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Interval observer design for uncertain discrete-time linear systems

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ABSTRACT

This paper proposes a novel interval observer design method for discrete-time linear systems with unknown but bounded disturbance and measurement noise. The proposed interval observer has a new structure that provides more design degrees of freedom. A direct method based on H_∞ technique is used to improve the accuracy of interval estimation. The design conditions are formulated into linear matrix inequalities, which can be efficiently solved. Two numerical examples are given to illustrate the design and validate the performance of the interval observers.

1. Introduction

State estimation is important in practical control applications as measuring all state variables is often difficult. Therefore, observer design and filter design have received considerable attention over the past decades [1–3]. For instance, [4] proposes a generalized state observer for an array of Markovian coupled networks under the round-robin protocol and redundant channels. An H_∞ filter is designed in [5] to achieve fault detection for networked systems with uncertainties and incomplete information. In [6], a codesign method comprising event-triggered and distributed H_∞ filtering is proposed and applied to a suspension system. Based on a general assumption that the disturbance is unknown but bounded, interval observer can estimate the admissible bounds of the state variables using available information. This practical estimation technique has found applications in a wide range of areas including biological systems and bioreactors [7,8], nonlinear systems control [9], and fault detection and diagnostics [10,11].

In recent years, a number of interval observer design methods have been proposed [7,10,12–14]. The most commonly used technique is the cooperative error approach based on the monotone system theory [15]. The main idea of this method is to design the observer such that the error system is both cooperative and stable. For instance, consider a discrete-time error system $e_{k+1} = (A - LC)e_k$. The interval observer design requires not only Schur stability for $A - LC$ but also that all the elements of $A - LC$ are non-negative. Note that the conventional observer design only requires $A - LC$ to be Schur stable. Compared with the design of a conventional observer, the interval observer design has extra constraints of non-negativity, which may lead to more theoretical difficulty and computational complexity in searching for a gain matrix L to simultaneously ensure the non-negativity and stability of the error system. Therefore, it is non-trivial to design an interval observer. To handle the design difficulty, several methods based on coordinate transformation have been proposed to obtain more relaxed design conditions [12,13,16]. For an interval observer, it is desirable to generate a robust cooperative error system such that the estimated lower and upper bounds are close to the state variables. As pointed out in [17], although the methods based on coordinate transformation can simplify design conditions, its limitation is that the coordinate transform matrix and the observer gain matrix cannot be simultaneously synthesized to fulfill the cooperative property and other performance such as robust constraint.

Interval observer design can be converted into solving a set of linear matrix inequalities (LMIs) or linear program, see e.g. [14,16], and [18]. However, the obtained LMIs are still restrictive and solutions may not exist for some systems. We further observe that most existing interval observer design methods focus on continuous-time systems, and there are limited results on the discrete-time cases [19–22]. In view of these major gaps, we propose a new interval observer design method for uncertain discrete linear systems. Motivated by [14], the proposed interval observer design method in this paper is based on LMIs. This study has two main contributions. First, we develop a new observer structure that not only provides more design degrees of freedom but also relaxes the design conditions. Second, the method takes into account the effects of process disturbance and measurement noise by incorporating H_∞ technique to attenuate uncertainties in order to obtain accurate interval estimation.

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The remainder of this paper is organized as follows. The studied problem and some preliminaries are briefly introduced in Section 2. Section 3 proposes an interval observer with new structure and presents an LMI-based interval observer design method. In Section 4, simulation results are given to illustrate the performance of the proposed method, and the conclusion is given in Section 5.

Notation. In this paper, I_n denotes the $n \times n$ identity matrix, $\|x\|$ represents the L_2-norm of a signal x_k, M^T stands for the matrix transposition of a matrix M and $M^\dagger$ denotes the pseudo-inverse of M. The max operator and the symbols $\geq, >, \leq$, and $<$ on vectors and matrices are applied elementwise. For a real symmetric matrix P, $P \succ 0$ ($P \prec 0$) indicates that P is positive definite (negative definite). An asterisk $*$ is used to represent a term induced by symmetry.

2. Problem formulation and preliminaries

Consider the following discrete-time linear system:

$$\begin{cases} x_{k+1} = Ax_k + Bw_k \\ y_k = Cx_k + v_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^m$, $w_k \in \mathbb{R}^p$ and $v_k \in \mathbb{R}^m$, are the state vector, the measured output vector, the unknown disturbance, and the measurement noise, respectively, and A, B and C are constant matrices of appropriate dimensions. It is assumed that the initial state x_0 is bounded and the disturbance w_k and measurement noise v_k are bounded as

$$w^- \leq w_k \leq w^+, \; v^- \leq v_k \leq v^+ \tag{2}$$

where w^-, w^+, v^- and v^+ are known constants.

Remark 1. For notational simplicity, the control input in system (1) is excluded. It should be noted that the proposed method can readily be applied to the system with control input u_k since u_k is usually known in an observer design.

We aim to develop an interval observer which consists of two dynamical systems to estimate the upper and lower bounds of state x_k, respectively. To this end, we first give the following lemmas.

Lemma 1 ([23]). *Given a vector x_k with $x_k^- \leq x_k \leq x_k^+$ and a matrix A, the following inequality holds:*

$$A^+x_k^- - A^-x_k^+ \leq Ax_k \leq A^+x_k^+ - A^-x_k^- \tag{3}$$

where $A^+ = \max\{0, A\}$ and $A^- = A^+ - A$.

Lemma 2 ([16]). *Any solution of the system*

$$x_{k+1} = Ax_k + w_k \tag{4}$$

with $w_k \in \mathbb{R}^n_+$ and a nonnegative matrix $A \in \mathbb{R}^{n\times n}_+$, is elementwise nonnegative for all $k \geq 0$, provided $x_0 \geq 0$.

