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Stabilization of a discrete time linear system over finite data-rate channel with noise attenuation performance by spherical polar coordinate quantizer

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ABSTRACT

This paper considers the problem of achieving the first moment stability and the given noise attenuation performance of a discrete time linear system with unbounded noise. Based on spherical polar coordinates, a quantizer is proposed with a definite relation between the quantized data and the corresponding quantization error. And a necessary and sufficient condition for the first moment stability of the system is presented. Further, based on the quantizer, two coding schemes are proposed for time-varying data rate and fixed data rate respectively, which achieve both the first moment stability and the noise attenuation performance of the system with unbounded noise.

1. Introduction

In this paper we formulate and solve a stabilization problem with a communication channel of finite data rate connecting the measurement sensor to the controller. Our task involves designing the coding scheme to stabilize a discrete time linear system with unbounded noise and achieving both the first moment stability of the system and the given noise attenuation performance.

The stabilization problem of this kind motivated by numerous applications is a very active and expanding research area, where communication between the plant and the controller is limited due to capacity or security constraints. Since the system in practice is harassed by external noise inevitably, much attention is paid to quantized feedback system with the noise . Some papers address the issue of noise, differing mainly in the stability property they aim to achieve and in their assumptions on the noise.

In the aspect of deterministic stability, [1] studies the input-to-state stability of quantized linear systems with bounded noise under feedback dropouts. [2] considers the problem of achieving input-to-state stability for system with unknown disturbance by zooming-in/out approach. [3] considers the same problem by the information approach to require fewer quantization regions. In the aspect of stochastic stability, [4,5], and [6] achieve state boundedness in the presence of bounded disturbance by using the knowledge of a disturbance bound. As to unbounded disturbance, [7] designs a controller to bound the plant's state in probability. In [8], mean square stability in the stochastic setting is obtained by utilizing statistical information about the disturbance (a bound on its appropriate moment). [9] presents a data rate theorem for mean square stabilization of a linear discrete-time system over a time-varying rate communication channel. The channel transmission rate is assumed constant over a block of time but changes independently from block to block according to a given probability distribution. Further, under the condition that the channel rate process is modeled as a homogeneous positive-recurrent Markov chain, [10] and [11] provide a data-rate theorem for a scalar system over stochastic time-varying rate channels. In [12], some stochastic stability results for adaptive quantizer for the system with unbounded noise are presented. [13] considers the problem of stabilizing a continuous-time system driven by Brownian motion process in both the forward channel and the reverse channel. In [14] a tight data rate bound is presented for the moment stability over erasure channels by proposing a general drift approach for verifying stochastic stability of Markov chains. [15] investigates a class of multi-sensor discrete-time systems with both plant and observation noise, and obtains a coding and control policy leading to the existence of a unique invariant distribution and finite second moment for the sampled state under fixed-rate communication requirements and provides tight bounds for moment stability. [16] considers the problem of stabilizing a system driven by a Gaussian disturbance using a feedback signal transmitted over a memoryless Gaussian communication channel. [17] develops a framework for discrete-time control systems subject to average data-rate constraints. In contrast to these works, we are concerned with the

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first moment stability and the noise attenuation performance of discrete time linear system.

The objective of this paper is to design coding schemes for achieving both the first moment stability of discrete time linear system with unbounded noise and the given noise attenuation performance. Here the first moment stability is referred to as the boundedness of the upper limit of the expectation of the system state norm (see Definition 2.1) and the noise attenuation performance is measured by the upper limit of the rate of the expectation of the state norm to the expectation of the noise norm (see (4)). The coding schemes in the paper are based on spherical polar coordinates since a definite relation between the quantized data and the corresponding quantization error is developed to facilitate the stability analysis of the system [18]. The relation shows that the magnitude of the quantized data is proportional to an upper bound of the magnitude of the corresponding quantization error. The contributions of the paper are as follows:

(1) Different from the existing zooming-in/out methods, the parameter L_k of the quantizer, which is proportional to the radius of quantization region, is updated under the action of a designed input r_k (see (7)) for rather complicated task. The input is either time-varying and updated at each time step (for time-varying data rate) or constant and designed in advance (for fixed data rate).
(2) Under the proposed zooming-in/out method, the bounds of the expectation and the variance of L_k are obtained in a recursive form (see Lemma 4.3), which are used for the design of the coding schemes. This means that the bounds of the expectation and the variance of the radius of quantization region are also obtained.
(3) A necessary and sufficient condition of guaranteeing the first moment stability with respect to unbounded noise for system with quantized state measurements is presented. Based on the condition, two coding schemes are proposed for time-varying data rate and fixed data rate respectively, which achieve both the first moment stability and the noise attenuation performance of the system.

