For the purpose of sampled-data *fault detection*, we generate the discrete-time residual signal $r(k)$ to reflect the occurrence of a fault:

$$\|r(k)\| = \begin{cases} \text{zero or very small}, & v(t) = 0 \\ \text{nonzero or large}, & v(t) \neq 0, \end{cases} \quad \forall u, y, d \tag{2}$$

Regardless which approach is followed for residual generation (see [6] for survey of various schemes), the *fundamental problem of residual generation (FPRG)* imposes the following critical conditions on the residual $r(k)$:

(i) Robustness to the exogenous input $d(t)$
(ii) Sensitivity to the fault vector $v(t)$
(iii) Convergence of $r(k)$ in the presence of exogenous inputs for the fault-free case, i.e., $v = 0$

The exact discrete-time model of (1)(a)–(b) is given by

$$x(k+1) = x(k) + \int_{kT}^{(k+1)T} f(x(\tau), u(k), d(\tau), v(\tau))\mathrm{d}\tau$$
$$=: F^e_T(x(k), u(k), d[k], v[k]) \tag{3}$$
$$y(k) = g(x(k), u(k), d(k), v(k)) \tag{4}$$

where F^e_T is not available in an explicit closed form for most practical cases. Instead, we will rely on an approximate discrete-time model $x(k+1) = F^a_{T,h}(x(k), u(k), d[k], v[k])$ to develop our results, where h is the integration period used to refine the model. The discrepancy of the exact and approximate models is evaluated via the following consistency property [9].

Definition 1. The approximate model $F^a_{T,h}$ is said to be one step consistent with F^e_T if there exist a class-$\mathcal{K}$ function $\rho(\cdot)$ and $T_1 > 0$ such that given any strictly positive numbers $(\delta_1, \delta_2, \delta_3, \delta_4)$ and each fixed $T \in (0, T_1]$, there exists $h_1 \in (0, T]$ such that

$$|F^e_T(x, u, \bar{d}, \bar{v}) - F^a_{T,h}(x, u, \bar{d}, \bar{v})| \leq T\rho(h) \tag{5}$$

for all $|x| \leq \delta_1$, $|u| \leq \delta_2$, $\|d\|_\infty \leq \delta_3$, $\|v\|_\infty \leq \delta_4$ and $h \in (0, h_1]$.

Note that the one-step consistency can be verified using sufficient conditions presented in [9,10] based on the Euler approximation without knowing the exact model F^e_T.