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Distributed rotating consensus of second-order multi-agent systems with nonuniform delays

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ABSTRACT

In this paper, we study a distributed rotating consensus problem of second-order multi-agent systems with nonuniform delays. A distributed algorithm is adopted to drive all agents to reach consensus while moving around a common point. Based on a frequency domain approach, an upper bound on the maximum delay is given for the consensus stability of the system. Finally, a numerical simulation result is included to illustrate the obtained results.

1. Introduction

As an unavoidable issue in practical applications, delays have been frequently considered in the study of distributed consensus problems [1–15]. From the view point of the final dynamics of the consensus point, the consensus problems with delays can be categorized into two classes: the static consensus and the moving consensus. For the static consensus case, the study has already been very mature and numerous results have been obtained. For example, articles [1,2] used frequency domain approaches to address the consensus stability of the multi-agent system with nonuniform delays and give different conditions to ensure the system consensus. For the moving consensus case, current works are often limited to the flocking-like consensus problem, e.g., [14,15], where all agents finally move along a line. This might be due to the reason that for the case of the static consensus point, the system can be transformed into a reduced-order equivalent one which is completely decoupled with the component corresponding to the zero eigenvalue, while for the case of the moving consensus point, the components of the system corresponding to the eigenvalues with negative real-parts cannot be decoupled with the components corresponding to the eigenvalues with nonnegative real-parts, which together with delays makes the analysis much more complicated than the case of the static consensus point.

In this paper, we investigate a rotating consensus problem of second-order multi-agent systems with nonuniform delays. The meaning of "rotating consensus" is that all agents should not only reach a consensus but also finally move together along a circle around a common point. It has potential important applications in many fields, including satellite formation flight, spacecraft docking and unmanned underwater robots. In contrast to the flocking-like consensus problem with delays, where the line moving mode has no effects on the agent relative positions, the rotating consensus problem contains a cyclic moving mode, making the agent relative positions time-varying before consensus, and hence the analysis of the rotating consensus problems is more complicated than that of the flocking-like consensus problem due to the coupling of the time-varying relative positions and the delays. Though some results about the rotating consensus problem have been obtained, e.g., [16–21], most of these results only considered the case without delays. To this end, our objective is to extend the existing results about the rotating consensus to take nonuniform delays into account. First, we introduce a distributed rotating consensus algorithm with nonuniform delays. Then we transform the original system into an equivalent one. Based on this equivalent system, we use a frequency domain approach to analyze the critical condition for the system, and an upper bound on the maximum delay for the consensus stability is obtained.

2. Graph theory

Let $\mathcal{G}(\mathcal{I}, \mathcal{E}, \mathcal{A})$ represent an undirected graph, where $\mathcal{I} = \{1, \ldots, n\}$ denotes the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edge set, and $\mathcal{A} = [a_{ij}]$ denotes a weighted adjacency matrix. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (i, j)$. It is assumed that $(i, i) \notin \mathcal{E}$ for all i. For the weighted adjacency matrix $\mathcal{A}$, $a_{ij} > 0$ if $e_{ij} \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The neighbor set of node i is denoted by

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$N_i = \{j \in \mathcal{I} : (i, j) \in \mathcal{E}\}$. The Laplacian of the graph $\mathcal{G}$ is defined as $L = [l_{ij}]$, where $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. A path is an edge sequence having the form $(i_1, i_2), (i_2, i_3), \ldots$, where $i_j \in \mathcal{I}$. The graph is connected if each node has at least one path to every other node.

