Systems & Control Letters 115 (2018) 26–33

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Parameters and fractional differentiation orders estimation for linear continuous-time non-commensurate fractional order systems

Zehor Belkhatir *, Taous Meriem Laleg-Kirati

Division of Computer, Electrical and Mathematical Sciences and Engineering (CEMSE), King Abdullah University of Science and Technology (KAUST), Saudi Arabia

ARTICLE INFO

Article history:
Received 12 February 2017
Received in revised form 9 February 2018
Accepted 19 February 2018

Keywords:
Linear fractional order systems
Non-commensurate orders
Parameters and fractional differentiation orders estimation
Modulating functions method

ABSTRACT

This paper proposes a two steps algorithm for the joint estimation of parameters and fractional differentiation orders of a linear continuous-time fractional system with non-commensurate orders. The proposed algorithm combines the modulating functions and the first-order Newton methods. Sufficient conditions ensuring the convergence of the method are provided. Moreover, the method is extended to the joint estimation of smooth unknown input and fractional differentiation orders. A potential application of the proposed algorithm consists in estimating the fractional differentiation orders of a fractional neurovascular model along with the neural activity considered as an input for this model. To assess the performance of the proposed method, different numerical tests are conducted.
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1. Introduction

Fractional calculus, which is the generalization of the integer-order integration and differentiation operators, origins back to the seventeenth century. Thanks to the interesting properties of non-locality and memory of the fractional derivative, it has been used to model complex phenomena in different fields of science and engineering [1–3]. However, fractional differentiation orders, parameters and/or input of those models are usually unknown. So, they have to be estimated from available measurements. However, solving parameter and unknown input estimation problems for fractional order systems (FOSs) is not straightforward, and it is more challenging when information about the fractional differentiation orders is missing.

Several estimation methods for FOSs have been proposed in the literature. For instance, a fractional version of the extended Kalman filter has been proposed in [4] to estimate the unknown parameters and the fractional differentiation orders of a discrete FOS. A fractional version of Zak–Walcott sliding mode observer is introduced in [5] for the joint estimation of the pseudo-states and the unknown input of FOS. A higher order sliding mode observer with unknown input is proposed in [6]. Victor et al. [7] associated a simplified refined instrumental variable method with a Gauss–Newton approach to estimate the unknown parameters and the fractional differentiation orders. A combined gradient/least-squares method is proposed for the simultaneous estimation of the parameters and the commensurate fractional order of linear FOSs in [8].

The modulating functions (MFs) method has been introduced in the fifties for parameters' estimation of integer-order systems [9]. It has been extended to FOSs in [10] where the fractional differentiation order has been reduced to an integer order in the noise-free case. In [11], it has been used to estimate the parameters of FOSs with known fractional differentiation orders. Moreover, the MFs method has been combined with an iterative method to estimate both the coefficients and a single differentiation order of a fractional advection–dispersion partial differential equation [12]. More recently, the MF method has been used to estimate the pseudo-state, and the multiple point sources of linear FOSs in [13] and [14], respectively.

In this paper, we propose a hybrid algorithm which combines the MF method and the first-order Newton method to jointly estimate the fractional differentiation orders and the parameters of fractional ordinary differential equations (FODEs). The approach is also extended to solve the estimation problem of smooth unknown input simultaneously with fractional differentiation orders. In addition, the present work considers a proper initialization of the fractional derivative operator. Furthermore, the proposed MF-based method offers some interesting advantages. It transfers the fractional differentiation orders from the output, which is often noisy, to the MFs whose fractional derivative can be analytically computed; it is robust against noise; and it helps in characterizing the Jacobian matrix of Newton's method at each iteration which reduces the computational cost.

The remainder of the paper is organized as follows. Section 2 recalls definitions and concepts on fractional calculus and MFs.

