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Input/output-to-state stability of impulsive switched systems[☆]

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ABSTRACT

This paper studies the input/output-to-state stability (*IOSS*) of impulsive switched systems. With the help of Lyapunov and average dwell-time (ADT) methods, some sufficient conditions for *IOSS* of impulsive switched systems are obtained, where both types of impulses, stabilizing impulses and destabilizing impulses, are considered. It is shown that when all of the modes are *IOSS*, a switched system under an ADT scheme is *IOSS* even if there exist destabilizing impulses, and when none of the modes is *IOSS*, *IOSS* can still be achieved under the designed ADT scheme coupled with stabilizing impulses. In particular, for a special case in which an impulsive switched system is composed of some *IOSS* modes and some non-*IOSS* modes, a relationship is established among the ADT scheme, impulses, and the total dwell time between non-*IOSS* and *IOSS* modes such that the impulsive switched system is *IOSS*. Two examples are provided to illustrate the applications of our results.

1. Introduction

Switched systems are a special class of hybrid systems that consist of two components — a family of subsystems and a switching signal. In switched systems, we often call each subsystem a mode. The switching signal selects an active mode at every instant of time. As a special class of hybrid systems, switched systems arise in a variety of applications, such as biological systems [1], automobiles and locomotives with different gears [2], DC–DC converters [3], manufacturing processes [4], and shrimp harvesting methods [5]. Many interesting results for switched systems have been reported in the literature [6–11], among which [11,12] addressed switched systems with unstable modes. Another important type of hybrid system is the impulsive system, which is composed of three parts: a continuous process, which governs the continuous evolution of the system between impulses; a discrete process, which governs the way in which the system state is suddenly changed at impulse times; and an impulsive law, which determines when the impulses occur. Earlier works on impulsive systems can be found in [13–15]. These works have successfully built a theory with formal definitions and initial assumptions on the impulsive systems. As a control method, impulsive control has received much attention because it is discontinuous and usually has a simple structure, and only discrete control is needed to achieve the desired performance [16–19]. Applications of impulsive control include sampled data control systems [20], networked control systems [21,22], multi-agent systems [23], biological systems and ecological systems [24–26]. On the other hand, some switched systems in the real world, such as optimal control models in economics, frequency-modulated signal processing systems, bursting rhythm models in pathology, and flying object motions, display a certain kind of dynamics with impulsive effects at the switching points [15,27,28]. These kinds of switched systems correspond to a more comprehensive model known as the impulsive switched system, which cannot be well described by purely continuous or purely discrete models. A fundamental analysis of the properties of impulsive switched systems can be found in [15]. During the last decade, impulsive switched systems have received considerable attention; see [29] and [30].

When investigating the dynamics of a nonlinear system, it is necessary to characterize the effects of external inputs. The concepts of input-to-state stability (*ISS*), integral-input-to-state stability (*iISS*), and input/output-to-state stability (*IOSS*) [31–33] have proved very useful in characterizing the effects of external inputs. Apart from being a tool for analysis, the theory of *ISS* has had a central role in the design of nonlinear control systems, in particular in robust stabilization for nonlinear systems, tracking design, small-gain theorems for nonlinear systems, and the design of nonlinear observers [11,33,34]. It is known that an *ISS* system

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satisfies the asymptotic gain property (*AG*). This implies that regardless of the size of the initial state, the state will eventually tend to a neighborhood of the origin, the size of which is proportional to the magnitude of the input. *IOSS*, which considers the effect of output, satisfies the input/output-asymptotic gain property (*IO-AG*) [35]. This implies that regardless of the initial state, if the inputs and the outputs are small, then the state of the system will eventually become small. Similar to *ISS* and *iISS*, the notation of *IOSS* was originally introduced for continuous-time systems [33,36,37] and then developed for discrete-time systems [38,39] and switched systems [11]. Note that the *ISS* and *iISS* properties of impulsive systems have been studied in [18,19,29,40–42]. The possibility of impulse effects on *IOSS*, however, has not been addressed in these works [11,33,36–41]. It is still an open problem and remains challenging.