Lemma 1 ([22]). *Suppose that the graph $\mathcal{G}$ is connected. The Laplacian L of the graph $\mathcal{G}$ has a simple eigenvalue at 0 associated with the eigenvector $\mathbf{1}_n$ and all its other $n-1$ eigenvalues are positive, where $\mathbf{1}_n$ denotes an n-dimensional vector each entry of which is 1.*

3. Model and algorithm

Suppose that the multi-agent system under consideration consists of n agents in a plane. Each agent is regarded as a node in an undirected graph $\mathcal{G}$. Each edge $(\mathrm{v}_j, \mathrm{v}_i) \in \mathcal{E}(\mathcal{G})$ or $(\mathrm{v}_i, \mathrm{v}_j) \in \mathcal{E}(\mathcal{G})$ corresponds to an available information channel between agent v_i and agent v_j at time t. The set of the neighbors of the ith agent associated to the graph $\mathcal{G}$ is denoted by N_i, and the Laplacian of the graph $\mathcal{G}$ is denoted by L. Each agent updates its current state based upon the information received from its neighbors.

Let $x_i \in \mathbb{C}$ and $v_i \in \mathbb{C}$ be the position and velocity states of agent i. Suppose that each agent has the following dynamics:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \end{aligned} \tag{1}$$

where $u_i(t) \in \mathbb{C}$ is the control input (or algorithm) at time t and the initial conditions of $x_i(s)$ and $v_i(s)$ satisfy the dynamics (1) for all $s \leq 0$.

In this paper, we are interested in considering the scenario where all agents reach a consensus while moving together around a common point along a circle with nonuniform delays. First, we give a mathematical definition about this motion. We say the algorithm u_i asymptotically solves a rotating consensus problem, if and only if the states of agents satisfy

$$\begin{aligned} &\lim_{t\to+\infty} [x_i(t) - x_j(t)] = 0, \\ &\lim_{t\to+\infty} [v_i(t) - v_j(t)] = 0, \\ &\lim_{t\to+\infty} [\dot{v}_i(t) - \tilde{\mathrm{i}} v_i(t)] = 0 \end{aligned}$$

for all $i, j \in \mathcal{I}$, where $\tilde{\mathrm{i}}$ denotes the imaginary unit. The first two limits mean that all agents finally converge to a point while the third limit means that all agents finally move along a circle according to the knowledge of the circular motion. Rotating consensus has important potential applications in many fields including spacecraft docking, formation flight and unmanned underwater robots. Though some results have been obtained on the rotating consensus problem, e.g., [16]-[21], most of these results only considered the case without delays.

To study the rotating consensus problem with nonuniform delays, we consider the following algorithm:

$$\begin{aligned} u_i(t) = \tilde{\mathrm{i}} v_i(t) &+ \sum_{\mathrm{v}_j \in N_i} a_{ij}(x_j(t-\tau_{ij}) - x_i(t-\tau_{ij})) \\ &+ \sum_{\mathrm{v}_j \in N_i} a_{ij}(v_j(t-\tau_{ij}) - v_i(t-\tau_{ij})), \end{aligned} \tag{2}$$

where $\tau_{ij} = \tau_{ji}$ are the communication delays between the ith and the jth agents. Suppose that there are M different delays, denoted by $\tau_m \in \{\tau_{ij}, i, j \in \mathcal{I}\}$ $(m = 1, 2, \ldots, M)$. Let $\xi(t) = [x_1(t), v_1(t), \ldots, x_n(t), v_n(t)]^T$. The network dynamics is summarized as

$$\dot{\xi}(t) = (I_n \otimes A)\xi(t) - \sum_{m=1}^{M} (L_m \otimes B)\xi(t - \tau_m) \tag{3}$$

where $\otimes$ denotes the Kronecker product, $\xi(s) = \xi(0), s \in (-\infty, 0]$, $A = \begin{bmatrix} 0 & 1 \\ 0 & \tilde{\mathrm{i}} \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $L_m \otimes B$ is the coefficient matrix of the variable $\xi(t - \tau_m)$ for $m = 1, \ldots, M$. Clearly, $L = \sum_{m=1}^{n} L_m$.

