

# Nonlinear network dynamics for interconnected micro-grids

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## ABSTRACT

This paper deals with transient stability in interconnected micro-grids. The main challenge is to understand the impact of the connectivity of the graph and model nonlinearities on transient and steady-state behavior of the system as a whole. The contribution of this paper is three-fold. First, we provide a robust classification of transient dynamics for different intervals of the parameters for a single micro-grid. We prove that underdamped dynamics and oscillations arise when the damping coefficient is below a certain threshold which we calculate explicitly as function of the product between the inertia coefficient and the synchronization parameter. Second, for interconnected micro-grids, under the hypothesis of homogeneity, we prove that the transient dynamics mimics a consensus dynamics. We provide bounds on the damping coefficient characterizing underdamped and overdamped consensus. Third, we extend the study to the case of disturbed measurements due to hacking or parameter uncertainties. We introduce model nonlinearities and prove absolute stability.

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## 1. Introduction

This paper investigates transient stability of interconnected micro-grids. First, we develop a model for a single micro-grid combining swing dynamics and synchronization, inertia and damping parameters. We focus on the main characteristics of the transient dynamics especially the insurgence of oscillations in underdamped transients. The analysis of the transient dynamics is then extended to multiple interconnected micro-grids. By doing this, we relate the transient characteristics to the connectivity of the graph. We also investigate the impact of the disturbed measurements (due to hacking or parameter uncertainties) on the transient.

### 1.1. Main theoretical findings

The contribution of this paper is three-fold. First, for the single micro-grid, we identify intervals for the parameters within which the behavior of the transient stability has similar characteristics. This shows robustness of the results and extends the analysis to cases where the inertia, damping and synchronization parameters are uncertain. In particular, we prove that underdamped dynamics and oscillations arise when the damping coefficient is below a certain threshold which we calculate explicitly. The threshold is obtained as function of the product between the inertia coefficient and the synchronization parameter.

Second, for interconnected micro-grids, under the hypothesis of homogeneity, we prove that the transient stability mimics a consensus dynamics and provide bounds on the damping coefficient for the consensus value to be overdamped or underdamped. This result is meaningful as it sheds light on the insurgence of topology-induced oscillations. These bounds depend on the topology of the grid and in particular on its maximum connectivity, namely, the maximum number of links over all the nodes of the network. We also observe that the consensus value changes dramatically with increasing damping coefficient. This implies that the micro-grid, if working in islanding mode, can synchronize to a frequency which deviates from the nominal one of 50 Hz. This finding extends to smart-grids with different inertia but same ratio between damping and inertia coefficient.

Third, we extend the analysis to the case where both frequency and power flow measurements are subject to disturbances. Using a traditional technique in nonlinear analysis and control, we isolate the nonlinearities in the feedback loop, and analyze stability under some mild assumptions on the nonlinear parameters. The obtained result extends also to the case where the model parameters like synchronization coefficient, inertia and damping coefficients are uncertain. This adds robustness to our findings and proves validity of the results even under modeling errors. The nonlinear framework accommodates also output limits assuming that they can be modeled using first and third quadrant sector nonlinearities.

To corroborate our theoretical findings, a case study from the Nigerian distribution network is discussed.

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## 1.2. Related literature

This study leverages on previous contributions of the authors in [1] and [2]. In [1] the author studies flexible demand in terms of a population of smart thermostatically controlled loads and shows that the transient dynamics can be accommodated within the mean-field game theory. In [2] the author extends the analysis to uncertain models involving both stochastic and deterministic (worst-case) analysis approaches. The analysis of interconnected micro-grids builds on previous studies provided in [3]. Here the authors link transient stability in multiple electrical generators to synchronization in a set of coupled Kuramoto oscillators. We refer the reader to the survey [4]. The connection between Kuramoto oscillators and consensus dynamics is addressed in [5]. A game perspective on Kuramoto oscillators is in [6], where it is shown that the synchronization dynamics admits an interpretation as game dynamics with equilibrium points corresponding to Nash equilibria. The observed deviation of the consensus value from the nominal mains frequency in the case of highly overdamped dynamics can be linked to inefficiency of equilibria as discussed in [7]. This study has benefited from some graph theory tools and analysis efficiently and concisely exposed in [8]. The model used in this paper, which combines swing dynamics with synchronization, inertia and damping parameters has been inspired by [9]. The numerical analysis has been conducted using data provided in [10].

This paper is organized as follows. In Section 2, we model a single micro-grid. In Section 3, we turn to multiple interconnected micro-grids. In Section 4, we analyze the impact of measurement disturbances. In Section 5, we provide numerical studies on the Nigerian grid. Finally, in Section 6, we provide conclusions.

## 2. Model of a single micro-grid

Consider a single micro-grid connected to the network, refer to it as the  $i$ th micro-grid. Let us denote by  $P_i$  the power flow into the  $i$ th micro-grid. Also let  $f_i$  be the frequency deviation of micro-grid  $i$  and  $\phi$  a virtual signal representing the frequency of the mains. From [11, Chapter 3] and [12, Chapter 3.9.2], by applying dc approximation, the power  $P_i$  evolves according to

$$\dot{P}_i = T_{ij}(\phi - f_i) = T_{ij}e_{ij}, \quad (1)$$

where  $T_{ij}$  is the synchronizing coefficient. This coefficient is obtained as the inverse of the transmission reactance between micro-grid  $i$  and the mains. In other words, the power  $P_i$  depends on the frequency error  $e_{ij} = \phi - f_i$ . The physical intuition of this is that in response to a positive error we have power injected into the  $i$ th micro-grid. Vice versa, a negative error induces power out from micro-grid  $i$ .

