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Controllability of multi-agent systems with directed and weighted signed networks[☆]

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ABSTRACT

This paper studies the controllability of multi-agent systems with signed networks, which are represented by directed weighted signed graphs. The adjacency weights of network depict the property of interactions. Positive weight means cooperative interaction while negative weight antagonistic interaction. First, by using switching equivalent transformation, the controllability of three kinds of signed networks is studied: structurally balanced, anti-balanced, and strictly unbalanced. It is shown that the controllability of the structurally balanced network is equivalent to that of the associated underlying network. Then, a graph-theoretic necessary condition for controllability is given by virtue of almost equitable partition of directed weighted signed networks. Furthermore, some necessary and sufficient conditions are given for the controllability of generic linear multi-agent systems. In addition, several examples are presented to illustrate the theoretical results.

1. Introduction

In recent years, cooperative control of multi-agent systems (MASs) has been extensively studied, which finds various applications in areas like mobile vehicle cooperative formation, satellite clusters attitude alignment, multiple robots flocking, and so on. Many basic and important issues have been studied in cooperative control of MASs, including consensus [1,2], formation control [3,4], and stabilizability [5,6], to name a few.

Controllability is a fundamental and important research topic in the cooperative control of MASs, and has drawn much attention of researchers from various scientific communities [7–26]. The MAS controllability was first introduced in [7], where a nearest-neighbor leader–follower framework was proposed and a necessary and sufficient condition was established through the eigenspectrum of the resulting Laplacian submatrix from the algebraic viewpoint. Afterwards, researchers investigated MAS controllability from the graph-theoretic perspective. Specifically, various concepts and properties of graph partitions were employed to study MAS controllability, such as equitable partition [8], almost equitable partition [9–12], distance partition [11–13] (see also surveys [14–16] for more details). Recently, some special graphs were investigated and their controllability issues were partially solved. Examples include the paths and cycles [17], trees [18], grids [19], multi-chain [20]. Additionally, there are some other works addressing the MAS controllability with switching topology, heterogeneous dynamics, protocols design, time-delay (see [21–25] and references therein).

It is noteworthy that the aforementioned results on controllability are derived for MASs with only cooperative interactions among agents. However, in reality, the interactions are not always cooperative, antagonistic (competitive) interactions can also exist among agents especially in the social, biological and information fields [26–28]. In social networks, individual relationships may be friendly or hostile. In international community, the relationship between countries could be cooperative or antagonistic. In biochemical and gene regulatory networks, the interactions among cells are either activations or inhibitions. In the field of information, users can express trust or distrust to other users. In practice, the interactions among agents are generally described by signed graphs, where the positive and negative weights represent the cooperative and antagonistic interactions, respectively. Recently, some researchers have addressed the issue of complex dynamic behaviors of MASs, such as bipartite consensus, interval bipartite consensus, bipartite containment [29–31]. In particular, the controllability problem for multi-agent systems with antagonistic interactions was studied in [26], where the networks were described by an undirected unweighted signed graph. In fact, however, the direction and link weights, which in the process of

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information transmission and exchange among the agents, have an important effect on the behavior of MASs. In general, a directed weighted graph is much more complicated than an undirected unweighted graph, e.g., the adjacency matrix and Laplacian matrix of the directed weighted graph are asymmetric matrices. For the MAS controllability issues, how do the directed weighted antagonistic interactions affect the controllability of MASs? What is the relationship between the controllability with directed weighted antagonistic interactions and the controllability with cooperative interactions? What is the difference between controllability of MASs with single integrator dynamics and controllability of MASs with general linear dynamics under the directed weighted antagonistic interactions? These challenging questions deserve further study.