In this paper we do not focus on the tight data rate bounds but on the coding scheme design for achieving the first moment stability and the noise attenuation performance. The data rate bounds obtained in the paper are not tight and more than those in [11,14] and [15] for the stochastic stability of their considered systems. This is mainly because: the data rate bounds which guarantee both stochastic stability and noise attenuation performance are evidently more than the bounds which guarantee only stochastic stability; to highlight main results and avoid tedious derivation, this paper treats the system matrix as a whole by the matrix norm instead of analyzing its each eigenvalue, which leads to the large date rate bounds; although the proposed quantizer facilitates the stability analysis of the system, the number of quantization blocks based on spherical polar coordinates is not minimum, that is, the data rate is not lowest.

In what follows, I and O denote respectively the identity matrix and the null matrix of appropriate dimensions. $\|\cdot\|$ denotes the Euclidean norm for a vector and the corresponding matrix-induced norm for a matrix, that is, the maximum singular value of a matrix, $\mathcal{E}(\cdot)$ and $\mathcal{D}(\cdot)$ denote the expectation and the variance of random variable respectively, $\delta_{\min}(\cdot)$ denotes the minimum singular value of a matrix and $\lceil\cdot\rceil$ the ceiling function.

2. Problem statement

We consider the following linear discrete time invariant system

$$X_{k+1} = AX_k + BU_k + \omega_k \tag{1}$$

where $X \in \mathbb{R}^d$ and $U \in \mathbb{R}^m$ are the state and the input respectively, $\omega \in \mathbb{R}^d$ is the unbounded disturbance, ω_k is assumed to be independent on X_k and Lebesgue-measurable, and satisfies $\underline{\omega} \le \mathcal{E}(\|\omega_k\|) \le \overline{\omega} < \infty$ with known bounds $\underline{\omega}$ and $\overline{\omega}$, and A and B are system matrices with appropriate dimensions and $\|A\| > 1$.

A noiseless digital channel connects sensor with controller, which can transmit one code word at each time step. The quantization value $\bar{X}_k$ of the state X_k is $Q(\Delta X_k) + \hat{X}_k$, where $Q(\cdot)$ is the function of the quantizer (see Section 3.1), $\Delta X_k = X_k - \hat{X}_k$, and $\hat{X}_k$ is the state of the quantizer which is updated as

$$\hat{X}_{k+1} = A(Q(\Delta X_k) + \hat{X}_k) + BU_k. \tag{2}$$

We use the quantized state feedback controller $U_k = K\bar{X}_k = K(Q(\Delta X_k)+\hat{X}_k)$ for the system, where the matrix K, quantized state feedback gain, makes $A + BK$ Schur stable.

By (1) and (2) we have

$$\Delta X_{k+1} = A(\Delta X_k - Q(\Delta X_k)) + \omega_k. \tag{3}$$

In this paper, $\{X_k, k \ge 0\}$ and $\{\Delta X_k, k \ge 0\}$ are defined on a probability space $(\Omega, \mathcal{F}, P)$, where Ω is the sample space, $\mathcal{F}$ a sigma field of subsets of Ω, and P a probability measure.

Definition 2.1. The system (1) is said to be first moment stable if $\limsup_{k\to\infty}\mathcal{E}(\|X_k\|) < \infty$.

The aim of this paper is to design coding schemes such that the considered system is first moment stable and the upper limit of the rate of the expectation of state norm to the expectation of the disturbance norm is less than a given number γ, that is,

$$\limsup_{k\to\infty} \frac{\mathcal{E}(\|X_k\|)}{\mathcal{E}(\|\omega_k\|)} < \gamma. \tag{4}$$

3. Quantizer based on spherical polar coordinates and main results

The encoder and the decoder in this paper will be restricted to use the quantizer based on spherical polar coordinates since under spherical polar coordinate the coding scheme facilitates the stability analysis of systems [18]. Let the vector $X = [x_1 \quad x_2 \quad \cdots \quad x_{d-1} \quad x_d]^{\mathrm{T}} \in \mathbb{R}^d$,where the notation "T" means transpose. Then we call the column $[x_1 \quad x_2 \quad \cdots \quad x_{d-1} \quad x_d]^{\mathrm{T}}$ as the Cartesian rectangular coordinate of x. The vector can also be represented using spherical polar coordinate

$$\begin{bmatrix} r \\ \theta_1 \\ \vdots \\ \theta_{d-2} \\ \theta_{d-1} \end{bmatrix} \in \mathbb{B}^d := \left\{ \begin{bmatrix} r \\ \theta_1 \\ \vdots \\ \theta_{d-2} \\ \theta_{d-1} \end{bmatrix} : 0 \le r < \infty, \right.$$

$$\left. 0 \le \theta_1, \theta_2, \ldots, \theta_{d-2} \le \pi, 0 \le \theta_{d-1} \le 2\pi \right\}$$

via the coordinate transformation