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Safe economic model predictive control of nonlinear systems[☆]

Zhe Wu [a], Helen Durand [b], Panagiotis D. Christofides [a,c,*]

[a] *Department of Chemical and Biomolecular Engineering, University of California, Los Angeles, CA, 90095-1592, USA*
[b] *Department of Chemical Engineering and Materials Science, Wayne State University, Detroit, MI 48202, USA*
[c] *Department of Electrical and Computer Engineering, University of California, Los Angeles, CA 90095-1592, USA*

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ABSTRACT

This work focuses on the design of a new class of economic model predictive control (EMPC) systems for nonlinear systems that address simultaneously the tasks of economic optimality, safety and closed-loop stability. This is accomplished by incorporating in the EMPC an economics-based cost function and Control Lyapunov-Barrier Function (CLBF)-based constraints that ensure that the closed-loop state does not enter unsafe sets and remains within a well-characterized set in the system state-space. The new class of CLBF-EMPC systems is demonstrated using a nonlinear chemical process example.

1. Introduction

Since operational efficiency and increasing energy consumption are becoming crucially important issues in the chemical and petrochemical industry, a model-based feedback control strategy, economic model predictive control (EMPC), has been proposed as an efficient method to address process control problems integrated with dynamic economic optimization of the process (e.g., [1–4]). EMPC allows the chemical process to be operated in a time-varying fashion (off steady-state) to dynamically optimize process economic performance, and incorporates constraints that guarantee closed-loop stability and feasibility within an explicitly-defined estimate of the closed-loop stability region under an appropriate control law (e.g., a Lyapunov-based feedback control law).

On the other hand, process operational safety is of significant importance in the chemical process industries due to the disastrous consequences unsafe operation has for both lives and property [5,6]. Despite the widely-used safety protection instruments applied in industry (e.g., alarm systems, emergency shut-down systems, and safety relief devices), the potential for unsafe process operation caused by multi-variable interactions motivates the development of improved process design and process operational safety methods. Several recent works have proposed a method that combines control and safety within a systems framework using a function termed the Safeness Index that indicates the relative safeness of the process states (e.g., [7]). It has been shown that under the EMPC integrated with Safeness Index function-based constraints, closed-loop stability and operational safety of nonlinear chemical process systems can be achieved. At this stage, however, the problem of incorporating safety region constraints in EMPC to deal with process operational safety and ensuring the feasibility of the resulting EMPC a priori has not been studied.

Recently, a control method termed Control Lyapunov-Barrier Function (CLBF)-based control (e.g., [8,9]) has been proposed for the control system design that accounts for both closed-loop stability and safety. Typically, CLBFs can be formulated through the weighted average of a Control Lyapunov Function (CLF) and a Control Barrier Function (CBF), and therefore they possess similar stabilizability and safety properties to those associated with the CLF and CBF from which they can be derived. In a recent work [10], a CLBF was combined with tracking MPC to drive the state of a closed-loop nonlinear system to its set point while avoiding the unsafe region in state-space. At this stage, however, it remains an open issue to incorporate a CLBF into EMPC design to obtain closed-loop stability, process operational safety, and optimal economic benefits simultaneously.

These safety and stability considerations motivate the development of CLBF-based EMPC that integrates a Control Lyapunov-Barrier Function with EMPC to account for input constraints, safety considerations, and the stability of the closed-loop system. The proposed methodology is applied to a nonlinear chemical process example to demonstrate the ability of the CLBF-EMPC to operate the process in an economically optimal manner while avoiding an unsafe region in the state-space.

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* Corresponding author at: Department of Chemical and Biomolecular Engineering, University of California, Los Angeles, CA 90095-1592, USA.
E-mail addresses: wuzhe@g.ucla.edu (Z. Wu), helen.durand@wayne.edu (H. Durand), pdc@seas.ucla.edu (P.D. Christofides).