* Corresponding author.
E-mail addresses: zehor.belkhatir@kaust.edu.sa (Z. Belkhatir), taousmeriem.laleg@kaust.edu.sa (T.M. Laleg-Kirati).

https://doi.org/10.1016/j.sysconle.2018.02.012
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In Section 3, the estimation problem is formulated. Section 4 describes the proposed two-stage algorithm. Section 5 presents sufficient conditions that ensure the local convergence of the synthetized algorithm. Section 6 extends the proposed method to the case of unknown smooth input and fractional differentiation orders estimation. The performance of the algorithm is shown in Section 7 through different numerical examples.

2. Preliminaries

In this section, some definitions and properties of the initialized fractional derivative, and the MFs are presented.

2.1. Fractional derivative

Definition 1 ([15]). The initialized Riemann–Liouville (RL) fractional derivative is defined as

$$D_t^\alpha f(t) = d_t^\alpha f(t) + \Psi(f_i, -p, -t_0, 0, t), \qquad t > 0, \tag{1}$$

where $\alpha = n - p$ with $n \in \mathbb{N}$ and $p, \alpha \in \mathbb{R}_+^*$. The terms $d_t^\alpha f(t)$ are the uninitialized αth order RL derivative and $\Psi(f_i, -p, -t_0, 0, t)$ is the initialization function given as follows:

$$\begin{cases} d_t^\alpha f(t) = \dfrac{d^n}{dt^n} \dfrac{1}{\Gamma(p)} \displaystyle\int_0^t (t-\tau)^{p-1} f(\tau)\, d\tau, \\ \Psi(f_i, -p, -t_0, 0, t) = \dfrac{d^n}{dt^n} \dfrac{1}{\Gamma(p)} \displaystyle\int_{-t_0}^0 (t-\tau)^{p-1} f_i(\tau)\, d\tau, \end{cases} \tag{2}$$

where $\Gamma(.)$ is the gamma function and $f_i(t)$ is the initialization function defined for $t \in [-t_0, 0],\ t_0 \in \mathbb{R}_+$. This definition assumes that the history-function for $t \in (-\infty, -t_0)$ is zero.

The following lemma provides an analytic closed form for the derivative of $D_t^\alpha f(t)$ with respect to α.

Lemma 1. *If the αth initialized RL derivative of a function $f(t)$ given in (1) exists, then the derivative of $D_t^\alpha f(t)$ with respect to $n-1 \le \alpha < n$ is given by*

$$\frac{\partial}{\partial \alpha}\left[D_t^\alpha f(t)\right] = \psi_0(p) D_t^\alpha f(t) - \frac{1}{\Gamma(p)} \frac{d^n}{dt^n}\left[\int_0^t (t-\tau)^{p-1} \ln(t-\tau) f(\tau)\, d\tau + \int_{-t_0}^0 (t-\tau)^{p-1} \ln(t-\tau) f_i(\tau)\, d\tau\right], \tag{3}$$

where $p = n - \alpha$ and $\psi_0(.) = \frac{\Gamma'(.)}{\Gamma(.)}$ is the standard digamma function.

Proof. The proof is completed by differentiating (1) with respect to α. □

2.2. Modulating functions

Definition 2 ([16]). A function $\phi(t)$, defined on $[0, T]$, is an MF of order n if it satisfies the following properties:

$(P1)\quad \phi \in C^n([0, T])$,
$(P2)\quad \phi^{(i)}(0) = \phi^{(i)}(T) = 0,\ \ i = 0, 1, \ldots, n-1$,

where $C^n([0, T])$ denotes the space of n times differentiable functions over $[0, T]$.

The MF $\phi(t)$ of order n satisfies the following properties [17]:

$(P3)\quad D_t^\alpha \phi(t)$ exists, $\ \forall\, 0 \leqslant \alpha \leqslant n$;
$(P4)\quad D_t^\alpha \phi(t)|_{t=0} = 0; \quad \forall\, 0 \leqslant \alpha \leqslant n.$

A generalization of an integration by parts-like formula proposed in [11] for uninitialized RL fractional derivative is given in the following lemma for the case of initialized RL derivative with a constant initialization function.