Lemma 3 ([24]). *Given matrices $\mathcal{X} \in \mathbb{R}^{a\times b}$, $\mathcal{Y} \in \mathbb{R}^{b\times c}$, and $\mathcal{Z} \in \mathbb{R}^{a\times c}$, with rank$(\mathcal{Y}) = c$. The general solution of $\mathcal{X}\mathcal{Y} = \mathcal{Z}$ is*

$$\mathcal{X} = \mathcal{Z}\mathcal{Y}^\dagger + \mathcal{S}(I - \mathcal{Y}\mathcal{Y}^\dagger) \tag{5}$$

where $\mathcal{S} \in \mathbb{R}^{a\times b}$ is an arbitrary matrix.

Lemma 4 ([25]). *Given a scalar $\gamma > 0$. The discrete-time system described by*

$$\begin{cases} x_{k+1} = \mathcal{A}x_k + \mathcal{B}d_k \\ z_k = \mathcal{C}x_k + \mathcal{D}d_k \end{cases} \tag{6}$$

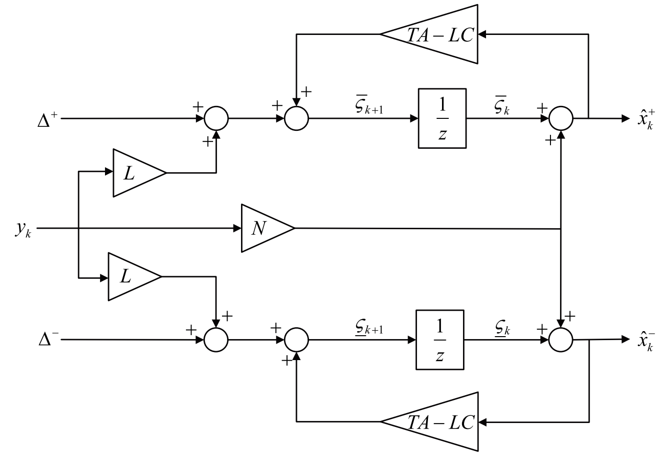

Fig. 1. The block diagram of the proposed interval observer.

is stable and satisfies $\|z\| < \gamma\|d\|$, if and only if there exists a matrix $P \succ 0$ such that

$$\begin{bmatrix} \mathcal{A}^TP\mathcal{A} + \mathcal{C}^T\mathcal{C} - P & \mathcal{A}^TP\mathcal{B} + \mathcal{C}^T\mathcal{D} \\ \mathcal{B}^TP\mathcal{A} + \mathcal{D}^T\mathcal{C} & \mathcal{B}^TP\mathcal{B} + \mathcal{D}^T\mathcal{D} - \gamma^2 I \end{bmatrix} \prec 0 \tag{7}$$

3. Interval observer design

We propose the following interval observer for system (1):

$$\begin{cases} \bar{\zeta}_{k+1} = TA\hat{x}_k^+ + L(y_k - C\hat{x}_k^+) + \Delta_k^+ \\ \hat{x}_k^+ = \bar{\zeta}_k + Ny_k \\ \underline{\zeta}_{k+1} = TA\hat{x}_k^- + L(y_k - C\hat{x}_k^-) + \Delta_k^- \\ \hat{x}_k^- = \underline{\zeta}_k + Ny_k \end{cases} \tag{8}$$

where $\bar{\zeta}_k \in \mathbb{R}^n$ and $\underline{\zeta}_k \in \mathbb{R}^n$ are intermediate variables, $\hat{x}_k^+ \in \mathbb{R}^n$ and $\hat{x}_k^- \in \mathbb{R}^n$ are the estimated upper and lower bounds of x_k, respectively, and $\Delta_k^+ \in \mathbb{R}^n$ and $\Delta_k^- \in \mathbb{R}^n$ are given as

$$\Delta_k^+ = (TB)^+w^+ - (TB)^-w^- + L^+v^+ - L^-v^- + N^+v^+ - N^-v^-, \tag{9}$$

$$\Delta_k^- = (TB)^+w^- - (TB)^-w^+ + L^+v^- - L^-v^+ + N^+v^- - N^-v^+. \tag{10}$$

In (8), $T \in \mathbb{R}^{n\times n}$, $N \in \mathbb{R}^{n\times m}$ and $L \in \mathbb{R}^{n\times m}$ are the matrices to be designed, with T and N satisfying

$$T + NC = I_n. \tag{11}$$

Remark 2. The structure of observer (8) is inspired by [26]. Note that the proposed observer is causal since no future measurements are used in its implementation. Fig. 1 shows the block diagram of the proposed interval observer.

By defining the estimation error as

$$e_k^+ = \hat{x}_k^+ - x_k, \; e_k^- = \hat{x}_k^- - x_k \tag{12}$$

and using (1), (8) and (11), we obtain the following error system:

$$\begin{cases} e_{k+1}^+ = (TA - LC)e_k^+ + \Delta_k^+ - TBw_k + Lv_k + Nv_{k+1} \\ e_{k+1}^- = (TA - LC)e_k^- + \Delta_k^- - TBw_k + Lv_k + Nv_{k+1} \end{cases} \tag{13}$$

For brevity, we define

$$d_k^+ = \begin{bmatrix} \Delta_k^+ - TBw_k \\ v_k \\ v_{k+1} \end{bmatrix}, \; d_k^- = \begin{bmatrix} \Delta_k^- - TBw_k \\ v_k \\ v_{k+1} \end{bmatrix} \tag{14}$$