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Zero-error tracking control with pre-assignable convergence mode for nonlinear systems under nonvanishing uncertainties and unknown control direction

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ABSTRACT

This work addresses the tracking control problem for a class of high-order multi-input multi-output (MIMO) nonlinear systems with unknown control direction and nonparametric uncertainties. By integrating a matrix (rather than scalar) rate transformation with Nussbaum gain, we develop an accelerated robust adaptive control that exhibits several attractive features: (1) it is able to achieve full-state zero-error tracking despite unknown control direction and non-vanishing uncertainties; (2) with the proposed control scheme, the whole tracking process seamlessly consists of the first phase of steering the tracking error into an adjustable small residual region with accelerated convergence rate and the second phase of further driving the error to zero; (3) before reaching the residual region, each component of the tracking error is forced to decay at an accelerated rate that can be pre-assigned; and (4) the resultant control action is continuous differentiable everywhere without involving excessively large initial driving effort.

1. Introduction

Most important systems (such as robotic systems, high speed trains, spacecraft etc.) can be modeled by the following dynamic equations,

$$\begin{cases} \dot{x}_i = x_{i+1} \\ \dot{x}_n = F(x,p) + G(x,p)u + D(x,p,t) \\ y = x_1 \end{cases} \tag{1}$$

for $i = 1, 2, \ldots, n-1$, where $x_j = [x_{j1}, \ldots, x_{jm}]^T \in R^m$, $j = 1, \ldots, n$, and $x = \left[x_1^T, \ldots, x_n^T\right]^T \in R^{mn}$ is the state vector; $p \in R^r$ represents the unknown parameter vector inseparable from system nonlinearities (i.e., $F(x,p)$, $G(x,p)$, and $D(x,p,t)$ are nonparametric uncertainties); $u = [u_1, \ldots, u_m]^T \in R^m$ is control input vector of the system; $y \in R^m$ is the output vector; $F(\cdot) = [f_1(\cdot), \ldots, f_m(\cdot)]^T \in R^m$ is a smooth but uncertain nonlinear function vector; $G(x,p) \in R^{m\times m}$ is the control gain matrix and $D(x,p,t) = [d_1(\cdot), \ldots, d_m(\cdot)]^T \in R^m$ denotes all the other system modeling uncertainties and external disturbances. Define state tracking error as $E = x_1 - y_d = [e_1, \ldots, e_m]^T$ and $E^{(i)} = \left[e_1^{(i)}, \ldots, e_m^{(i)}\right]^T$, $i = 1, \ldots, n-1$, where $y_d = [y_{d1}, \ldots, y_{dm}]^T \in R^m$ and $y_d^{(i)} = \left[y_{d1}^{(i)}, \ldots, y_{dm}^{(i)}\right]^T$ are the known reference signal and its ith derivatives. In this work, we seek for a tracking control approach capable of achieving the following three objectives:

(O_1) All the internal signals are bounded; the control action is continuous differentiable; and no excessively initial large control effort is involved;

(O_2) Full-state zero-error tracking is obtained despite unknown control direction and nonparametric uncertainties arising from $F(x,p)$, $G(x,p)$, and $D(x,p,t)$; and

(O_3) The tracking process is pre-designable in that each component of the tracking error, before reaching the residual set, has its own pre-assigned convergence mode and convergence rate.

It poses significant challenge to realize the above-mentioned objectives (O_1)–(O_3) simultaneously. As a matter of fact, although there is a rich collection of tracking control results for system (1), very few have been able to achieve Objectives (O_1) and (O_2) concurrently. This is because in the presence of nonparametric uncertainties, it is rather difficult to drive the tracking error to zero with continuous (not to mention continuous differentiable) control action [1,2]. The underlying problem becomes even more challenging if the control direction is not known a priori, which has gained increasing attention from control community during

