



Necessary condition of linear distributed parameter systems with exact controllability[☆]

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ABSTRACT

In this paper, we studied the necessary condition of distributed parameter system with exact controllability. Let Φ_0^t be the control mapping. We introduce new a class of control operators that is called the I-class control which satisfy $\mathcal{R}(\Phi_0^t) \cap \mathcal{R}(T(t))$ is closed set for $t > 0$. If the system is exactly controllable in finite time τ , then the semigroup $T(t)$ must have closed range. In particular, if the generator of $T(t)$ has compact resolvent, its spectra distribute in a strip parallel to imaginary axis. As an application we assert that the systems associated with the immediately norm-continuous semigroups are never exact controllable for zero-class or I-class controls.

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1. Introduction

Let $\mathbb{X}$ and $\mathbb{U}$ be Banach spaces. Let linear operator $A : D(A) \subset \mathbb{X} \rightarrow \mathbb{X}$ generate a C_0 semigroup $T(t)$ on $\mathbb{X}$. Denote by $\mathbb{X}_{-1}$ the completion space of $\mathbb{X}$ under the norm

$$\|x\|_{-1} = \|R(\beta, A)x\|_{\mathbb{X}}, \text{ for some } \beta \in \rho(A).$$

$\mathbb{X}_{-1}$ is a Banach space and $\mathbb{X} \subset \mathbb{X}_{-1}$ is dense set. Here we mention that the different choices of $\beta \in \rho(A)$ lead to equivalent norms in the interpolation space $\mathbb{X}_{-1}$. Denote by $T_{-1}(t)$ the continuous extension of the semigroup $T(t)$ in $\mathbb{X}_{-1}$, that is also a C_0 semigroup.

Denote by $L_{loc}^p(\mathbb{R}_+, \mathbb{U})$ the set composed of all local p -integrable $\mathbb{U}$ -valued functions, which is a linear space. Let $B : \mathbb{U} \rightarrow \mathbb{X}_{-1}$ be a bounded linear operator. Let us consider a linear dynamic system in $\mathbb{X}$ or $\mathbb{X}_{-1}$

$$\begin{cases} \frac{dw(t)}{dt} = Aw(t) + Bu(t) \\ w(0) = w_0 \in \mathbb{X} \end{cases} \quad (1.1)$$

where $u(t)$ is a $\mathbb{U}$ -valued measurable and locally p -integrable function, briefly $u \in L_{loc}^p(\mathbb{R}_+, \mathbb{U})$, which is called the control (see, [1]). In the sequel, we will denote by (A, B) the system (1.1).

For system (1.1), the controllability problem is proposed as:

For any given $w_0, w_1 \in \mathbb{X}$, can one find a time τ and a control $u(t)$ such that the solution of (1.1), $w^u(t) \in \mathbb{X}$, has property that $w^u(0) = w_0$ and $w^u(\tau) = w_1$?

If the answer is yes, one says that the system (A, B) is completely controllable. If there is a time τ such that (A, B) is completely controllable, then the system is said to be exactly controllable in finite time τ . The controllability presents an ability of control affecting the system, it is a structural feature of the control system.

Since the range of B is in $\mathbb{X}_{-1}$, A has an extension in $\mathbb{X}_{-1}$, the perturbation theory of semigroup of bounded linear operators [2] asserts that the system (1.1) has a mild solution in $\mathbb{X}_{-1}$

$$w^u(t) = T(t)w_0 + \int_0^t T_{-1}(t-s)Bu(s)ds \in \mathbb{X}_{-1} \quad (1.2)$$

The controllability of (A, B) is studied in space $\mathbb{X}$ but $\mathbb{X}_{-1}$, so Weiss in [3] introduced a concept of admissibility of B . The operator B is said to be admissible for $T(t)$ if there exists a $t > 0$ such that for any $u(\cdot) \in L_{loc}^p(\mathbb{R}_+, \mathbb{U})$, it holds that

$$\Phi_0^t(u) = \int_0^t T_{-1}(t-s)Bu(s)ds \in \mathbb{X}$$

and there exists a positive constant m_t such that

$$\|\Phi_0^t(u)\|_{\mathbb{X}}^p \leq m_t^p \int_0^t \|u(s)\|_{\mathbb{U}}^p ds, \quad \forall u(\cdot) \in L_{loc}^p(\mathbb{R}_+, \mathbb{U}).$$

Clearly, the admissibility of B is equivalent to the $\Phi_0^t(u)$ is a bounded linear operator from $L^p([0, t], \mathbb{U})$ to $\mathbb{X}$.

