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Advances on adaptive learning control: The case of non-minimum phase linear systems

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ABSTRACT

The adaptive learning control theory is extended to include the case of non-minimum phase (time-invariant) linear systems with uncertain parameters. The existence of two approximate solutions to the related output tracking problem is proved. Exponential tracking of periodic output reference signals is guaranteed, along with the exponential estimation of the constant system parameters and of the constant coefficients characterizing the truncated Fourier series expansion for the periodic input reference signal.

1. Introduction

Global or semi-global exponential output tracking of sufficiently smooth T-periodic signals (with known period T) can be achieved by the adaptive learning control for special classes of uncertain linear and nonlinear time-invariant systems (see [1] and [2] and references therein for the most relevant results). Such special classes include uncertain minimum phase linear systems [3]. The reader is also referred to: [4] for the related identification approach; [5–7] (and references therein) for the most recent results and comparative discussions on the memory-mirrored counterpart 'iterative/repetitive learning'; [8–11] for a combined adaptive/iterative learning control approach. The adaptive learning control strategy relies on the following design steps: (i) the uncertain T-periodic input reference, which guarantees perfect output tracking for compatible initial conditions, is viewed as a disturbance signal affecting the tracking error dynamics; (ii) such T-periodic disturbance is developed in Fourier series, with an explicit estimate of the approximation error being defined; (iii) an adaptive control is designed, which, on the basis of the available tracking error, estimates a finite number of Fourier coefficients to attenuate the effect of the above disturbance signal on the closed loop error system. Persistency of excitation conditions are always satisfied, owing to the orthogonality of the related basis functions. The output tracking error is then guaranteed *to exponentially converge into a residual set (containing the origin), whose diameter may be arbitrarily reduced by increasing the number of the estimated Fourier coefficients* (this convergence property, in this paper, will be referred to as $\mathcal{CP}$). However, the adaptive learning control problem is yet to be successfully solved for non-minimum phase linear systems. In this case, it does not suffice to confine the design steps to the lower part of the system that dynamically describes the output and its time derivatives (see [3] or even [5]), while the estimation of system parameters is reasonably foreseeable to be involved.

The aim of this brief is to extend the adaptive learning control theory to linear (time-invariant) systems with uncertain parameters, without resorting to the minimum phase assumption. We start in Section 3 from following a design methodology that is reminiscent of the adaptive pole placement technique for linear systems.[1] The problem is suitably recast to comply with the internal model principle scenario. However, differently from [19–23], here the uncertain input reference is not restricted to be generated by a finite-dimensional exosystem: an explicit approximation error (reducing with the number of estimated Fourier coefficients) is taken into account in the related stability analysis, in order to preserve property $\mathcal{CP}$. Anyway, difficulties arise with this approach when the matrix characterizing the extended system becomes relatively large. This is due to the involved use of a relatively large number of terms in the truncated Fourier series expansion. The resulting introduction of poles on the stability boundary leads to numerical issues in the presence of implementation with finite precision arithmetics. Such difficulties are overcome by the

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1 The reader is referred to [12] (see also [13–17]) for the global adaptive pole-placement control of possibly non-minimum phase linear systems and to [18] for the adaptive state feedback control of weakly non-minimum phase linear systems.

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adaptive learning controller of Section 4, whose design extends the one presented in [24] to the case of uncertain system parameters. It is a feed-forward algorithm (see [25] for a similar approach to discrete-time systems) that uses an external model to generate the disturbance and two estimation processes to recover the uncertain parameters. Property $\mathcal{CP}$ is again definitely achieved. Key-design steps – common to both the proposed approaches – are: the use of the new local Persistency of excitation lemma in the Appendix; the use of a local "separation principle" in the involved stability analyses[2] ; persistency of excitation conditions that only depend on the specific *a priori* defined output/input reference signals not varying with the system trajectories. The prices to be paid for such theoretical improvements are constituted by the local nature of the results, as well as by the required run-time availability of the output and its reference.

2. Problem statement

Consider a time-invariant linear system described by the transfer function ($s \in \mathbb{C}$)

$$W(s) = \frac{g_{n-1}s^{n-1} + \cdots + g_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0}, \tag{1}$$

whose parameters $g_0, \ldots, g_{n-1}, a_0, \ldots, a_{n-1}$ are uncertain and whose numerator and denominator polynomials are coprime. Denote by (A, B, C) the minimal realization of (1) in reachability canonical form and thus consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned} \tag{2}$$

in which: $x \in \mathbb{R}^n$ is the state vector; $u \in \mathbb{R}$ is the scalar input; $y \in \mathbb{R}$ is the scalar output to be controlled. The system uncertainties are inherited by the elements of the uncertain triple (A, B, C). Let $y_* \in \mathcal{C}^{p_y}$ ($p_y \in \{m \in \mathbb{N}, m \gg 1\}$) denote any sufficiently smooth T-periodic output reference signal with known period T and define the corresponding output tracking error $\tilde{y} = y - y_*$. We will show that the adaptive learning control theory can be extended to include systems (2) that are not restricted to be minimum-phase. In other words, we will prove that adaptive learning controls can be designed, on the basis of the (y, y_*)-measurements, in order to achieve the aforementioned property $\mathcal{CP}$. The existence of the periodic input reference signal (namely, $u_*(t)$) is related to the following rather natural requirement that we introduce to guarantee the problem solvability.