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the past decades. The pioneering work that addresses unknown control direction is from Nussbaum [3] for a class of first-order linear systems, which has motivated various efforts on using the Nussbaum-type function to tackle the difficulty of unknown control direction for nonlinear systems [4,5]. In addition, some other tools are also utilized for dealing with the problem of unknown control directions. For example, Ref. [6] is based on periodic switching function to cope with the unknown control direction, whereas the method in [7] is based on monitoring function to deal with such problem; Ref. [8] addresses the design of a sliding mode controller for a class of uncertain nonlinear plants with unknown sign of the high frequency gain. However, the aforementioned results are based on SISO nonlinear (or linear) systems, thus the control gain is just a scalar constant or time-varying function. Due to in the MIMO nonlinear systems the control gain is matrix form and the control direction is not known a priori, adaptive control of such systems becomes rather difficult. In [9], an iterative learning control for a class of MIMO uncertain nonlinear systems with unknown control direction is proposed, in which a gain-selector combined with a Nussbaum gain was used to probe the correct gain matrix from the candidates in the unmixing matrix set; however the gain matrix must remain constant. In [10], by using a matrix decomposition technique, an adaptive controller is developed for MIMO nonlinear systems to drive the tracking error to zero asymptotically, but the assumption about the availability of the partial derivatives of the control gain is imposed. In [11–13] with the help of the newly-constructed Nussbaum analysis tool, a promising analysis framework is established to pave the way for tackling multiple unknown disturbances and identical control directions. However, the proposed method should employ multiple Nussbaum functions for controller design, which complicate the stability analysis and control.

Furthermore, it is also nontrivial, although highly desirable, to achieve Objective (O_3) in practice. For example, in traffic control systems, to prevent traffic jam at an intersection, it is desired that each of the group vehicles should arrival at the intersection in different time and/or pass through at different speed, rather than arriving at the same time and same speed. There are some works on rate of convergence control, such as [14] for fully exponential convergence, [15,16] for prescribed non-zero tracking performance guaranteeing that the tracking error converges to an arbitrarily small residual set with prespecified exponential convergence rate, and [17] for possibly non-exponential convergence, but for more general case as in (O_3) that requires each component of the tracking error, before reaching the residual region, has its own and different convergence mode and rate and such mode and rate can be explicitly and arbitrarily pre-assigned, there is no result in the literature, to our best knowledge.

In this work, we develop a control solution for system (1) to achieve (O_1)–(O_3) simultaneously. First, in order to deal with the unknown control direction and asymmetric yet uncertain gain matrix, we convert original gain matrix equivalently into a symmetric part and skew-symmetric part; Second, as Nussbaum gain technique is normally applicable for scalar (rather than matrix) control gain [5], we make use of Lemma 1 to circumvent this difficulty; Third, to avoid the excessive control effort in the startup point, a special value $N(\chi(t_0)) = 0$ is utilized in the control scheme to render $u(t_0) = 0$; Fourth, to achieve zero-error tracking in the presence of time-varying control gain and non-vanishing uncertainties, a special structure of $\dot{\chi}$ (the rate of the parameter χ in Nussbaum function) is defined in the control scheme to ensure that the tracking error is square integrable; Finally, to ensure that each component of the tracking error has its own (different) converging mode and decaying rate, a diagonal matrix rate function $\beta \in R^{m\times m}$ is introduced for tracking error transformation, which allows the tracking error, priori to reaching a small and adjustable residual region, to have different convergence rates that can be pre-specified by the control designer. Throughout this paper, $\|\cdot\|$ represents the Euclidean norm of a vector or the induced matrix norm. Let R denote the real numbers, R_+ denote the nonnegative real numbers, and I be the unit matrix. C^n denotes the set of functions that have continuous derivatives up to the order n and $\boldsymbol{e}^{\iota t}$ denotes the exponential function.

2. Problem formulation

2.1. Setting and conditions

In order for the system (1) to admit a feasible tracking control solution, the following conditions are imposed.

Assumption 1. The known desired trajectory y_d, as well as its up to $(n+1)$th derivatives, are bounded. The system states are available for control design.

Assumption 2 (*[18,19]*)**.** There exists some nonnegative constant a and nonnegative scalar and computable smooth function $\phi(x)$ such that

$$\|F(x,p)+D(x,p,t)\| \le a\phi(x) \tag{2}$$

where $\phi(x)$ is bounded if x is bounded. In addition, $F(x,p)$, $G(x,p)$, and $D(x,p,t)$ are bounded if x is bounded.