This study is devoted to addressing the issue of MAS controllability with directed weighted signed networks. Firstly, the switching equivalent transformation is employed to study the controllability of structurally balanced, anti-balanced, and strictly unbalanced networks, respectively. Secondly, by virtue of almost equitable partition of directed weighted signed networks, the controllable subspaces are quantitatively studied. Finally, the controllability of generic linear MASs with directed weighted antagonistic interactions is investigated. The main contributions of this work include: (i) The relationship between controllability with antagonistic interactions and controllability with cooperative interactions is addressed. The results show that the controllability of structurally balanced networks is equivalent to that of traditional networks whose adjacency edge weights are all positive. (ii) The graph-theoretic necessary condition for controllability is proposed by virtue of almost equitable partition of directed weighted signed networks. The result provides a quantitative characterization of MAS controllability. (iii) Several necessary and sufficient conditions for controllability of generic linear MASs with directed weighted antagonistic interactions are established. The results indicate that the controllability of generic linear MASs is congruously determined by the interaction network and the agent dynamics.

Compared with the existing works, the advantages and novelties of this study lie in the following aspects. (1) Taking consideration of the directed weighted signed networks, the theoretical results obtained in this paper generalize the ones in [26], where undirected unweighted signed networks were considered. Moreover, anti-balanced and strictly unbalanced signed networks are further investigated compared to [26]. These results not only strengthen the understanding of the differences between the MAS controllability with antagonistic interactions and cooperative interactions, but also gain new insights into the graph-theoretic characterization of controllability with antagonistic interactions. (2) We generalize the concept of almost equitable partitions to directed weighted signed graphs and derive useful properties of almost equitable partitions. This concept includes as special cases the traditional almost equitable partitions of undirected and/or unsigned graphs [9–33] and provide a new perspective to the characterization of directed weighted signed graphs. We also provide a graph-theoretic necessary condition for controllability. The result gives a quantitative analysis for controllability. In addition, not limited to connected [8,9,11,26] and strongly connected [10] networks, our results can also be applied to arbitrary weighted signed networks. (3) Agents are assumed to take single-integrator dynamics in [8–26], while we consider the scenario of general linear agents, which brings new features for the study of the controllability problem. Additionally, some precise necessary and sufficient conditions on the controllability of general linear MASs with antagonistic interactions are obtained. Compared with the recent results (e.g., Theorem 1 of [11], Theorem 1 of [24]), these conditions here do not involve in the eigenvalue of Laplacian matrix and control protocol design. Therefore, it is more direct and easier to verify.

The rest of this paper is organized as follows. In Section 2, the problem of MAS controllability is formulated, and preliminary signed graph concepts are also presented. Section 3 presents the controllability with switching equivalence. The controllability with signed graph partitions is studied in Section 4. In Section 5, the controllability of generic linear MASs is investigated. Finally, the conclusions are given in Section 6.

Notation: Let I_n be the $n \times n$ identity matrix and $diag\{a_1, a_2, \dots, a_n\}$ be a diagonal matrix with a_i, $i = 1, 2, \dots, n$ being the diagonal entries. $\mathbf{1}$ denotes an all-1 vector or matrix and $\mathbf{0}$ represents an all-zero vector or matrix. Let $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{N}$ represent the set of real numbers, complex numbers and natural numbers, respectively. For $i \in \mathbb{N}$, e_i denotes the standard basis vectors. Suppose $A \in \mathbb{R}^{n\times n}$, $\lambda \in \mathbb{C}$ and $\beta \in \mathbb{R}^n$, if $Av = \lambda v$, then (λ, v) is said to be an eigenpair of A. $\sigma(A)$ denotes the set of all the eigenvalues of A. For any matrix B, $img(B)$ denotes the column space of B. $\otimes$ denotes the Kronecker product, $|X|$ denotes the cardinality of a set X, $X \setminus Y$ denotes the set $\{x|\ x \in X, x \notin Y\}$, and $sign(x)$ denotes the sign function of a scalar $x \in \mathbb{R}$. Let V be a vector space and let $T : V \to V$ be a linear operator. A vector space $W \subseteq V$ is T-invariant, if $TW \subseteq W$.

