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A unified Razumikhin-type criterion on input-to-state stability of time-varying impulsive delayed systems

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ABSTRACT

This paper discusses the problem of input-to-state stability (ISS) of time-varying impulsive delayed systems. By introducing a switching parameter and using the notion of average impulsive interval, a unified Razumikhin-type criterion on ISS which is simultaneously effective for stabilizing impulses and destabilizing impulses is derived. The condition which requires the coefficient of the estimated upper bound of the derivative of a Lyapunov function to be constant in the existing results on ISS of impulsive systems is weakened. The results in this paper allow the coefficient of the derivative of a Lyapunov function to be time-varying function which can be both positive and negative and may even be unbounded. Furthermore, the impulsive intervals of an impulsive sequence are allowed to have arbitrarily small lower bound and large enough upper bound simultaneously. As a by-product, a unified criterion on ISS for time-varying impulsive delay-free systems is also presented. Two examples are presented to illustrate the effectiveness of our results.

1. Introduction

It is well known that the concept of input-to-state stability (ISS) which was introduced by Sontag in [1] has played important role in stability analysis and control system design. In the past nearly thirty years, various extensions of the ISS properties have since been made for many types of dynamical systems such as discrete systems [2], delayed systems [3–6], impulsive systems [7–9], impulsive delayed systems [8,10–15], etc.

In this paper, we are concerned with the ISS properties of time-varying impulsive delayed systems (TVIDSs), with input signals affecting both the continuous dynamics and the state impulse map. In general, there are two kinds of impulses in terms of stability in impulsive systems. An impulsive sequence is said to be destabilizing if the impulsive effect can suppress the stability of the impulsive system. An impulsive sequence is said to be stabilizing if a corresponding impulsive effect can enhance the stability of the impulsive systems. In the current literature, there are some unified criteria on ISS which are simultaneously valid for these two kinds of impulses based on average impulsive interval (see [7,8,14]). There are also some results on ISS which are devoted to investigated these two kinds of impulses separately by using the lower bound of impulsive intervals for destabilizing impulses and the upper bound of impulsive intervals for stabilizing impulses (see [10–13]). However, all of the results require the coefficients of the estimated upper bound for the time-derivative of a Lyapunov function or a Lyapunov functional to be constant numbers. As shown by examples in Section 4, the existing results cannot be applied to analyze such systems. Hence, it is necessary and interesting to generalize the existing theory by allowing derivative associated with a Lyapunov function or a Lyapunov functional to satisfy a weaker assumption, which involves time-varying coefficients. Recently, Lyapunov stability and ISS properties are investigated by using Lyapunov function or Lyapunov functional whose time-derivative can take both positive and negative values for continuous time-varying dynamical systems with and without delays [5,6,16–20]. However, as far as we know, there are few results on ISS of impulsive systems by Lyapunov functions with indefinite derivatives.

This paper aims to establish a unified Razumikhin-type criterion on ISS of TVIDSs which is simultaneously effective for stabilizing impulses and destabilizing impulses under average impulsive interval condition. With the help of notions of a scalar uniformly exponentially stable function (UESF) introduced in [18] and average impulsive interval, the criterion of this paper allows the estimated upper bound of the derivative of the Lyapunov function to have time-varying coefficient (the coefficient may be sign-changing or unbounded function), and the impulsive sequence is allowed to have arbitrarily small lower bound and sufficiently big upper bound for impulsive intervals simultaneously. As a by-product, a unified criterion on ISS for time-varying impulsive delay-free systems can also be provided easily.

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This paper is organized as follows. In Section 2, the time-varying impulsive delayed system is presented, together with some definitions and two lemmas. A new Razumikhin-type criterion is obtained to ensure the ISS properties of TVIDSs in Section 3, and a criterion on ISS for time-varying impulsive systems without time-delays is also given. Finally, two examples are presented to illustrate the effectiveness of our results.

