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Couple-group consensus conditions for general first-order multiagent systems with communication delays

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ABSTRACT

In this paper, we establish group consensus conditions for first-order systems with communication delays using the relationship between time-delay parameter and the closed-loop characteristic roots. At first, we derive necessary conditions for the delay-free case, then we proceed to the delay-induced case. Contrary to most studies on group consensus where the conditions are derived considering the Laplacian of the entire multiagent network, we derive the group consensus conditions based on the Laplacian matrices that describe each subgroup and the inter-connection among sub-groups. Simulation studies are used to validate the derived conditions.

1. Introduction

Coordination control is an important subject amongst researchers as we continue to evolve into the age of automation and robotics. It is required that autonomous machines demonstrate some intelligent coordination behavior exhibited by human or animal counterparts. Many applications of coordination control can be found in Unmanned Aerial Vehicles (UAV), Unmanned Ground Vehicles (UGV) and Unmanned Underwater Vehicles (UUV). Recently, some applications are being developed for static (non-mobile) systems such as power and renewable energy systems. Other applications may be found in biology, chemistry, economics and even politics. Some of the interesting topics in coordination control of autonomous machines include; consensus, formation, alignment, rendezvous, containment, circumnavigation, swarming and flocking control.

A network of multiagent systems (MAS) is said to reach a *consensus* when the states of the agents within the network reach an agreement about a given objective. Consensus control can be seen as a special case for each of the aforementioned forms of coordination control because they all require the states of the agents reaching agreements depending on the specified group objective. Numerous investigations have been conducted by various researchers on consensus control problems based on different approaches such as leader–follower [1,2], graph theory [3,4], behavior-based [5,6] and virtual structure [7] based approaches. Recently, group consensus control has attracted the attention of researchers [8–21]. In group consensus, the agents within a subgroup achieve asymptotic consensus to a terminal state which may differ from the terminal state of another subgroup. Group consensus problem of MAS under switching topology was studied in [21], authors in [19] studied the group consensus problem for systems with communication delay in discrete-time instants using nonnegative matrix theory and graph theory. Group consensus tracking control problem was studied in [10] for second-order multiagent systems with directed fixed topology. Usually, consensus protocols are designed for MAS in infinite-time. Recently, authors in [22] studied group consensus using finite-time analysis. Specifically, a nonlinear distributed protocol using local information was designed for a leader–follower based multiagent system under directed topology. Also, the authors proposed an approach for estimating the settling time of the multi-agent system. Furthermore, fixed-time consensus tracking control for MAS with inherent nonlinear dynamics was discussed in [23]. According to the authors, the consensus protocol proposed does not require inter-group balance conditions and interactions are allowed between leaders and followers in different subgroups. Also, an approach to designing controller gains to achieve fixed time group tracking was proposed.

2. Related works

In this section, we review some related works by earlier researchers on group consensus control problem. Necessary and sufficient consensus conditions for general second-order multi-agent systems with communication delays were established in [24] using the relationship between time delay parameter and the roots

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of the characteristic equations. The effect of coupling strengths amongst groups of multi-agent systems was investigated in [12]. Using simulation examples only, the authors in [12] deduced an upper bound for the coupling strength between subgroups without an explicit criterion for computing these bounds. In [20], necessary and sufficient conditions for group consensus were derived for multi-agent systems via a rotation matrix approach. Group consensus problem for linearly coupled multi-agent systems was studied in [13]. Authors in [25], discuss group consensus for general linear multiagent systems and stated that by designing appropriate control gains, group consensus can be achieved for any magnitude for the coupling strengths amongst agents. In [10], the authors studied group consensus tracking control for second-order multi-agent systems with directed fixed topology. The authors extended their results to χ-group consensus tracking case. Authors in [14] discussed group consensus control for MAS with nonlinear input constraints. The authors proposed consensus protocols for discrete and continuous-time cases and the consensus conditions were derived using graph-theory, Lyapunov and LaSalle's invariance principles. Using Mao's stability analysis for stochastic differential equations, authors in [26] studied derived group consensus conditions for leader–follower based multiagent systems under noises and time-delays in directed networks. According to the authors, this approach ensures consensus can be achieved almost certainly and exponentially fast.

In this paper, we derive some necessary conditions for group-consensus control of first-order multi-agent systems with and without communication delays using the roots of the characteristic equations of the closed-loop system. Using graph theoretic methods, we establish some relationships between the inter-group and intra-group coupling strength that are required to be satisfied for group-consensus to be achieved. Contrary to previous studies in group consensus where the relationships between subgroups is neglected and the consensus protocols and control gains are chosen using information about the Laplacian describing the entire multi-agent network, the contributions of this paper are as follows:

- The approach for designing the consensus conditions takes into consideration the relationship between eigenvalues of the Laplacian describing each subgroup, Laplacian describing inter-group relations, the intra and inter group coupling strengths.
- We establish some necessary consensus conditions which are necessary in designing the coupling strengths or control gains for the delay free case.
- We extend the conditions and discussions in the delay free instance to the case where communication delays are inherent in the network.