Now we suppose that B is admissible for $T(t)$. The exact controllability of (A, B) in finite time τ is equivalent to the operator $\Phi_0^\tau(u)$ from $L^2([0, \tau], \mathbb{U})$ to $\mathbb{X}$ is surjective, i.e.,

$$\Phi_0^\tau L^p([0, \tau], \mathbb{U}) = \left\{ \int_0^\tau T_{-1}(\tau-s)Bu(s)ds \mid u(s) \in L^p([0, \tau], \mathbb{U}) \right\} = \mathbb{X}. \quad (1.3)$$

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In analysis of exact controllability of (A, B) , the major problems are

(Q1) If A and B are given, is the system (A, B) exactly controllable in finite time τ ? How to determine or estimate the time τ ?

(Q2) If there exists a B such that (A, B) is exactly controllable in finite time τ , what is the necessary condition for A or $T(t)$?

(Q3) If (A, B) is exactly controllable in finite time τ , what is the necessary condition for B ? What is the sufficient condition for A and B ?

The question (Q2) is different from (Q1), it discusses the controllability problem from system classification point of view, namely, the necessary condition of a system with the exact controllability. In the present paper we mainly study the question (Q2). The key point of the problem is how to transform property of $\Phi_0^t(u)$ to $T(t)$ or A .

There are great deal of results on the controllability in the existing literature, including concrete models and abstract results. Here we refer to the book [4] by Staffans for the admissibility of control operator and L^p well-posed of the system, and book [5] by Tucsnak and Weiss for the controllability and observability for general semigroup, the book [6] by Komornik for the exact controllability and stabilization of various systems described by Partial Differential Equations (briefly, PDEs).

In the earlier research, one mainly studied the question (Q1), i.e., controllability (or its dual property, observability) of the single system or concrete models. For example, Triggiani in [7–9] showed that if $\mathbb{X}$ is an infinite dimensional separable Banach space, and B is a compact linear operator from $\mathbb{U}$ to $\mathbb{X}$, then the system (A, B) is never exact controllability in finite time, in particular, if A generates a compact C_0 semigroup for $t > 0$, then the system is not exactly controllable for any bounded linear operator B . Russell [10], Lions [11] studied the exact controllability of some concrete models. Ahmed [12], Louis and Wexler [13] presented some sufficient conditions for controllability of the linear control system. For more recent works on the exact controllability of the systems we refer to [14–17] and some works on the robustness of exact controllability [18–22]. For more concepts of controllability we refer to [23]. At same time we also observed there were many works studying the approximate controllability of the dynamic systems, for instance, [24–26], a nature question is: are these systems the exactly controllable? Indeed, such a question is related to the question (Q2).

In the investigation of the exact controllability of the control system (or its dual property – exact observability), Russell and Weiss in [27] observed firstly that the exact observability of system might have a requirement for the system. Later Liu and Russell [28] observed that under reasonable conditions, exact controllability of a linear system in a Hilbert space with bounded control might imply time reversibility. These observations proposed a question: whether do the exact controllability and observability have a requirement for the system? This directly leads to the question (Q2). However such a question was not studied for a long time. Ten years later, Xu et al. in [29] studied the necessary condition of the system with exact observability in Hilbert space $\mathbb{X}$. They showed if C is a zero-class of observation operator, which means that the output mapping $\Psi_\eta(x) = Ce^{At}x$, $0 \leq t \leq \eta$ satisfies condition $\lim_{\eta \rightarrow 0} \|\Psi_\eta\| = 0$, and (A, C) is exactly observable, then the semigroup $T(t)$ must be left-invertible. As a consequence the necessary condition of (A^*, C^*) being exactly controllable is that $T^*(t)$ is right-invertible. Xu and Shang in [30] studied characteristic of the left-invertible semigroups, and proved that the left-invertible semigroup is a C_0 group if A has a compact resolvent. Such a result answered the conjectures in [27,28]. Haak in [31] extended the result of [29] from C being the zero-class to the backward-forward conditioning (BFC) system, namely, (A, C) is exactly observable in time τ and there exists $0 < \eta < \tau$ such that $\|\Psi_\tau^{-1}\| \|\Psi_\eta\| < 1$. Different from [29,31], F. Zahrae et al. in [32] studied the exact

observability of (A, C) in finite time by the property of CA^{-1} , they showed that if (A, C) is exactly observable in finite time, then CA^{-1} is compact if and only if A^{-1} is. The results mentioned above answered partially the question (Q2). In the present paper, we continue to investigate question (Q2) and find a new necessary condition of the system with the exact controllability.