Notation: Throughout this paper, unless otherwise specified, we use the following notations. Let $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{N} = \{1, 2, \ldots\}$, $\mathbb{N}_{a,b} = \{a, a+1, \ldots, b\}$ for $a, b \in \mathbb{N}$ and $a \leqslant b$. If x and y are real numbers, then $x \vee y$ and $x \wedge y$ denote the maximum and minimum of x and y, respectively. If A is a vector or matrix, its transpose is denoted by A^T. For $x \in \mathbb{R}^n$, let $|x| = \sqrt{x^T x}$ be the Euclidean vector norm. For $-\infty < a < b < \infty$, we use the notation $\mathbb{PC}([a, b]; \mathbb{R}^n)$ to denote the class of functions from $[a, b]$ to $\mathbb{R}^n$ satisfying the following: (i) it has at most a finite number of jump discontinuities on $(a, b]$, i.e., points at which the function has finite-valued but different left-hand and right-hand limits; (ii) it is continuous from the right at all points in $[a, b)$. For $\psi \in \mathbb{PC}([a, b]; \mathbb{R}^n)$, its norm is defined as $\|\psi\|_{[a,b]} = \sup_{a \leqslant s \leqslant b} |\psi(s)|$. We say that a function $\psi : [a, \infty) \to \mathbb{R}^n$ belongs to the class $\mathbb{PC}([a, \infty); \mathbb{R}^n)$, if $\psi|_{[a,b]}$ (ψ restricted on $[a, b]$) is in $\mathbb{PC}([a, b]; \mathbb{R}^n)$ for all $b > a$. Given $\tau > 0$, a norm on $\mathbb{PC}([-\tau, 0]; \mathbb{R}^n)$ is defined as $\|\phi\| = \|\phi\|_{[-\tau,0]}$ for $\phi \in \mathbb{PC}([-\tau, 0]; \mathbb{R}^n)$. For simplicity, $\mathbb{PC}_\tau$ is used for $\mathbb{PC}([-\tau, 0]; \mathbb{R}^n)$ for the rest of this paper. Given $x \in \mathbb{PC}([t_0-\tau, \infty); \mathbb{R}^n)$ and for each $t \in [t_0, \infty)$, let x_t be an element for $\mathbb{PC}_\tau$ defined by $x_t(\theta) := x(t + \theta)$, $-\tau \leqslant \theta \leqslant 0$. Let $\mathcal{K}$ represent the class of continuous strictly increasing function $\chi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\chi(0) = 0$. $\mathcal{K}_\infty$ is the subset of $\mathcal{K}$ functions that are unbounded. A function $\varpi : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is said to be the class $\mathcal{KL}$, if $\varpi(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $t > 0$ and $\varpi(s, t)$ decreases to 0 as $t \to \infty$ for each fixed $s \geqslant 0$.

2. Preliminaries

Consider the time-varying impulsive system of the following form:

$$\begin{cases} x'(t) = f(t, x_t, u_c(t)), & t \neq t_k, \ t \geqslant t_0 \\ x(t_k) = I_k(t_k, x(t_k^-), u_d(t_k^-)), & t = t_k, \ k \in \mathbb{N} \\ x(t_0 + s) = \phi(s), & -\tau \leqslant s \leqslant 0 \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $x'(t)$ denotes the right-hand derivative of $x(t)$, $u_c \in \mathbb{PC}([t_0, \infty); \mathbb{R}^{m_1})$ denotes the disturbance input, $u_d \in \mathbb{PC}([t_0, \infty); \mathbb{R}^{m_2})$ denotes the impulsive disturbance input. $f : \mathbb{R}^+ \times \mathbb{PC}_\tau \times \mathbb{R}^{m_1} \to \mathbb{R}^n$. The initial function $\phi \in \mathbb{PC}_\tau$. The impulsive functions $I_k : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^{m_2} \to \mathbb{R}^n (k \in \mathbb{N})$, and the impulsive instants $t_k (k = 1, 2, \ldots)$ satisfy $0 \leqslant t_0 < t_1 < t_2 < \cdots$ and $\lim_{k\to\infty} t_k = \infty$. We define $g(t, \psi) = f(t, \psi, u_c(t))$ and suppose that $g : \mathbb{R}^+ \times \mathbb{PC}_\tau \to \mathbb{R}^n$ is composite-$\mathbb{PC}$ (i.e., for any function $x \in \mathbb{PC}([t_0 - \tau, \infty); \mathbb{R}^n)$, the composite function $t \mapsto g(t, x_t)$ is in $\mathbb{PC}([t_0, \infty); \mathbb{R}^n)$), quasibounded, and locally Lipschitz in its second variable so that system (1) has a unique solution $x(t, t_0, \phi)$ which exists in a maximal interval $[t_0 - \tau, b)$, where $t_0 < b \leqslant \infty$ (see [21]). In addition, we assume that $f(t, 0, 0) \equiv 0$ and $I_k(t_k, 0, 0) \equiv 0$ for all $t \geqslant t_0$ so that system (1) admits a trivial solution $x(t) \equiv 0$.