2. Preliminaries and problem formulation

2.1. Signed graph

A (weighted) signed graph $\mathbb{G} = (\mathcal{G}, \theta)$ consists of an unsigned (weighted) graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and a signal mapping $\theta : \mathcal{E} \to \{+, -\}$, where $\mathcal{G}$ is said to be the underlying graph of $\mathbb{G}$. The edge set $\mathcal{E} = \mathcal{E}_+ \cup \mathcal{E}_-$, where $\mathcal{E}_+ = \{(j, i)|\ a_{ij} > 0\}$ and $\mathcal{E}_- = \{(j, i)|\ a_{ij} < 0\}$ denote the sets of positive and negative edges, respectively. Let $\mathcal{N}_i = \mathcal{N}_{i_+} \cup \mathcal{N}_{i_-}$ denote the neighborhood set of vertex i, where $\mathcal{N}_{i_+} = \{j \in \mathcal{V}|\ (j, i) \in \mathcal{E}_+\}$ and $\mathcal{N}_{i_-} = \{j \in \mathcal{V}|\ (j, i) \in \mathcal{E}_-\}$ represent the positive and negative neighborhood set of vertex i, respectively. A path is a sequence of distinct vertices such that any two consecutive vertices are an edge of the graph. A path with identical starting and ending vertices is called as a cycle. A cycle is positive (negative) if $|\mathcal{E}_-|$ is even (odd). Given a signed graph $\mathbb{G}$, for every vertex i, $c_i = \sum_{j\in\mathcal{N}_i} |a_{ij}|$ denotes the degree of i. The Laplacian of $\mathbb{G}$ is defined as $L = C - \mathcal{A}$, where $C = diag\{c_1, c_2, \dots, c_n\}$ and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ are, respectively, the degree matrix and the adjacency matrix of $\mathbb{G}$. The signature matrix set is denoted by $\mathcal{D} = \{diag\{\delta_1, \delta_2, \dots, \delta_n\}|\ \delta_i \in \{\pm 1\}\}$. Two matrices $M_1, M_2 \in \mathbb{R}^{n\times n}$ are said to be signature similar if $\exists D \in \mathcal{D}$ such that $M_2 = DM_1D$.

Definition 1 (*[29,32]*)**.** Let $\mathbb{G}$ be a signed graph with the vertex set $\mathcal{V}$.

1. $\mathbb{G}$ is structurally balanced (simply, balanced) if it has a bipartition of the vertices $\mathcal{V}_1, \mathcal{V}_2$, with $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, such that $a_{ij} \geqslant 0$ for $\forall v_i, v_j \in \mathcal{V}_q$ $(q \in \{1, 2\})$ and $a_{ij} \leqslant 0$ for $\forall v_i \in \mathcal{V}_q, v_j \in \mathcal{V}_r, q \neq r$ $(q, r \in \{1, 2\})$.
2. $\mathbb{G}$ is structurally anti-balanced (simply, anti-balanced) if it has a bipartition of the vertices $\mathcal{V}_1, \mathcal{V}_2$, with $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, such that $a_{ij} \leqslant 0$ for $\forall v_i, v_j \in \mathcal{V}_q$ $(q \in \{1, 2\})$ and $a_{ij} \geqslant 0$ for $\forall v_i \in \mathcal{V}_q, v_j \in \mathcal{V}_r, q \neq r$ $(q, r \in \{1, 2\})$.
3. $\mathbb{G}$ is strictly structurally unbalanced (simply, unbalanced) if $\mathbb{G}$ is neither balanced nor anti-balanced.

2.2. Problem formulation

Consider a MAS consisting of n single integrator dynamics agents, which are labeled by set $\mathcal{V} = \{1, 2, \dots, n\}$. Naturally, we assume that the set of the first m nodes, (i.e., leaders) is denoted by $\mathcal{V}_l$ $(\mathcal{V}_l = \{1, 2, \dots, m\})$, and the set of the rest agents, (i.e., followers) is denoted by $\mathcal{V}_f = \mathcal{V} \setminus \mathcal{V}_l$. The leaders can be actuated by external inputs, and the followers obey distributed