Systems & Control Letters 116 (2018) 56–65

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Dissipative delay range analysis of coupled differential–difference delay systems with distributed delays

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A R T I C L E I N F O

Article history:
Received 2 June 2017
Received in revised form 9 April 2018
Accepted 18 April 2018

Keywords:
Coupled differential–difference systems
Range stability
Dissipativity
Sum of square programming

A B S T R A C T

This paper proposes methods to handle the problem of delay range stability analysis for a linear coupled differential–difference system (CDDS) with distributed delays subject to dissipative constraints. The model of linear CDDS contains many models of linear delay systems as special cases. A novel Lyapunov–Krasovskii functional with non-constant matrix parameters, which are related to the delay value polynomially, is applied to derive stability conditions. By constructing this new functional, sufficient conditions in terms of robust linear matrix inequalities (LMIs) can be derived, which guarantee range stability of a linear CDDS subject to dissipative constraints. To solve the resulting robust LMIs numerically, we apply the technique of sum of squares programming together with matrix relaxations without introducing any potential conservatism to the original robust LMIs. Furthermore, the proposed methods can be extended to solve delay margin estimation problems for a linear CDDS subject to prescribed dissipative constraints. Finally, numerical examples are presented to demonstrate the effectiveness of the proposed methodologies.

1. Introduction

Functional differential equations [1] are able to characterize a dynamical process whose behavior is affected by its past values, i.e a dynamical system conditioned by delay effects. To analyze the stability property of such system, however, is non-trivial due to its infinite dimensional nature. Two major directions, which are based on either time [2] or frequency domain [3], have been investigated to provide solutions to characterize how delays affect the stability of a system.

For a linear delay system, the information of its stability can be obtained by analyzing its corresponding spectrum. Many different approaches [2,4] have been developed in frequency domain, which can provide almost a complete stability characterization when the delay systems exhibit certain structures. For more complex delay structures such as distributed delays with general kernels, the numerical schemes in [5–7] can produce reliable results verifying system stability with given point-wise delay values, which suffer almost no conservatism. Furthermore, the method in [8] allows one to calculate the value of $\mathcal{H}^\infty$ norm of a delay system with known point-wise delay values. However, to the best of our knowledge, none of the existing spectral based approaches can handle the problem of delay range stability analysis subject to performance objectives [9] for linear delay systems. Namely, to test whether a delay system is stable and simultaneously dissipative [10] for all delay value $r \in [r_1, r_2]$ with respect to a supply function, where the exact delay value r is unknown but bounded by $r_1 \le r \le r_2$ with known values $r_2 > r_1 > 0$.

On the other hand, constructing Krasovskii functional [2,10] has been applied as a standard approach in time domain to analyze the stability of a delay system. Many different functionals (see [2,10,11] and the references therein) have been proposed among existing literature [12,13] to analyze the problem of point-wise delay stability. Compared to its frequency domain counterparts, time domain approaches may be more adaptable to handle range stability analysis with performance objectives, though only sufficient conditions can be derived. In [14,15], the results concerning the range stability of a linear discrete delay system are presented based on the principle of quadratic separation. On the other hand, the solutions of the same problem have been proposed based on constructing Krasovskii functionals in [16,17]. However, no results, based on the Krasovskii approach, concerning range stability analysis have been proposed when distributed delays are considered.[1] On the other hand, almost all existing Krasovskii functionals in literature are based on constant matrix parameters, which is a very conservative choice when it comes to range stability

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[1] The methods proposed in [18] can handle polynomials distributed delay kernels. However, the approaches in [18] are derived not based on Krasovskii functionals, but the principle of robust control (Quadratic Separation).

https://doi.org/10.1016/j.sysconle.2018.04.008
0167-6911/

analysis. This motivates one to propose new functionals to specifically tackle the problem of range stability analysis considering performance objectives or even further potential constraints.

In this paper, we propose methodologies which allow one to conduct range stability analysis for a linear coupled differential–difference system (CDDS) [19,20,21] subject to dissipative constraints. The linear CDDS model considered in this paper contains distributed delay terms with polynomials kernels, which is able to incorporate many models of time delay systems as special cases. A novel Lyapunov–Krasovskii functional, with delay-dependent matrix parameters, is applied to be constructed together with a quadratic supply function to derive stability conditions. The resulting sufficient conditions, expressed in robust LMIs, are able to ensure the range stability and dissipativity of the linear CDDS over a known delay interval. To solve the robust LMIs numerically, we apply SoS programming [22,23] with the relaxation technique in [24] to equivalently transfer the original polynomials optimization problem into semidefinite programs with finite dimensions, without introducing any potential conservatism. Furthermore, the proposed scenario is extended to handle the problem of estimating the margin of a stable delay interval with given dissipative constraints. Finally, we also prove that the resulting stability conditions in this paper exhibit a hierarchical feasibility enhancement similar to the one in [16].