Definition 1 (*[10]*). For the prescribed impulsive sequence $\{t_k, k \in \mathbb{N}\}$, system (1) is said to be input-to-state stable (ISS), if there exist functions $\varpi \in \mathcal{KL}$ and $\gamma_c, \gamma_d \in \mathcal{K}_\infty$, such that for every $\phi \in \mathbb{PC}_\tau$ and every pair of input (u_c, u_d), the solution $x(t)$ of (1) is globally and satisfies

$$\begin{aligned} |x(t)| \leqslant\ & \varpi(\|\phi\|, t - t_0) + \gamma_c(\|u_c\|_{[t_0,t]}) \\ & + \gamma_d(\max_{t_k \in (t_0,t]} \{|u_d(t_k^-)|\}), \quad t \geqslant t_0. \end{aligned} \tag{2}$$

Definition 2 (*[7,22]*). For an impulsive sequence $\{t_k, k \in \mathbb{N}\}$, let $N(t, s)$ be the number of instant t_k in the semi-open interval $(s, t]$. If

$$\frac{t-s}{T_a} - N_0 \leqslant N(t, s) \leqslant \frac{t-s}{T_a} + N_0 \tag{3}$$

for $T_a > 0, N_0 > 0$, then T_a and N_0 are called the average impulsive interval (AII) and the elasticity number, respectively.

Remark 1. In [22], the authors gave a specific impulsive sequence $\{t_k, k \in \mathbb{N}\}$ satisfying (3) as shown in the following form:

$$t_k - t_{k-1} = \begin{cases} \epsilon & \text{if } \mathrm{mod}(k, N_0) \neq 0, \\ N_0(T_a - \epsilon) + \epsilon & \text{if } \mathrm{mod}(k, N_0) = 0 \end{cases} \tag{4}$$

where ϵ and T_a are positive numbers satisfying $\epsilon \leqslant T_a$, and $N_0 \in \mathbb{N}$. We have $\inf_{k\in\mathbb{N}}\{t_k - t_{k-1}\} = \epsilon$ and $\sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} = N_0(T_a - \epsilon) + \epsilon$. When ϵ is sufficiently small and N_0 is sufficiently large, $\inf_{k\in\mathbb{N}}\{t_k - t_{k-1}\}$ will be small, and $\sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\}$ will be large.

Definition 3. A function $V : [t_0 - \tau, \infty) \times \mathbb{R}^n \to \mathbb{R}^+$ is said to belong to class ν_0, if V is continuous on each of the sets $[t_{k-1}, t_k) \times \mathbb{R}^n$, $\lim_{(t,y)\to(t_k^-,x)} V(t, y) = V(t_k^-, x)$ exists. For $V \in \nu_0$, the upper right-hand derivative of V with respect to system (1) is defined by $D^+V(t, \psi(0)) = \limsup_{h\to 0^+} \frac{1}{h}[V(t + h, \psi(0) + hf(t, \psi, u_c(t))) - V(t, \psi(0))]$, for $t \in [t_{k-1}, t_k)$ $(k \in \mathbb{N})$, $\psi \in \mathbb{PC}_\tau$.

We introduce the concept of uniformly exponentially stable function (UESF) proposed in [18]. Consider the following scalar linear time-varying (LTV) system:

$$y'(t) = \mu(t)y(t), \quad t \in J = [t^\#, \infty), \tag{5}$$

where $y : J \to \mathbb{R}$ is the state variable and $\mu \in \mathbb{PC}(J; \mathbb{R})$.

Definition 4 (*[18]*). The function $\mu \in \mathbb{PC}(J; \mathbb{R})$ is said to be uniformly exponentially stable with guaranteed decay rate α(UES [WGDR α]) if system (5) is UES [WGDR α], namely, there exist constants $k > 0$ and $\alpha > 0$ such that

$$|y(t)| \leqslant k|y(t_0)| \exp(-\alpha(t - t_0)), \quad \forall t \geqslant t_0 \in J. \tag{6}$$

Lemma 1 (*[18]*). *The scalar function $\mu \in \mathbb{PC}(J; \mathbb{R})$ is UES [WGDR α] if and only if there exist constants $\alpha > 0$ and $\beta \geqslant 0$ such that*

$$\int_{t_0}^{t} \mu(s)\mathrm{d}s \leqslant -\alpha(t - t_0) + \beta, \quad \forall t \geqslant t_0 \in J. \tag{7}$$

The following lemma will also be needed to obtain our main results.

Lemma 2. *Assume that the function $\gamma \in \mathcal{K}$. The inequality*

$$\gamma(x + y + z) \leqslant \gamma(3x) + \gamma(3y) + \gamma(3z)$$

holds for all $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$.

Proof. Noting that $\gamma \in \mathcal{K}$, for any non-negative numbers x, y and z, we have

$$\gamma(x + y + z) \leqslant \begin{cases} \gamma(3x) & \text{if } x \geqslant y \vee z \\ \gamma(3y) & \text{if } y \geqslant z \vee x \\ \gamma(3z) & \text{if } z \geqslant x \vee y. \end{cases}$$

Therefore

$$\begin{aligned} \gamma(x + y + z) &\leqslant \max\{\gamma(3x), \gamma(3y), \gamma(3z)\} \\ &\leqslant \gamma(3x) + \gamma(3y) + \gamma(3z) \end{aligned}$$

for all $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$.