Systems & Control Letters 117 (2018) 1–10

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

On exploiting inexact scheduling parameters for gain-scheduled control of linear parameter-varying discrete-time systems

Arash Sadeghzadeh

Faculty of Electrical Engineering, Shahid Beheshti University, Tehran, Iran

ARTICLE INFO

Article history:
Received 17 March 2017
Received in revised form 29 March 2018
Accepted 21 April 2018

Keywords:
Gain-scheduled controller
Discrete-time system
Parameter-varying system
Bounded variation rate
Inexact scheduling parameters
Induced l_2-gain performance

ABSTRACT

This paper is concerned with the problem of gain-scheduled controller design for parameter-varying discrete-time systems. In practice, the scheduling parameters are only available with limited accuracy due to the measurement errors. In the present paper, these uncertainties are systematically taken into account for the gain-scheduled controller synthesis to achieve improved performance in practical situations. It is assumed that all the matrices of the state-space model of the plant have a homogeneous polynomial dependency of arbitrary degree on the scheduling parameters. The merit of the proposed method is its capability in dealing with the measurement error less conservatively than available approaches while it can also encompass the traditional methods employing the exact scheduling parameters in a less conservative manner. A comprehensive numerical example demonstrates the effectiveness of the proposed method.

1. Introduction

Gain-scheduling is a methodology to tackle the problem of designing controller for a nonlinear system with widespread practical engineering applications ranging from aerospace to process control [1–3]. As a special case, this strategy is applicable for the control of linear parameter-varying (LPV) systems, a subclass of nonlinear systems which are modeled as parameterized linear systems whose parameters are time-varying. Some key results related to gain-scheduled control for continuous-time systems can be found in [4–10]. In this note, the gain-scheduled control design for discrete-time systems is investigated. In most of the existing approaches, the scheduling parameters are modeled through polytopic representation, a convex hull of known vertices. Then, the synthesis procedure is constructed based on the well-known Lyapunov theory [11–16]. Due to this theory, the performance of a closed-loop system comprising of an LPV system and a gain-scheduled controller can be evaluated by some parameter-dependent Linear Matrix Inequalities (LMIs) [17]. From the controller design point of view, generally an invertible nonlinear transformation is employed to cast the design problem in terms of solutions to a set of LMIs [11,12]. In this case, one of the most critical issues in the control design for LPV systems is the choice of the Lyapunov function. It is a trivial fact that employing parameter-dependent Lyapunov functions considerably reduces the related conservatism but at the expense of more computational complexity. Besides that, by introducing auxiliary (slack) variables in the LMI setting of the performance assessment conditions through reciprocal application of Elimination lemma [18], the related conservatism can efficiently be decreased, see e.g. [13]. In its general form, the obtained performance conditions are infinite-dimensional LMI problems whose solutions are known to be NP-hard. At the expense of some conservatism, one possible way is to impose a special structure on the parameter-dependent Lyapunov matrix $P(\theta)$ to obtain a tractable optimization problem [14]. In this way, a systematic procedure to tackle the robust stability analysis for uncertain linear time-invariant systems has been introduced in [19], and subsequently the same method is employed to study the control problem of LPV systems [11,13] where by exploiting the geometric properties of polytopic domains, homogeneous polynomially parameter-dependent solutions of arbitrary degree on the scheduling parameters are sought.

In reality, the exact scheduling parameters are not available. Although, this fact has been neglected in most of the previously mentioned methods. The scheduling parameters are either measured or estimated and thus there always exist some inherent uncertainties which potentially could have a great effect on the closed-loop performance. Several researchers have already directed their attention to this problem. Most of the recent publications which consider the scheduling parameter errors are devoted to the continuous-time LPV systems, see e.g. [9,10] and references therein. Under the existence of uncertainties in the available scheduling parameters for discrete-time LPV systems, gain-scheduled state feedback controller [20], fixed-structure controller design [15], and gain-scheduled output feedback controller synthesis [12,21] have been proposed. The stabilization problem is tackled in [21] where