The paper is organized as follows. In Section 2 we formulate the linear CDDS model to be analyzed in this paper. Subsequently, theoretical preliminaries are presented in Section 3 which provide necessary tools to derive the main results in the following section. In Section 4, the main results on range stability analysis incorporating dissipative constraints are presented, including remarks and detailed explanations. Finally, we present several numerical examples in Section 5 to demonstrate the advantage of our proposed schemes.

Notation

The notations in this paper follow standard rules. In addition, we introduce certain new symbols for the sake of having efficient presentations. We define $\mathbb{T} := \{x \in \mathbb{R} : x \geq 0\}$ and $\mathbb{S}^n := \{X \in \mathbb{R}^{n\times n} : X = X^\top\}$. We frequently apply the notations of universal quantifier $\forall$ and the existential quantifier $\exists$ throughout the paper. $\mathcal{X}^{\mathcal{Y}}$ stands for the set containing all possible functions defined from $\mathcal{Y}$ onto $\mathcal{X}$. The notations $\|\mathbf{x}\|_q = \left(\sum_{i=1}^n |x_i|^q\right)^{\frac{1}{q}}$ and $\|f(\cdot)\|_p = \left(\int_{\mathbb{R}} |f(t)|^p \mathrm{d}t\right)^{\frac{1}{p}}$ and $\|\boldsymbol{f}(\cdot)\|_p = \left(\int_{\mathbb{R}} \|\boldsymbol{f}(t)\|_2^p \mathrm{d}t\right)^{\frac{1}{p}}$ are the norms/semi-norms associated with $\mathbb{R}^n$ and Lebesgue integrable functions spaces $\mathbb{L}^p(\mathbb{R} ⨾ \mathbb{R})$ and $\mathbb{L}^p(\mathbb{R} ⨾ \mathbb{R}^n)$, respectively. $\mathbf{Sy}(X) := X + X^\top$ is the sum of a matrix with its transpose. A column vector containing a sequence of objects is defined as $\mathbf{Col}_{i=1}^n x_i := \left[\mathbf{Row}_{i=1}^n x_i^\top\right]^\top = \left[x_1^\top \cdots x_i^\top \cdots x_n^\top\right]^\top$. The symbol $*$ is applied to denote $[*]YX = X^\top YX$ or $X^\top Y[*] = X^\top YX$. We use $\mathsf{O}_{n\times n}$ to indicate a $n \times n$ zero matrix with the abbreviation O_n, whereas $\mathbf{0}_n$ denotes a $n \times 1$ column vector. The symbols $\prec$ and $\succ$ are used to denote the relations of positive and negative definiteness, respectively, whereas $<$ and $>$ indicate point-wise orders. (The corresponding partial order relations of the aforementioned relations follow the same rules). The diagonal sum of two matrices and n matrices are defined as $X \oplus Y = \mathsf{Diag}(X, Y)$, $\bigoplus_{i=1}^n X_i = \mathsf{Diag}_{i=1}^n(X_i)$, respectively. Furthermore, $\otimes$ stands for the Kronecker product. Moreover, we assume the order of matrix operations as *matrix (scalars) multiplications* $> \otimes > \oplus >$ *matrix (scalar) additions*. Finally, the notion of empty matrix, which follows the same definition in Matlab (see https://au.mathworks.com/help/matlab/ref/zeros.html?requestedDomain=www.mathworks.com), is applied in this article to render our results more adaptable to the handling of different problems. All the matrix operations concerning empty matrices follow the same rules in Matlab.

2. Problem formulation

In this paper the linear CDDS

$$\begin{aligned}
\dot{\boldsymbol{x}}(t) &= A_1\boldsymbol{x}(t) + A_2\boldsymbol{y}(t-r) + \int_{-r}^{0} A_3(r)L_d(\tau)\boldsymbol{y}(t+\tau)\mathrm{d}\tau + D_1\boldsymbol{w}(t)\\
\boldsymbol{y}(t) &= A_4\boldsymbol{x}(t) + A_5\boldsymbol{y}(t-r) \qquad (1)\\
\boldsymbol{z}(t) &= C_1\boldsymbol{x}(t) + C_2\boldsymbol{y}(t-r) + \int_{-r}^{0} C_3(r)L_d(\tau)\boldsymbol{y}(t+\tau)\mathrm{d}\tau + D_2\boldsymbol{w}(t)\\
&\mathbf{Col}\,(\boldsymbol{x}(0), \boldsymbol{y}(0+\cdot)) = \mathbf{Col}\,(\boldsymbol{\xi}, \boldsymbol{\phi}(\cdot)) \in \mathbb{R}^n \times \widehat{\mathbb{C}}([-r,0) ⨾ \mathbb{R}^\nu)
\end{aligned}$$