In this paper, we study the *IOSS* of impulsive switched systems, where two types of impulses, the stabilizing impulse and the destabilizing impulse, are considered. By the Lyapunov method and the ADT method, some sufficient conditions ensuring *IOSS* are obtained. The main contributions of this paper lie in two facts. First, we establish a relationship among the ADT scheme, impulses, and the decay or growth of system trajectories such that the impulsive switched system is *IOSS* for the following two cases: all the modes are *IOSS* with destabilizing impulses and none of the modes is *IOSS* with stabilizing impulses. Second, we find the relationship for the total dwell time between non-*IOSS* and *IOSS* modes such that the impulsive switched system including *IOSS* and non-*IOSS* modes is *IOSS* for both types of impulses. The rest of this paper is organized as follows. In Section 2, the problem is formulated, and some notations and definitions are given. In Section 3, the main results are presented. Two examples are given in Section 4, and conclusions are given in Section 5.

Notations. Let $\mathbb{Z}_+$ denote the set of positive integer numbers, $\mathbb{R}$ the set of real numbers, $\mathbb{R}_+$ the set of all nonnegative real numbers, and $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ the n-dimensional and $m \times n$-dimensional real spaces equipped with the Euclidean norm $|\cdot|$, respectively. $a \wedge b$ and $a \vee b$ are the minimum and maximum of a and b, respectively. $P = \{1, 2, \dots, m\}$, $m \in \mathbb{Z}_+$ is an index set. For $J \subseteq \mathbb{R}$ and $S \subseteq \mathbb{R}^k$ with $1 \le k \le n$, let $C(J, S) = \{\varphi : J \to S, \varphi \in C_0\}$ and $\mathcal{F} = \{\varphi : [t_0, \infty) \to P, \varphi \in F_0\}$, where C_0 is the set of continuous functions, and F_0 is the set of piecewise constant functions.

2. Preliminaries

Consider the following impulsive switched system,

$$\begin{cases} \dot{x} = f_{\sigma(t)}(x, u),\ t \neq t_n,\ n \in \mathbb{Z}_+, \\ x(t) = g_{\sigma(t^+)}(x(t^-)),\ t = t_n,\ n \in \mathbb{Z}_+, \\ y = h_{\sigma(t)}(x(t)), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is a measurable locally bounded disturbance input, $y \in \mathbb{R}^l$ is the output, $\dot{x}$ denotes the right-hand derivative of x, and $\sigma \in \mathcal{F}$ denotes a piecewise constant signal, which is called a switching signal. When $\sigma(t) = i$, $1 \le i \le m$, the mode $\dot{x} = f_i$ and impulse jump $x = g_i$ are activated. A sequence of discrete times, $\{t_n\}$, $n \in \mathbb{Z}_+$, known as the set of impulse-switching times, determines when the switching and the impulse occur. Throughout this paper, we assume that $0 \le t_0 < t_1 < \cdots < t_k \to +\infty$ as $k \to +\infty$ (t_1 is the first impulse-switching time). In particular, we exclude the case where $\{t_k\}$ has a finite accumulation point (often referred to as Zeno behavior [13,14]), implying that a switching signal σ has at most finitely many switching times and impulse times over a finite time interval. We also assume that $f_\sigma \in C(\mathbb{R}^n \times \mathbb{R}^m, \mathbb{R}^n)$ is locally Lipschitz, $g_\sigma \in C(\mathbb{R}^n, \mathbb{R}^n)$, $h_\sigma \in C(\mathbb{R}^n, \mathbb{R}^l)$, and $f_\sigma(0, 0) = g_\sigma(0) = h_\sigma(0) \equiv 0\ \forall \sigma \in \mathcal{F}$. All signals in this paper, including the state x and the input u, are assumed to be right continuous and to have left limits at all times. In view of this, we denote by $(\cdot)^-$ and $(\cdot)^+$ the left-limit and right-limit operators, respectively, i.e., $x(t^-) = \lim_{s\nearrow t} x(s)$ and $x(t^+) = \lim_{s\searrow t} x(s)$. We assume that the solution of system (1) with initial condition $x(t_0) = x_0$ uniquely exists on $[t_0, +\infty)$.