Assumption 1. There exist smooth T-periodic reference signals $(x_*(t), u_*(t))$ that, for compatible initial conditions $x_*(0)$, comply with the reference system

$$\begin{aligned} \dot{x}_*(t) &= Ax_*(t) + Bu_*(t) \\ y_*(t) &= Cx_*(t). \end{aligned} \tag{3}$$

Remark 1. Assumption 1 necessarily requires that no zeros of $W(s)$ belong to $\Lambda = \{0, \pm jl2\pi/T, l = 1, 2, \ldots\}$ (j denotes the imaginary unit). In the special case (typically addressed by the internal model theory-based context) in which the periodic signal $y_*(t)$ is described, for some zero or even $N_* \in \mathbb{N}_0$, by the finite Fourier expansion (see [26]): $y_*(t) = \sum_{i=0}^{N_*}\rho_i\varphi_i(t)$, with $\varphi_0(t) = 1, \varphi_{2l}(t) = \sqrt{2}\cos\left(lt\frac{2\pi}{T}\right)$, $\varphi_{2l-1}(t) = \sqrt{2}\sin\left(lt\frac{2\pi}{T}\right)$ ($l = 1, \ldots, N_*/2$), the output reference $y_*(t)$ can be equivalently viewed as the output $y_*(t) = -q^{\mathrm{T}}w(t)$ of the finite-dimensional exosystem: $\dot{w}(t) = Rw(t)$, $w(0) = w_0$, with: q^{T} being a suitable row vector; w_0 being a suitable initial condition depending on the Fourier coefficients ρ_i in the y_*-expansion; the spectrum $\sigma(R)$ of the square matrix R being represented by $\sigma(R) = \{0, \pm jl2\pi/T,\ 1 \le l \le N_*/2\}$. Well-known necessary and sufficient conditions for the solution to the corresponding regulator problem (see [19,21–23] for related results) are: (i) (A, B) is stabilizable; (ii) (A, C) is detectable; (iii) the rank condition

$$\mathrm{rk}\begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} = n + 1 \tag{4}$$

holds for any $\lambda \in \sigma(R)$, i.e. the zeros of the transfer function $W(s)$ do not coincide with the elements of $\sigma(R)$. In particular the above conditions guarantee that there exist a matrix Γ and a row vector γ^{T} satisfying $\Gamma R = A\Gamma + B\gamma^{\mathrm{T}}$, $C\Gamma + q^{\mathrm{T}} = 0$, so that the signals required by Assumption 1 are actually constituted by $x_* = \Gamma w$ and $u_* = \gamma^{\mathrm{T}}w$.

3. Method I: adaptive learning control through adaptive pole placement

We briefly start from following a design methodology that is reminiscent of the adaptive pole placement technique for linear systems. The problem is suitably recast to comply with the internal model principle scenario. However, here the uncertain input reference is not restricted to be generated by a finite-dimensional exosystem and an explicit approximation error (reducing with the number of estimated terms in the Fourier series expansion for the uncertain periodic input reference $u_*(t)$) is taken into account in the related stability analysis.

3.1. Recasting the problem

Let us define the tracking error $\tilde{x} = x - x_*$, whose dynamics are given by:

$$\dot{\tilde{x}} = A\tilde{x} + B(u - u_*), \tag{5}$$

whereas $\tilde{y}$ satisfies $\tilde{y} = C\tilde{x}$. According to Assumption 1, we can write ($N \in \mathbb{N}_0$ is a null or even design parameter)

$$u_*(t) = \sum_{i=0}^{N}\bar{\rho}_i\varphi_i(t) + \varepsilon(t), \quad |\varepsilon(t)| \le \varepsilon_N, \tag{6}$$

where[3] : $\varepsilon(t)$ either is identically zero (when $u_*(t)$ admits a finite Fourier series expansion and N is, accordingly, sufficiently large) or generally constitutes a T-periodic signal given by the difference $u_*(t) - \sum_{i=0}^{N}\bar{\rho}_i\varphi_i(t)$; the bound ε_N on the approximation error is given by

$$\varepsilon_N = \left[\left(\frac{2\pi}{T}\right)^{\tau}(N-1)^{\frac{\tau-1}{2}}\right]^{-1}2^{\frac{\tau-1}{2}}B_{\nu\tau}, \tag{7}$$

where $\tau \in \mathbb{N}$ is the (sufficiently large) order of the time derivative $\frac{\mathrm{d}^\tau u_*(t)}{\mathrm{d}t^\tau}$ which the upper bound $B_{\nu\tau}$ on $\left|\frac{\mathrm{d}^\tau u_*(t)}{\mathrm{d}t^\tau}\right|$ corresponds to. Such bound ε_N on the approximation error $\varepsilon(t)$ in (6) reduces with the number N of sinusoidal terms used to recover the uncertain periodic input reference $u_*(t)$.

With this in mind and differently from [3] and related papers (resorting to minimum-phase requirements) - while following ideas similar to the ones in [22] (though in [22] the disturbance cancellation problem is restricted to be addressed for known linear

[2] The use of such local "separation principle" makes the adaptive learning control worthy to be here preferred to the repetitive one.

[3] See [26]; see also [2] for more stringent theoretical approximation results.