The rest of this paper is organized as follows; Section 3 presents the formulation of the group consensus problem and reviews some basic graph theory terminologies. In Sections 4 and 5 we establish conditions for group consensus for first-order multi-agent systems without and with communication delays respectively. Simulation studies were conducted in Section 6.

Notations $Im(\chi)$ is used to denote imaginary parts of χ. $Re(\chi)$ is used to denote real parts of χ. I_n represent identity matrix of size n. $\det(A)$ is the determinant of matrix A.

3. Problem formulation

3.1. Algebraic graph theory

Graph theory is a standard framework for representing connections and interactions between networked, distributed or multi-agent systems. A graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is defined as a pair consisting of vertices $\mathcal{V}$ and edges $\mathcal{E}$. $\mathcal{V}(\mathcal{G})$ represents the set of vertices in $\mathcal{G}$ and $\mathcal{E}(\mathcal{G})$ is the edge set of $\mathcal{G}$. A graph is said to be undirected when the edge between any pair of vertices has no orientation. Conversely, in a directed graph or *digraph*, each edge $e \in \mathcal{E}(\mathcal{G})$ is directed between any pair of vertices, that is, the edge $e = v_i v_j$, originates at vertex v_i and terminates at vertex v_j. In a *simple* graph, there are no self-loops or multiple edges between vertices. In a *complete* graph, every pair of vertices is connected.

Some special matrices are used to describe the properties and information in a graph. These matrices include, *degree*, *adjacency*, *incidence*, and *laplacian* matrices. For a graph on n vertices and m edges, the degree matrix $\Delta(\mathcal{G}) \in \mathcal{R}^{n\times n}$, is a diagonal matrix, with elements on the diagonal representing the degree $d(v_i)$ of each vertex. $d(v_i)$ is the sum of edges incident to the vertex v_i. The adjacency matrix $\mathcal{A}(G)$ is a symmetric $n \times n$ matrix describing the adjacency relationship in $\mathcal{G}$. Each $a_{ij} \in \mathcal{A}(\mathcal{G})$ assume 1 if $v_i v_j \in \mathcal{E}(\mathcal{G})$ and 0 otherwise. The Laplacian matrix in an undirected graph $\mathcal{L}(\mathcal{G}) = \Delta(\mathcal{G}) - \mathcal{A}(G)$. The incidence matrix $\mathcal{W}$ of a directed graph $\mathcal{D}$, is defined as $\mathcal{W} = [w_{ij}]$. $w_{ij} = -1$ if v_i is the tail of e_j, $w_{ij} = 1$ if v_i is the head of e_j and $w_{ij} = 0$ if v_i is not adjacent to e_j The Laplacian matrix of a directed graph $\mathcal{D}$ is $\mathcal{L}(\mathcal{D}) = \mathcal{W}(\mathcal{D})\mathcal{W}(\mathcal{D})^T$.

3.2. Consensus

Consider a network of multiagent systems (MAS) described by $\mathcal{G}$ consisting of n agents described by the following first-order dynamics:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \dots, n \tag{1}$$

where x_i, and u_i represent the states and control inputs of each agent in the network.

Definition 1. The MAS described by $\mathcal{G}$ with dynamics (1) achieves consensus if for any $x_i(0)$,

$$\lim_{t\to\infty} |x_i(t) - x_j(t)| = 0 \quad \forall i, j = 1, 2, \dots n$$

and the MAS asymptotically solves average-consensus problem when,

$$\lim_{t\to\infty} |x_i(t)| = \frac{1}{n}\sum_{j=1}^{n} x_j(0) \ \forall i, j = 1, 2, \dots N.$$

Now, suppose the control protocol $u_i(t)$ is chosen as:

$$u_i(t) = -\sum_{j=1}^{n} a_{ij}(x_i(t) - x_j(t)) \ \ j = 1, 2, \dots, n \tag{2}$$

the closed loop system (1) under protocol (2) is given as:

$$\dot{x} = -\mathcal{L}x. \tag{3}$$

Lemma 1 (*[27]*)**.** *Suppose that $\mathcal{L} = [l_{ij}] \in \mathcal{R}^{n\times n}$ satisfy $l_{ij} < 0, i \neq j$ and $\sum_{j=1}^{n} l_{ij} = 0, \quad i = 1, 2, \dots, n$, then the following conditions are equivalent:*

- *$\mathcal{L}$ has a simple zero eigenvalue and all other eigenvalues have positive real parts.*
- *$\mathcal{L}x = 0$ implies that $x_1 = x_2 = \cdots x_n$.*
- *Consensus is reached asymptotically for the system $\dot{x} = -\mathcal{L}x$.*
- *the directed graph of $\mathcal{L}$ has a directed spanning tree*
- *the rank of $\mathcal{L}$ is $n - 1$.*

3.3. Group consensus

Consider a network consisting of $n + m$ multiagent systems belonging to subgroup χ_1 and χ_2 respectively, described by the following first-order dynamics.

$$\dot{x}_i(t) = ax_i + bu_i(t) \tag{4}$$