is considered, where $\boldsymbol{x}(t) \in \mathbb{R}^n$ and $\boldsymbol{y}(t) \in \mathbb{R}^\nu$ are the solution of the coupled differential–difference equations in (1), $\boldsymbol{w}(\cdot) \in \mathbb{L}^2(\mathbb{T} ⨾ \mathbb{R}^q)$ represents disturbance, $\boldsymbol{z}(t) \in \mathbb{R}^m$ is the regulated output. Furthermore, $\boldsymbol{\xi} \in \mathbb{R}^n$ and $\boldsymbol{\phi}(\cdot) \in \widehat{\mathbb{C}}([-r,0) ⨾ \mathbb{R}^n)$ are the initial conditions where $\widehat{\mathbb{C}}(\mathcal{X} ⨾ \mathbb{R}^n)$ stands for the Banach space of bounded right piecewise continuous functions with a uniform norm $\|\boldsymbol{f}(\cdot)\|_\infty := \sup_{\tau\in\mathcal{X}}\|\boldsymbol{f}(\tau)\|_2$. The dimensions of the state space matrices in (1) are determined by the indexes $n; \nu \in \mathbb{N}$ and $m; q \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$. Moreover, $L_d(\tau) := \boldsymbol{\ell}_d(\tau) \otimes I_\nu$ where $\boldsymbol{\ell}_d(\tau) \in \mathbb{R}^{d+1}$ contains polynomials at each row up to degree $d \in \mathbb{N}_0$. $A_3(r) \in \mathbb{R}^{n\times\varrho}$ and $C_3(r) \in \mathbb{R}^{m\times\varrho}$ are polynomials matrices of r with $\varrho = (d+1)\nu$. r is a constant but with unknown and bounded values as $r \in [r_1, r_2]$, where the values of $r_2 > r_1 > 0$ are known. Finally, it is assumed $\rho(A_5) < 1$ which ensures the input to state stability of $\boldsymbol{y}(t) = A_4\boldsymbol{x}(t) + A_5\boldsymbol{y}(t-r)$ [19] where $\rho(A_5)$ stands for the spectra radius of A_5. Since $\rho(A_5) < 1$ is independent from r, thus this condition ensures the input to state stability of $\boldsymbol{y}(t) = A_4\boldsymbol{x}(t) + A_5\boldsymbol{y}(t-r)$ for all $r > 0$.

Remark 1. Many delay related systems can be modeled by (1). See [13,19] and the references therein. In comparison with the CDDS model in [19], (1) takes disturbances into account and incorporates distributed delay terms with polynomials kernels at both the state and output. In terms of real-time applications, the structures of $A_3(r), C_3(r)$ can be justified by the fact that the distributed delay gain matrices can be related to the numerical values of r [25]. See a representative example by the Example 2 in [12].

3. Preliminaries

3.1. Legendre polynomials

Without losing generalities, we assume in this paper that $\boldsymbol{\ell}_d(\tau) = \mathbf{Col}_{i=0}^d \ell_i(r,\tau)$ in (1) consists of Legendre polynomials [16,26,27,28]

$$\ell_d(r,\tau) := \sum_{k=0}^{d} \binom{d}{k}\binom{d+k}{k}\left(\frac{\tau}{r}\right)^k = \sum_{k=0}^{d} \binom{d}{k}\binom{d+k}{k}\tau^k r^{-k} \qquad (2)$$

$\forall d \in \mathbb{N}_0, \forall \tau \in [-r, 0]$ where $\int_{-r}^0 \boldsymbol{\ell}_d(\tau)\boldsymbol{\ell}_d^\top(\tau)\mathrm{d}\tau = \bigoplus_{k=0}^d \frac{r}{(2k+1)}$. Note that the form of (2) is derived from the structure of Jacobi polynomials [29] with $\alpha = \beta = 0$ in [28].

Some properties of Legendre polynomials are summarized as follows.

Property 1. *Given $d \in \mathbb{N}_0$ and $\boldsymbol{m}_d(\tau) := \mathbf{Col}_{i=0}^d \tau^i$, then the following three properties hold for all $r > 0$.*

- $$\begin{aligned}&\exists! \mathsf{L}_d(\cdot) \in \left(\mathbb{R}_{[d+1]}^{(d+1)\times(d+1)}\right)^{\mathbb{R}_+}, \exists! \Lambda_d \in \mathbb{R}_{[d+1]}^{(d+1)\times(d+1)}\\ &\forall \tau \in \mathbb{R}, \quad \boldsymbol{\ell}_d(\tau) = \mathsf{L}_d(r)\boldsymbol{m}_d(\tau) = \Lambda_d\left[\bigoplus_{i=0}^d r^{-i}\right]\boldsymbol{m}_d(\tau)\end{aligned} \qquad (3)$$