Lemma 2. *Consider a function $f(t)$ and let $\phi(t)$ be an MF of order n, with $n \in \mathbb{N}^*$. Assume that the αth initialized RL fractional derivative of $f(t)$, with $\alpha \in \mathbb{R}_+^*$, exists. Let the initialization function be a constant $f_0(t) = c_0$, defined for $t \in [-t_0, 0],\ \ t_0 \in \mathbb{R}_+$. Then, the following expression holds:*

$$\int_0^T D_t^\alpha f(t)\, \phi(T-t)\, dt = \int_0^T f(T-t)\, D_t^\alpha \phi(t)\, dt + \frac{c_0}{\Gamma(1-\alpha)} \int_0^T \phi(T-t)(t+t_0)^{-\alpha}\, dt - c_0\, D_t^{\alpha-1} \phi(T). \tag{4}$$

Proof. The proof is provided in the Appendix. □

3. Problem statement

Let us consider the following non-commensurate FOS:

$$y(t) + \sum_{i=1}^N a_i\, D_t^{\alpha_i} y(t) = u(t), \quad t \in [0, T], \tag{5}$$

where $y(t) \in \mathbb{R}$ is the output, $u(t) \in \mathbb{R}$ is the input, $a_i \in \mathbb{R}$, for $i = 1, 2, \ldots, N$, are the unknown parameters and $\alpha_i \in \mathscr{P}_i = (n_i - 1, n_i)$, with $n_i \in \mathbb{N}^*$ and i = 1,2, ...,N, are the unknown fractional differentiation orders. They are assumed to be as follows: $0 < \alpha_1 < \alpha_2 < \cdots < \alpha_N$, i.e., $n_i < n_{i+1}$ for $i = 1, 2, \ldots, N-1$.

We denote the vectors $\theta \in \mathbb{R}^N$ and $\alpha \in \mathscr{P} = \prod_{i=1}^N \mathscr{P}_i$,[1] as

$$\theta = \begin{pmatrix} a_1 & a_2 & \cdots & a_N \end{pmatrix}^{tr}, \ \alpha = \begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_N \end{pmatrix}^{tr}, \tag{6}$$

and their estimates as

$$\hat{\theta} = \begin{pmatrix} \hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_N \end{pmatrix}^{tr}, \ \hat{\alpha} = \begin{pmatrix} \hat{\alpha}_1 & \hat{\alpha}_2 & \cdots & \hat{\alpha}_N \end{pmatrix}^{tr}, \tag{7}$$

where $(.)^{tr}$ denotes the transpose of a row vector.

In this work, we are interested in solving the following estimation problem (EP 1):

(EP 1) *Given $y(t)$ and $u(t)$, for $t \in (0, T)$, find estimates $(\hat{\alpha}, \hat{\theta})$ for the unknown vectors of fractional differentiation orders and parameters.*

We will also show how to solve the estimation problem (EP 2) which is given as follows:

(EP 2) *Given $y(t)$, for $t \in (0, T)$, and knowing the value of the vector θ, find estimates $(\hat{\alpha}, \hat{u}(t))$ for the unknown vector of fractional differentiation orders and the unknown input signal.*

4. Main results

In this section, we introduce a hybrid method which combines the MFs approach and the first-order Newton method to solve (EP 1). The proposed method is iterative and consists of two steps. While the first step solves the parameters' estimation problem at each iteration using the MF method, the second step solves a nonlinear system of equations using Newton's method to find estimates for the fractional differentiation orders.

4.1. Proposed two-stage algorithm

Let the fractional derivatives of the output signal be initialized with a constant initialization function $f_0(t) = c_0,\ t \in [-t_0, 0]$, $t_0 > 0$.

[1] $\mathscr{P} = \prod_{i=1}^N \mathscr{P}_i = \{(a_1, a_2, \ldots, a_N) | \forall\, i = 1, 2, \ldots, N, a_i \in \mathscr{P}_i\}$ is the generalized Cartesian product of N sets $\mathscr{P}_1, \mathscr{P}_2, \ldots, \mathscr{P}_N$.

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