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Robust pole placement under structural constraints

João Fábio S. dos Santos, Paulo C. Pellanda, Alberto M. Simões *

Department of Electrical Engineering, Instituto Militar de Engenharia, Rio de Janeiro, RJ, 22290-270, Brazil

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ABSTRACT

A new controller synthesis technique is presented which allows the design of output feedback control systems achieving robust regional pole clustering in the presence of parametric uncertainties as well as satisfying prescribed structural constraints. Such features are rarely jointly present in currently available controller synthesis methods. The central idea in the proposed approach consists in reformulating the original robust pole placement problem into an equivalent robust stabilization problem involving highly structured controller and uncertainty. A numerical application corroborating the applicability of the proposed synthesis technique is also presented.

1. Introduction

Closed-loop robustness in the presence of parametric uncertainty represents a primary design objective in any modern control system synthesis method, e.g. sliding mode [1], adaptive [2] or robust [3] control. In the particular case of linear time invariant systems, an appealing robust controller synthesis technique is regional pole placement. As well-known, by clustering the closed-loop poles in appropriate regions of the complex plane, the design engineer can set bounds on the damping ratio, the decay rate or the undamped natural frequency of the closed-loop modes, and hence shape time-domain parameters of the system response, e.g. rise time, settling time or overshoot. As noted in [4], from an application viewpoint, regional pole clustering is more important than exact pole assignment. The reader is referred, e.g. to [5,4] and the references within for earlier research on the subject.

In the seminal work [6], sufficient conditions are obtained for pole placement in a general class of convex regions of the complex plane defined by linear matrix inequalities (LMI) constraints. Interestingly, the resulting controller synthesis problem can be solved very efficiently via semidefinite programming tools. Moreover, in the LMI framework, pole placement constraints can be considered simultaneously with other design criteria, e.g. H_∞ constraints. That technique has been expanded in [7] so as to address the so-called robust $\mathcal{D}$-stabilization problem, i.e. robust pole placement in LMI-regions.

Different approaches have been proposed subsequently in the literature dealing with robust $\mathcal{D}$-stabilization. See, e.g., [8,9] and references therein. Unfortunately, these techniques are limited to state feedback control and hence become inoperative in output feedback control problems.

In the output feedback case, a notorious limitation of LMI-based controller synthesis techniques is the difficulty in handling structural constraints on the control law itself. The LMI-based robust pole placement technique in [7], for instance, can only produce full-order output feedback controllers. In [10], an LMI technique is described to perform robust pole placement on second-order linear systems, but the only controller structure that can be handled is proportional-derivative control. The technique in [11] allows the design of fixed-order H_∞ controllers also achieving $\mathcal{D}$-stabilization, but the technique is limited to SISO systems.

The controller synthesis technique recently introduced in [3] allows the design of reduced-order output feedback controllers ensuring closed-loop robust $\mathcal{D}$-stability, hence potentially overcoming the difficulty of LMI-based techniques indicated above. However, it is stated by the author that only systems with small size can be handled, due to the associated computational burden. The numerical applications discussed in [3], for instance, involve only static controllers with a single tunable parameter.

The recurrent difficulty of LMI approaches in designing structured controllers is one of the main motivations behind the regain of interest seen in the last decade in controller synthesis techniques based on nonsmooth optimization, e.g. [12–19]. Another motivation lies in the numerical difficulties that LMI and bilinear matrix inequalities (BMI) [20] techniques tend to face for problems of moderate size. Such a difficulty is mainly due to the presence of Lyapunov variables, whose number grows quadratically with the order of the closed-loop system [13]. Nonsmooth controller synthesis techniques, on the other hand, can handle systems with dozens of states, even hundreds, see e.g. [21,22].

Pole placement constraints appearing initially in the aforementioned nonsmooth optimization-based synthesis techniques

* Corresponding author.
E-mail address: simoes@ime.eb.br (A.M. Simões).