Assumption 3. The control gain matrix $G(\cdot) \in R^{m\times m}$ is square but unnecessarily symmetric yet completely unknown. The only information available for control design is that $G_1 = \frac{G+G^T}{2}$ is either positive definite or negative definite, but not certain which one. Here it is assumed that there exist some unknown bounded constants $\bar{\lambda}$ and $\underline{\lambda}$ such that, with the minimum eigenvalue $\lambda_{\min}(t)$ and the maximum eigenvalue $\lambda_{\max}(t)$ of G_1, the following inequality holds $\underline{\lambda} \le \lambda_{\min}(t) < \lambda_{\max}(t) \le \bar{\lambda}$, where $\underline{\lambda}$ and $\bar{\lambda}$ have the same sign, i.e., both being positive or negative.

Remark 1. Note that the positive or negative definiteness of $G_1 = \frac{G+G^T}{2}$ ensures that for any given nonzero vector $\boldsymbol{x} \in R^m$, one gets that $\boldsymbol{x}^T G_1 \boldsymbol{x} > 0$ or $\boldsymbol{x}^T G_1 \boldsymbol{x} < 0$, which also implies that $\boldsymbol{x}^T G \boldsymbol{x} > 0$ or $\boldsymbol{x}^T G \boldsymbol{x} < 0$, i.e., $\operatorname{sgn}\{\boldsymbol{x}^T G \boldsymbol{x}\} = \operatorname{sgn}\{\boldsymbol{x}^T G_1 \boldsymbol{x}\}$, which is unknown. Furthermore, Assumption 3 on matrix $G(\cdot)$ is much less restrictive than that G is assumed to be symmetric and positive define [20] because this corresponds to the condition that the control direction is known. Although in [10] unknown control direction is considered, additional constricts on the partial derivatives of the control gain are imposed (see Assumptions 1 and 2 in [10]).

Lemma 1 (*[21]*)**.** *Let Γ be an $m\times m$ symmetric matrix and $\boldsymbol{x} \in R^m$ be a nonzero vector, denote that $\rho = \frac{\boldsymbol{x}^T\Gamma\boldsymbol{x}}{\boldsymbol{x}^T\boldsymbol{x}}$. Then, there is at least one eigenvalue of Γ in the internal $(-\infty, \rho]$ and at least one in $[\rho, \infty)$.*

With this Lemma, it becomes obvious from Assumption 3 and Remark 1 that for any given nonzero vector $\boldsymbol{x} \in R^m$, we have that $\boldsymbol{x}^T G_1 \boldsymbol{x} \ne 0$, denoting $\bar{\alpha}(t) = \frac{\boldsymbol{x}^T G_1 \boldsymbol{x}}{\boldsymbol{x}^T \boldsymbol{x}}$ then $\boldsymbol{x}^T G_1 \boldsymbol{x} = \bar{\alpha}(t)\boldsymbol{x}^T\boldsymbol{x}$, where $\bar{\alpha}(t) \ne 0$. According to Lemma 1 and Assumption 3, there exist two constants $\underline{\lambda}$ and $\bar{\lambda}$ such that $\underline{\lambda} \le \lambda_{\min}(t) \le \bar{\alpha}(t) \le \lambda_{\max}(t) \le \bar{\lambda}$. Moreover, if $\boldsymbol{x} = 0$, it holds that $\boldsymbol{x}^T G_1 \boldsymbol{x} = \vartheta \boldsymbol{x}^T \boldsymbol{x}$ for any nonzero constant $\vartheta \in [\underline{\lambda}, \bar{\lambda}]$. Therefore, we can conclude that for any given $\boldsymbol{x}$,

$$\boldsymbol{x}^T G_1 \boldsymbol{x} = \alpha(t)\boldsymbol{x}^T\boldsymbol{x}, \tag{3}$$

where $\alpha(t) = \begin{cases} \bar{\alpha}(t), & \text{if } \boldsymbol{x} \ne 0 \\ \vartheta, & \text{if } \boldsymbol{x} = 0 \end{cases}$, which is a useful property for the control design in the sequel. In addition, to cope with the unknown