The dynamics for  $f_i$  follows a traditional swing equation:

$$\dot{f}_i = -\frac{D_i}{M_i}f_i + \frac{P_i}{M_i}, \quad (2)$$

where  $M_i$  and  $D_i$  are the inertia and damping constants of the  $i$ th micro-grid, respectively. By denoting  $f_i = x_1^{(i)}$ ,  $P_i = x_2^{(i)}$ ,  $\phi = x_1^{(j)}$ , and by considering  $\phi$  as an exogenous input to the  $i$ th micro-grid, the dynamics of the  $i$ th micro-grid reduces to the following second-order system:

$$\begin{bmatrix} \dot{x}_1^{(i)} \\ \dot{x}_2^{(i)} \end{bmatrix} = \begin{bmatrix} -\frac{D_i}{M_i} & \frac{1}{M_i} \\ -T_{ij} & 0 \end{bmatrix} \begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \end{bmatrix} + \begin{bmatrix} 0 \\ T_{ij} \end{bmatrix} x_1^{(j)}. \quad (3)$$

**Theorem 1.** Dynamics (3) is asymptotically stable. Furthermore, let  $D_i > 2\sqrt{T_{ij}M_i}$  then the origin is an asymptotically stable node. Vice versa, if  $D_i < 2\sqrt{T_{ij}M_i}$  then the origin is an asymptotically stable spiral.

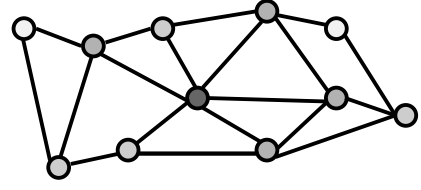


Fig. 1. Graph topology indicating smart-grids and interconnections.

**Proof.** For the first part, stability derives from  $\text{Tr}(A) = -\frac{D_i}{M_i}$ , where  $\text{Tr}(A)$  is the trace of matrix  $A$  and from  $\Delta(A) = \frac{T_{ij}}{M_i} > 0$ , where  $\Delta(A)$  is the determinant of matrix  $A$ . Let us recall that stability depends on the eigenvalues of  $A$  and that the expression of the eigenvalues is given by

$$\begin{aligned} \lambda_{1,2} &= \frac{\text{Tr}(A) \pm \sqrt{\text{Tr}(A)^2 - 4\Delta(A)}}{2} \\ &= \frac{1}{2} \left( -\frac{D_i}{M_i} \pm \sqrt{\left(\frac{D_i}{M_i}\right)^2 - 4\frac{T_{ij}}{M_i}} \right). \end{aligned} \quad (4)$$

As for the rest of the proof, we know that if  $D_i > 2\sqrt{T_{ij}M_i}$  then  $\text{Tr}(A)^2 > 4\Delta(A)$  and the origin is an asymptotically stable node.

The last case is when  $D_i < 2\sqrt{T_{ij}M_i}$  which implies  $\text{Tr}(A)^2 < 4\Delta(A)$  and therefore the origin is an asymptotically stable spiral.  $\square$

The above theorem identifies intervals for the parameters within which the behavior of the transient stability is unchanged. This provides robustness to our results and extends the analysis to cases where the inertia, damping and synchronization parameters are uncertain.

## 3. Multiple interconnected micro-grids

Let us now consider a network  $G = (V, E)$  of interconnected smart-grids, where  $V$  is the set of nodes, and  $E$  is the set of arcs. Fig. 1 displays an example of interconnection topology. Nodes represent smart-grids units and arcs represent power lines interconnections. We use shades of gray to emphasize different levels of connectivity of the smart-grids. The connectivity of a grid is indicated by the degree of the node. We recall that for undirected graphs the degree of a node is the number of links with an extreme in node  $i$ . We denote by  $d_i$  the degree of node  $i$ .

Building on model (3) developed for the single grid, we derive the following macroscopic dynamics for the whole grid:

$$\underbrace{\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix}}_A = \underbrace{\begin{bmatrix} -\text{Diag}\left(\frac{D_i}{M_i}\right) & \text{Diag}\left(\frac{1}{M_i}\right) \\ -L & 0 \end{bmatrix}}_A \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \quad (5)$$

In the above set of equations, the block matrix  $L = [l_{ij}]_{i,j \in \{1, \dots, n\}}$  is the graph-Laplacian matrix:

$$\begin{aligned} L &:= \begin{bmatrix} T_{11} & \dots & -T_{1n} \\ & \ddots & \\ -T_{n1} & \dots & T_{nn} \end{bmatrix}, \\ l_{ij} &= \begin{cases} -T_{ij} & \text{if } i \neq j, \\ \sum_{h=1, h \neq i} T_{ih} & \text{if } i = j. \end{cases} \end{aligned} \quad (6)$$

Its row-sums are zero, its diagonal entries are nonnegative, and its non-diagonal entries are nonpositive. We also recall that

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