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involved essentially half-plane constraints via the spectral abscissa function [12,23,13,21]. More recently, however, a more general region of the complex plane has been considered in [17] specifically for pole placement. The synthesis technique in [17] allows the design of structured controllers achieving regional pole placement, but it unfortunately presents a serious inconvenience: similarly as in [6], robustness of the pole clustering is not explicitly addressed, but instead handled indirectly via an additional H_∞ constraint. As well known, unscaled H_∞ constraints are not the most appropriate way to deal with structured uncertainties, e.g. parametric uncertainties [24].

In the present work, a new controller synthesis technique is introduced that allows the design of output feedback controllers satisfying prescribed structural constraints as well as achieving robust pole placement in the presence of parametric uncertainties. The central idea in the proposed approach is to reformulate the original robust pole placement problem into an equivalent robust stabilization problem, which can be interpreted as a particular μ-synthesis [25] problem involving highly structured uncertainty and controller. The resulting synthesis problem can then be solved efficiently by means of a recently introduced parametric robust structured control design technique [18].

The paper is organized as follows. In Section 2, it is discussed how the pole clustering robustness can be assessed via an equivalent robust stability condition. In Section 3, the considered robust pole placement synthesis problem is formulated, and its solution is discussed. A numerical application illustrating the validity of the proposed approach is then presented in Section 4. Section 5 concludes the paper.

Notation: For a given real matrix $A \in \mathbb{R}^{n\times n}$, $\lambda_i[A]$ stands for the set of n eigenvalues of A, i.e. $\lambda_i[A] \triangleq \{\lambda \in \mathbb{C} : \det(\lambda I - A) = 0\}$, whereas $\alpha(A)$ denotes the spectral abscissa of A, i.e. $\alpha(A) \triangleq \max\{\mathrm{Re}(\lambda) : \lambda \in \lambda_i[A]\}$. If every eigenvalue of matrix A has strictly negative real part, then A is called a Hurwitz matrix. Symbol $\otimes$ stands for the Kronecker product. For a given matrix $H \in \mathbb{C}^{n\times m}$, $\overline{\sigma}(H)$ stands for its largest singular value. For a complex matrix P partitioned as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \in \mathbb{C}^{(p_1+p_2)\times(q_1+q_2)},$$

and a matrix $\Delta \in \mathbb{C}^{q_1\times p_1}$, notation $\Delta \star P$ stands for the classical upper linear fractional transformation (LFT) [24] given by

$$\Delta \star P \triangleq P_{22} + P_{21}\Delta(I - P_{11}\Delta)^{-1}P_{12}.$$

For two transfers $\mathcal{G}$ and $\mathcal{H}$, notation $(\mathcal{G}, \mathcal{H})$ stands for the closed-loop interconnection

$$\begin{cases} y = \mathcal{G}u, \\ u = \mathcal{H}y. \end{cases}$$

2. Robust $\mathcal{D}$-stability analysis

Consider the uncertain linear time-invariant system

$$\dot{x}(t) = (\Delta \star M)x(t), \quad (1)$$

where $M \in \mathbb{R}^{(r+n)\times(r+n)}$ represents the nominal state matrix and $\Delta \in \mathbf{\Delta} \subset \mathbb{R}^{r\times r}$ represents a block diagonal uncertainty matrix whose structure is given by

$$\mathbf{\Delta} \triangleq \left\{ \Delta = \mathrm{diag}(\delta_1^r I_{k_1}, \ldots, \delta_{m_r}^r I_{k_{m_r}}) : \delta_i^r \in \mathbb{R} \right\}. \quad (2)$$

The unit ball in $\mathbf{\Delta}$ is denoted by $\mathbf{B\Delta} \triangleq \{\Delta \in \mathbf{\Delta} : \overline{\sigma}(\Delta) \leq 1\}$.