Note that the properties of solutions to system (1) (e.g., *IOSS* properties to be investigated in this paper) are severely affected by the sequence $(\{t_n\}, \sigma)$, which is composed of impulse-switching times and a switching signal. We call this kind of sequence an impulsive switching signal. It is known that the average dwell-time (ADT) method proposed by Hespanha and Morse [43] for switching systems and Hespanha et al. [40] for impulsive systems has been used in many control and engineering problems. Based on the ADT idea, let $\mathscr{F}_+[\tau, N_0]$ denote a class of impulsive switching signals satisfying the average dwell-time (ADT) condition given by

$$N_\sigma(T, t) \le N_0 + \frac{T - t}{\tau},\ \forall T \ge t \ge t_0, \tag{2}$$

where $N_0 > 0$ is called the chatter bound, $\tau > 0$ denotes the ADT constant, and $N_\sigma(T, t)$ is the number of switches and impulses occurring in the interval $[t, T)$. Especially when $N_0 = 1$, this implies that consecutive impulses must be separated by at least τ units of time. Similarly, let $\mathscr{F}_-[\tau, N_0]$ denote a class of impulsive switching signals satisfying the reverse ADT condition given by

$$N_\sigma(T, t) \ge -N_0 + \frac{T - t}{\tau},\ \forall T \ge t \ge t_0, \tag{3}$$

where $\tau > 0$ denotes the reverse ADT constant. Throughout this paper, denote the impulse-switching times in the interval $(t, T]$ by $t_1, t_2, \dots, t_{N_\sigma(t,t_0)}$ and the index of the mode activated in the interval $[t_n, t_{n+1})$ by p_n.

A function $\alpha : [0, \infty) \to [0, \infty)$ is of class $\mathcal{K}$ if α is continuous, strictly increasing, and $\alpha(0) = 0$. In addition, if α is unbounded, it is of class $\mathcal{K}_\infty$. A function $\beta : [0, \infty) \times [0, \infty) \to [0, \infty)$ is of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $t \ge 0$ and $\beta(r, t)$ decreases to 0 as $t \to \infty$ for each fixed $r \ge 0$.

Given an impulsive switched signal $(\{t_n\}, \sigma)$, the system (1) is input/output-to-state stable (*IOSS*) if there exist functions γ_1, $\gamma_2 \in \mathcal{K}_\infty$ and $\beta \in \mathcal{KL}$ such that for each $t_0 \ge 0$, $x_0 \in \mathbb{R}^n$ and each input u, the solution satisfies

$$|x(t)| \le \beta(|x_0|, t - t_0) + \gamma_1(|u|_{[t_0,t]}) + \gamma_2(|y|_{[t_0,t]}), \tag{4}$$

for all $t \ge t_0$, where $|\cdot|_J$ denotes the supremum norm on the interval J; see [35] for detailed information. In the case in which there is no output term, that is, $\gamma_2 \equiv 0$, *IOSS* reduces to input-to-state stability (*ISS*). This definition depends on the choice of the impulsive switched signal; however, it is of interest to characterize *IOSS* or *ISS* properties that are uniform over a certain class of impulsive switching signals. The system (1) is uniformly input/output-to-state stable (*UIOSS*) over a given class $\mathscr{F}$ (of impulsive switching signal) if the system is *IOSS* for every sequence in $\mathscr{F}$ with functions β, γ_1 and γ_2 that are independent of the choice of the sequence.

Given a locally Lipschitz function $V_i : \mathbb{R}^n \to \mathbb{R}_+$, $i \in P$, the upper right-hand Dini derivative of V_i with respect to the ith mode of system (1) is defined by

$$D^+V_i(x) = \lim_{h\to 0^+} \sup \frac{1}{h}[V_i(x + hf_i(x, u)) - V_i(x)].$$

3. Main results

In this section, we present some sufficient conditions to ensure *IOSS* of the impulsive switched systems, where two types of impulses, stabilizing impulses and destabilizing impulses, are considered. We start with the situation in which every mode is *IOSS* or non-*IOSS*, and then, we consider the mixed case.