E-mail address: a_sadeghzadeh@sbu.ac.ir.

https://doi.org/10.1016/j.sysconle.2018.04.012
0167-6911/

the scheduling parameters are available with a finite accuracy. The presented method in [15] is an iterative design procedure constructed based on the concept of Strictly Positive Realness (SPRness) of transfer functions. The main drawback of the method is that a set of initial controllers should be provided for the initialization of the iterative procedure by any other method. The approach of [12] is an extension of the methods presented in [11,22] that employs a constant auxiliary variable G in the so-called dilated LMI characterization for performance analysis of discrete-time LPV systems.

Motivated by the above, we consider the problem of gain-scheduled output feedback control design for discrete-time LPV systems when the exact values of scheduling parameters are not available. Parameter-dependent Lyapunov function and auxiliary variables, with polynomial dependence of arbitrary degrees on the scheduling parameters, are exploited. An LMI-based method for the synthesis of a rationally parameter-dependent gain-scheduled controller is presented. Then, an iterative convex optimization scheme is provided to improve this controller. Contrary to the methods of [12,15], we assume that all of the matrices of the state space representation of LPV system to be polynomially parameter-dependent. Additionally, employing parameter-dependent auxiliary variables, contrary to the methods of [11,12], is obviously promising for achieving less conservative results. By rigorous theoretical proof, we have shown that some available approaches are special cases of the proposed method, namely, our conditions can reduce to those in [11,12] under special cases. This implies that the proposed conditions can give less conservative design.

The notation is fairly standard. In this paper, $0_{n\times m}$ is an $n \times m$ zero matrix and I_n is an $n \times n$ identity matrix. The subscript for the dimension may be dropped if the sizes of matrices are clear from the context. Moreover, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices and $\langle A\rangle$ is a shorthand notation for $A + A'$. In a symmetric matrix, $\star$ denotes the transpose of an off-diagonal block. For the ease of notation, $diag(\cdot)$ is employed to represent the block diagonal concatenation of input arguments. The corresponding 2-norm is defined as $\|x\|_2^2 = \sum_{k=0}^{\infty} x(k)'x(k)$.

2. Preliminaries

Consider the following finite-dimensional discrete-time LPV system:

$$\begin{aligned} x(k+1) &= A(\breve{\theta}(k))x(k) + B(\breve{\theta}(k))w(k), \quad x(0) = 0 \\ z(k) &= C(\breve{\theta}(k))x(k) + D(\breve{\theta}(k))w(k) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $w \in \mathbb{R}^m$ is the exogenous input, $z \in \mathbb{R}^q$ is the system output. All system matrices are appropriately dimensioned real matrices. They are assumed to be bounded for $k \geq 0$ and have a polynomial parameter dependency on the scheduling parameter $\breve{\theta}(k) \in \mathbb{R}^v$.

Suppose that the aforementioned system is exponentially stable. Then η is an upper bound on the induced l_2-gain performance of the LPV system (1) if

$$\sup_{w\neq 0,\ w\in l_2} \frac{\|z\|_2}{\|w\|_2} < \eta \tag{2}$$

for all allowable scheduling parameter trajectories. w is assumed to be a bounded energy signal. In virtue of bounded real lemma, an upper bound on the induced l_2-gain performance can be characterized. In what follows, the step index k is omitted for the clarity and for the ease of notation $\breve{\theta}(k+1)$ is denoted by $\breve{\theta}^+$.

To proceed further, we need the following lemma.

Lemma 1. *If there exist a scalar λ and parameter-dependent matrices $P(\breve{\theta}) = P(\breve{\theta})' > 0$ and $G(\breve{\theta})$ such that*

- $$\begin{bmatrix} -P(\breve{\theta}) + \langle G(\breve{\theta})\rangle & \star \\ \lambda G(\breve{\theta})' - A(\breve{\theta})G(\breve{\theta}) & P(\breve{\theta}^+) - \langle \lambda A(\breve{\theta})G(\breve{\theta})\rangle \end{bmatrix} > 0. \tag{3}$$

  holds for all allowable scheduling parameter trajectories, then the system given by (1) is exponentially stable.