Different from the approach used in [29,31,32], our start point is the control mapping

$$\Phi_0^t(u) = \int_0^t T_{-1}(t-s)Bu(s)ds \in \mathbb{X}, \quad \forall u \in L^p(\mathbb{R}_+, \mathbb{U}).$$

First we observed that, for any $t, \eta > 0$, it holds the equality

$$\Phi_0^{t+\eta}(u) = T(\eta)\Phi_0^t(u) + \Phi_0^\eta(u(\cdot + t)),$$

which means that the range of $\mathcal{R}(\Phi_0^{t+\eta})$ can be represented by the ranges $\mathcal{R}(T(\eta)\Phi_0^t)$ and $\mathcal{R}(\Phi_0^\eta)$, briefly,

$$\mathcal{R}(\Phi_0^{t+\eta}) \subseteq T(\eta)\mathcal{R}(\Phi_0^t) + \mathcal{R}(\Phi_0^\eta).$$

We now suppose that (A, B) is exactly controllable in finite time τ . Then for any $\eta > 0$,

$$\mathbb{X} = \mathcal{R}(\Phi_0^\tau) = \mathcal{R}(\Phi_0^{\tau+\eta}) = \mathbb{X} = T(\eta)\mathbb{X} + \Phi_0^\eta L^p([0, \eta], \mathbb{U}). \quad (1.4)$$

There are two special cases: one is $T(\eta)\mathbb{X} = \mathbb{X}$ for all $\eta > 0$, that occurs when B belongs to the zero-class control that is $\lim_{t \rightarrow 0} \|\Phi_0^t\| = 0$; Another is $\Phi_0^\eta L^p([0, \eta], \mathbb{U}) = \mathbb{X}$ for all $\eta > 0$, which occurs if the range of B is sufficiently large. Here we do not consider these special cases. We mainly pay our attention to the intersection set $T(\eta)\mathbb{X} \cap \Phi_0^\eta L^p([0, \eta], \mathbb{U})$. We introduce new a class of control operators.

Definition 1.1. Let $\mathbb{X}$ and $\mathbb{U}$ be the Banach spaces, $T(t)$ be a C_0 semigroup on $\mathbb{X}$, $B : \mathbb{U} \rightarrow \mathbb{X}_{-1}$ is admissible for $T(t)$ and

$$\Phi_0^t(u) = \int_0^t T_{-1}(t-s)Bu(s)ds, \quad u \in L^p([0, t], \mathbb{U}).$$

B is said to be an I-class control operator if $\mathcal{R}(T(t)) \cap \mathcal{R}(\Phi_0^t)$ is closed in $\mathbb{X}$ for any $t \geq 0$, i.e., condition (1.5).

If B is an I-class control operator, we hope to find out a condition of $T(t)$.

Our main results are as follows:

Theorem 1.1. Let $\mathbb{X}$ and $\mathbb{U}$ be Banach spaces, A be the generator of a C_0 semigroup $T(t)$ on $\mathbb{X}$, $B : \mathbb{U} \rightarrow \mathbb{X}_{-1}$ be admissible for $T(t)$. Suppose that B is an I-class control operator, i.e., for any $t > 0$,

$$T(t)\mathbb{X} \cap \Phi_0^t L^p([0, t], \mathbb{U}) \text{ are closed sets.} \quad (1.5)$$

If (A, B) is exactly controllable in finite time τ , then $\mathcal{R}(T(t))$ and $\mathcal{R}(\Phi_0^t)$ are closed sets in $\mathbb{X}$ for all $t \geq 0$.

In most of concrete models, A has a compact resolvent. For such a class system, we have the following result.

Theorem 1.2. Let $\mathbb{X}$ and $\mathbb{U}$ be Banach spaces, A be the generator of a C_0 semigroup $T(t)$ on $\mathbb{X}$. Suppose that A has a compact resolvent and $\sigma(A) = \{\lambda_n; n \geq 1\}$, and B is an I-class control operator. If (A, B) is exactly controllable in finite time τ , then there is a positive constant γ such that

$$-\gamma \leq \Re \lambda_n \leq \gamma, \quad \forall \lambda_n \in \sigma(A).$$

As a consequence, we have

Corollary 1.1. Let $\mathbb{X}$ and $\mathbb{U}$ be Banach spaces, A be the generator of a C_0 semigroup $T(t)$ on $\mathbb{X}$. Suppose that A satisfies the following conditions:

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