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2. Preliminaries

2.1. Notation

The notation $|\cdot|$ is used to denote the Euclidean norm of a vector. x^T denotes the transpose of x. The notation $L_f V(x)$ denotes the standard Lie derivative $L_f V(x) := \frac{\partial V(x)}{\partial x} f(x)$. Set subtraction is denoted by "\", i.e., $A \setminus B := \{x \in \mathbf{R}^n \mid x \in A, x \notin B\}$. $\emptyset$ signifies the null set. Given a set $\mathcal{D} \subset \mathbf{R}^n$, we denote the boundary of $\mathcal{D}$ by $\partial\mathcal{D}$, and the closure of $\mathcal{D}$ by $\overline{\mathcal{D}}$. $\mathcal{B}_\beta(\epsilon) := \{x \in \mathbf{R}^n \mid |x - \epsilon| < \beta\}$ is an open ball around ϵ with radius of β. The function $f(\cdot)$ is of class $\mathcal{C}^1$ if it is continuously differentiable in its domain. A continuous function $\alpha : [0, a) \to [0, \infty)$ is a class $\mathcal{K}$ function if it is strictly increasing and is zero only when evaluated at zero. A scalar function $V : \mathbf{R}^n \to \mathbf{R}$ is proper if the set $\{x \in \mathbf{R}^n \mid V(x) \le k\}$ is compact $\forall\, k \in \mathbf{R}$, or equivalently, V is radially unbounded [11].

2.2. Class of systems

The class of continuous-time nonlinear systems considered is described by the following system of first-order nonlinear ordinary differential equations:

$$\dot{x} = f(x) + g(x)u + h(x)w, \;\; x(t_0) = x_0 \tag{1}$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^m$ is the manipulated input vector, and $w \in W$ is the disturbance vector, where $W := \{w \in \mathbf{R}^q \mid |w| \le \theta, \; \theta \ge 0\}$. The control action constraint is defined by $u \in U := \{u_i^{\min} \le u_i \le u_i^{\max}, i = 1, \ldots, m\}$. $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are sufficiently smooth vector and matrix functions of dimensions $n \times 1$, $n \times m$, and $n \times q$, respectively. Throughout the manuscript, the initial time t_0 is taken to be zero ($t_0 = 0$), and it is assumed that $f(0) = 0$, and thus, the origin is a steady-state of the nominal (i.e., $w(t) \equiv 0$) system of Eq. (1) (i.e., $(x_s^*, u_s^*) = (0, 0)$, where x_s^* and u_s^* represent the steady-state state and input vectors, respectively).

2.3. Control Lyapunov-Barrier Function (CLBF)

In this work, we develop an economic model predictive control design that utilizes a Control Lyapunov-Barrier Function [12] in designing the constraints to maintain the closed-loop state in a safe operating region at all times in the following sense:

Definition 1. Consider the system of Eq. (1) with input constraints $u \in U$, and an open set $\mathcal{D}$ in state-space within which it is unsafe for the system to be operated. If there exists a control law $u \in U$ such that the state trajectories of the system for any initial state $x(0) = x_0 \in \mathcal{U} \subset \mathbf{R}^n$ satisfy $x(t) \in \mathcal{U}, \forall\, t \ge 0$, where $\mathcal{U} \cap \mathcal{D} = \emptyset$, we say that process operational safety is achieved in the sense that the control law u maintains the process state within a safe operating region $\mathcal{U}$ at all times.

Based on the original Control Lyapunov-Barrier Function (CLBF) [12] that was developed for the nominal system of Eq. (1) with $w(t) \equiv 0$, in this manuscript, we propose a constrained CLBF that accounts for the input constraints $u \in U$ in the system of Eq. (1), and is stabilizing even in the presence of small bounded disturbances $w(t)$. Specifically, the definition of a constrained CLBF is as follows:

Definition 2. Given a set of unsafe points in state-space $\mathcal{D}$ (i.e., the unsafe region), a proper, lower-bounded and $\mathcal{C}^1$ function $W_c(x)$: $\mathbf{R}^n \to \mathbf{R}$ is a constrained CLBF if it has a minimum at the origin and satisfies the following properties:

$$W_c(x) > \rho_c, \;\; \forall\, x \in \mathcal{D} \subset \phi_{uc} \tag{2a}$$

$$L_f W_c(x) < 0,$$
$$\forall\, x \in \{z \in \phi_{uc} \setminus (\mathcal{D} \cup \{0\} \cup \mathcal{X}_e) \mid L_g W_c(z) = 0\} \tag{2b}$$

$$\mathcal{U}_{\rho_c} := \{x \in \phi_{uc} \mid W_c(x) \le \rho_c\} \neq \emptyset \tag{2c}$$

$$\overline{\phi_{uc} \setminus (\mathcal{D} \cup \mathcal{U}_{\rho_c})} \cap \overline{\mathcal{D}} = \emptyset \tag{2d}$$