Let $\mathcal{D}$ denote the non-empty region of the complex left-half plane depicted in Fig. 1, constructed as the intersection of a disk, a half plane and a wedge, as follows:

$$\mathcal{D}(q, r, \tau, \theta, \gamma) \triangleq \Omega_c(q, r) \cap \Omega_{hp}(\tau) \cap \Omega_w(\theta, \gamma), \quad (3)$$

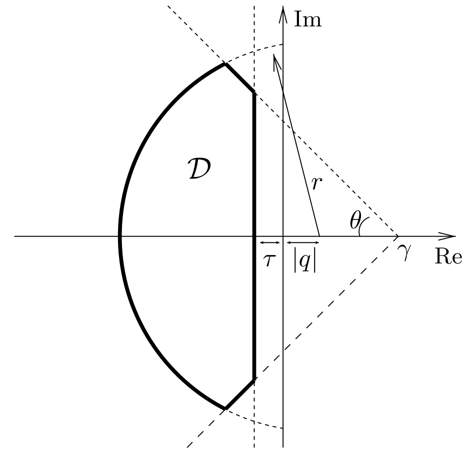

Fig. 1. Region $\mathcal{D}(q, r, \tau, \theta, \gamma)$ for robust pole placement.

with $\gamma, q \in \mathbb{R}$, $\tau, r \in \mathbb{R}_{>0}$, $\theta \in (0, \pi/2)$, and

$$\Omega_c(q, r) \triangleq \left\{ s \in \mathbb{C} : (\mathrm{Re}(s) + q)^2 + \mathrm{Im}(s)^2 < r^2 \right\}, \quad (4)$$

$$\Omega_{hp}(\tau) \triangleq \{ s \in \mathbb{C} : \mathrm{Re}(s) < -\tau \}, \quad (5)$$

$$\Omega_w(\theta, \gamma) \triangleq \{ s \in \mathbb{C} : \tan(\theta)(\mathrm{Re}(s) - \gamma) + |\mathrm{Im}(s)| < 0 \}. \quad (6)$$

It is to be recalled that the uncertain system (1) is said to be *robustly stable* if it is stable for all allowable uncertainty, or equivalently, if the eigenvalues of the state matrix $(\Delta \star M)$ lie in the open-left half plane for all $\Delta \in \mathbf{B\Delta}$. If, moreover, the poles of the uncertain system (1) lie in $\mathcal{D}$ for all allowable uncertainty, i.e., $\lambda_i[\Delta \star M] \in \mathcal{D}$ for all $\Delta \in \mathbf{B\Delta}$, then the uncertain system is said to be *robustly $\mathcal{D}$-stable*.

The following theorem, which states the main result of this section, provides a suitable necessary and sufficient condition for the robust $\mathcal{D}$-stability of the uncertain system (1).

Theorem 1. *Consider a non-empty region $\mathcal{D}(q, r, \tau, \theta, \gamma)$, with $q \neq r$, and let*

$$\Gamma(q, r) \triangleq \begin{bmatrix} \dfrac{q+r}{q-r} & \dfrac{1}{q-r} \\ \dfrac{-2r}{q-r} & \dfrac{-1}{q-r} \end{bmatrix} \otimes I_n. \quad (7)$$

Then, the uncertain system (1) is robustly $\mathcal{D}$-stable if and only if the system

$$\dot{x}(t) = A_\Delta x(t), \quad (8)$$

with

$$A_\Delta \triangleq diag\left(\Gamma \star (\Delta \star M), (\Delta \star M) + \tau I_n, \begin{bmatrix} \sin(\theta) & -\cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{bmatrix} \otimes ((\Delta \star M) + \gamma I_n) \right), \quad (9)$$

is robustly stable.

Proof. For a given matrix $A \in \mathbb{R}^{n\times n}$, let

$$A_c(q, r) \triangleq \Gamma(q, r) \star A, \quad (10)$$

$$A_{hp}(\tau) \triangleq A + \tau I_n, \quad (11)$$

$$A_w(\theta, \gamma) \triangleq \begin{bmatrix} \sin(\theta) & -\cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{bmatrix} \otimes (A + \gamma I_n). \quad (12)$$