- $$\begin{bmatrix} -P(\breve{\theta}) + \langle G(\breve{\theta})\rangle & \star & \star & \star \\ \lambda G(\breve{\theta})' - A(\breve{\theta})G(\breve{\theta}) & P(\breve{\theta}^+) - \langle \lambda A(\breve{\theta})G(\breve{\theta})\rangle & \star & \star \\ -C(\breve{\theta})G(\breve{\theta}) & -\lambda C(\breve{\theta})G(\breve{\theta}) & \eta I & \star \\ 0 & B(\breve{\theta})' & D(\breve{\theta})' & \eta I \end{bmatrix} > 0. \tag{4}$$

  holds for all allowable scheduling parameter trajectories, then in addition to the exponential stability, an upper bound $\eta > 0$ on the induced l_2-gain performance is guaranteed.

Proof. See Appendix. □

The aforementioned lemma provides so-called dilated LMI characterizations for stability and performance of discrete-time LPV systems. The dilation of the LMI characterizations by introducing auxiliary variables yields decoupling between the Lyapunov variables and the controller parameters that facilitates the use of parameter-dependent Lyapunov functions. Moreover, as it is shown in [22–24], the provided free dimensions in the solution space by the employed auxiliary variables may reduce the conservatism of the LMI optimization problems. In Lemma 1, both $G(\breve{\theta})$ and λ play the role of the auxiliary variables. In a special case by considering $\lambda = 0$, the conditions in Lemma 1 are converted to the provided dilated LMIs in [25]. This implies that Lemma 1 provides more free dimension in the solution space and may lead to less conservative results by performing a line search for λ.

3. System description and problem definition

Consider the following linear parameter-varying discrete-time system with v independent scalar parameters $\theta(k) \triangleq \begin{bmatrix}\theta_1(k) & \cdots & \theta_v(k)\end{bmatrix}'$ as follows:

$$\begin{aligned} x_p(k+1) &= A_p(\theta(k))x_p(k) + B_1(\theta(k))w(k) + B_2(\theta(k))u(k), \\ x_p(0) &= 0 \\ z(k) &= C_1(\theta(k))x_p(k) + D_1(\theta(k))w(k) + D_2(\theta(k))u(k) \\ y(k) &= C_2(\theta(k))x_p(k) + D_y(\theta(k))w(k) \end{aligned} \tag{5}$$

where $x_p(k) \in \mathbb{R}^n$ is the state vector, $w(k) \in \mathbb{R}^m$ is the disturbance input, $z(k) \in \mathbb{R}^q$ is the performance output, $y(k) \in \mathbb{R}^r$ is the measured output and $u(k) \in \mathbb{R}^p$ is the control input. It is assumed that all system matrices, with appropriate dimensions, are real and bounded and to be polynomial with respect to $\theta_i(k)$. The scheduling parameter $\theta(k)$ is assumed to lie in a hyper-rectangle, or equivalently

$$\underline{\theta}_i \leq \theta_i(k) \leq \overline{\theta}_i, \quad i = 1, \ldots, v \tag{6}$$

with the a priori known values of $\underline{\theta}_i, \overline{\theta}_i$ for $i = 1, \ldots, v$. Additionally, the parameter deviation for one sampling step is also assumed to be bounded as follows:

$$|\theta_i(k+1) - \theta_i(k)| \leq \Delta_i, \quad i = 1, \ldots, v \tag{7}$$

with the *a priori* known values of Δ_i for $i = 1, \ldots, v$. This directly leads to the fact that the admissible region for $(\theta_i(k), \theta_i(k+1))$ would be a polytope with six vertices as it is shown in [12]. Consequently, $\theta(k)$ and $\theta(k+1)$ can be modeled through multi-simplex framework, i.e. the Cartesian product of simplexes [26].