where $\rho_c \in \mathbf{R}$ and $\mathcal{X}_e := \{x \in \phi_{uc} \setminus (\mathcal{D} \cup \{0\}) \mid \partial W_c(x)/\partial x = 0\}$ is a set of states where $L_f W_c(x) = 0$ due to $\partial W_c(x)/\partial x = 0$. ϕ_{uc} is defined to be the union of the origin, $\mathcal{X}_e$ and the set where the time-derivative of $W_c(x)$ is negative with constrained input: $\phi_{uc} = \{x \in \mathbf{R}^n \mid \dot{W}_c(x(t), u(t)) = L_f W_c + L_g W_c u < 0, u = \Phi(x) \in U\} \cup \{0\} \cup \mathcal{X}_e$. $\Phi(x)$ is a nonlinear feedback control law, which will be discussed in detail in the next subsection.

2.4. Stabilization and safety via CLBF

We assume that there exists a feedback control law $u = \Phi(x) \in U$ (e.g., the universal Sontag control law [13]) such that the state of the closed-loop nominal system of Eq. (1) is bounded in a level set of $W_c(x)$ embedded in an open neighborhood D that includes the origin in its interior in the sense that there exists a $\mathcal{C}^1$ constrained Control Lyapunov-Barrier function $W_c(x)$ that has a minimum at the origin and where the following inequalities hold for all $x \in D$:

$$\alpha_1(|x|) \le W_c(x) - \rho_0 \le \alpha_2(|x|), \tag{3a}$$

$$\frac{\partial W_c(x)}{\partial x} F(x, \Phi(x), 0) \le 0, \tag{3b}$$

$$\left| \frac{\partial W_c(x)}{\partial x} \right| \le \alpha_4(|x|) \tag{3c}$$

where $\alpha_j(\cdot), j = 1, \; 2, \; 4$ are class $\mathcal{K}$ functions, and $W_c(0) = \rho_0$ is the global minimum value of $W_c(x)$ in D. $F(x, u, w)$ is used to represent the system of Eq. (1) (i.e., $F(x, u, w) = f(x) + g(x)u + h(x)w$). By continuity and the smoothness assumed for f, g and h, there exists a positive constant M such that $|F(x, u, w)| \le M$ holds for all $x \in \mathcal{U}_{\rho_c}$, $u \in U$ and $w \in W$. Also, there exist positive constants L_x, L_w, L'_x, L'_w such that the following inequalities hold for all $x, x' \in \mathcal{U}_{\rho_c}$, $u \in U$, and $w \in W$:

$$|F(x, u, w) - F(x', u, 0)| \le L_x|x - x'| + L_w|w| \tag{4a}$$

$$\left| \frac{\partial W_c(x)}{\partial x} F(x, u, w) - \frac{\partial W_c(x')}{\partial x} F(x', u, 0) \right|$$
$$\le L'_x|x - x'| + L'_w|w| \tag{4b}$$

The following theorem provides sufficient conditions under which the existence of a constrained CLBF of Eq. (2) for the system of Eq. (1) under the control law $u = \Phi(x) \in U$ guarantees that the solution of the system of Eq. (1) always stays in a safe operating region.

Theorem 1. *Consider that a constrained CLBF $W_c(x) : \mathbf{R}^n \to \mathbf{R}$ that has a minimum at the origin and meets the conditions of Eq. (2), exists for the nominal system of Eq. (1) with $w(t) \equiv 0$ subject to input constraints, defined with respect to a set of unsafe points $\mathcal{D}$ in state-space. The feedback control law $u = \Phi(x) \in U$ guarantees that the closed-loop state stays in $\mathcal{U}_{\rho_c}$ for all times for $x(0) = x_0 \in \mathcal{U}_{\rho_c}$,* and does not enter $\mathcal{D}$ for $x_0 \in \phi_{uc} \setminus \mathcal{D}$.

Proof. We first prove that if $x_0 \in \mathcal{U}_{\rho_c}$, the closed-loop state $x(t)$ is always bounded in $\mathcal{U}_{\rho_c}$ and never enters $\mathcal{D}$, for all $t \ge 0$. Based on the definition of ϕ_{uc}, it is trivial to show that $\dot{W}_c$ remains negative within the set $\mathcal{U}_{\rho_c} \setminus (\mathcal{X}_e \cup \{0\})$ using the controller $u = \Phi(x) \in U$.