4. Main results

For convenience of discussion, we make a model transformation. Let $\bar{v}_i(t) = x_i(t) + v_i(t)$ and $\phi(t) = [x_1(t), \bar{v}_1(t), \ldots, x_n(t), \bar{v}_n(t)]^T$. Then the system (3) can be written as

$$\dot{\phi}(t) = (I_n \otimes E)\phi(t) - \sum_{m=1}^{M} (L_m \otimes F)\phi(t - \tau_m) \tag{4}$$

where $E = \begin{bmatrix} -1 & 1 \\ -1-\tilde{\mathrm{i}} & 1+\tilde{\mathrm{i}} \end{bmatrix}$ and $F = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Clearly, when all delays are equal to zero, the system (4) can be written as

$$\dot{\phi} = (I_n \otimes E - L \otimes F)\phi(t). \tag{5}$$

Before the main results, we need first give some lemmas. Specifically, Lemma 2 proves that (4) has two simple eigenvalues at 0 and $\tilde{\mathrm{i}}$ and all its other $2n-2$ eigenvalues have negative real parts when $\tau_m = 0$ for all m. Lemmas 3 and 4 study the monotonicity of two specific functions that will be used in Theorem 1.

Lemma 2. *Suppose that the graph $\mathcal{G}$ is fixed and connected.*
(1) The matrix $I_n \otimes E - L \otimes F$ has simple eigenvalues at 0 and $\tilde{\mathrm{i}}$ and all its other $2n-2$ eigenvalues have negative real parts.
(2) The matrix $I_n \otimes E - \sum_{m=1}^{M} (L_m \otimes F) e^{-\tau_m \tilde{\mathrm{i}} w}$ has one eigenvalue at $w = 0$ and $w = 1$, respectively, where τ_m are nonnegative constants for all m.

Proof. See the Appendix.

Remark 1. From Lemma 2, there is one imaginary eigenvalue besides the zero eigenvalue in the system matrix of (5). This is different from most of the existing results, where the closed-loop system has only one zero eigenvalue. Moreover, the agents in this paper are in the form of second-order dynamics and considered in complex plane, different from the existing delay works, where the agents lie in the real plane and are essentially in the form of first-order dynamics. These two differences make the system analysis much more complicated than the existing works.

Lemma 3. *(1) Let $D(w) = \frac{\arctan w}{w}$. For $w \in (0, +\infty)$, $\frac{\mathrm{d}}{\mathrm{d}w} D(w) < 0$, and for $w \in (-\infty, 0)$, $\frac{\mathrm{d}}{\mathrm{d}w} D(w) > 0$.*
(2) Consider the equation $\tan w\tau - w = 0$, where $\tau > 0$ is a constant and $w \in \mathbb{R}$. Suppose that the equation $\tan w\tau - w = 0$ has nonzero roots and let $r_1 > 0$ and $r_2 < 0$ be, respectively, its positive root and negative root. For $w \in (r_2, 0)$, $\tan w\tau - w > 0$ and for $w \in (0, r_1)$, $\tan w\tau - w < 0$.

Proof. See the Appendix.

Lemma 4. *Consider the equation $\frac{(w^2-w)^2}{w^2+1} = \beta$, where $\beta > 0$ is a constant and $w \in \mathbb{R}$. It has only one root larger than 1 and its absolute value is no smaller than the other real roots. The function $F(w) = \frac{(w^2-w)^2}{w^2+1}$ is an increasing function of w in $(1, +\infty)$.*

Proof. See the Appendix.

Theorem 1. *Consider a network of agents with nonuniform delays. Suppose that the graph $\mathcal{G}$ is fixed and connected. The algorithm (2) solves the rotating consensus problem, if $\tau_{max} < \frac{\arctan(z)}{z}$, where z is the maximum root of $(\lambda_{max})^2 = \frac{(w^2-w)^2}{w^2+1}$, and τ_{max} and λ_{max} denote, respectively, the maximum delay and the maximum eigenvalue of L.*