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Admissibility and stabilization of stochastic singular Markovian jump systems with time delays

Guangming Zhuang [a], Qian Ma [b], Baoyong Zhang [b], Shengyuan Xu [b,*], Jianwei Xia [a]

[a] School of Mathematical Sciences, Liaocheng University, Liaocheng, Shandong 252059, PR China
[b] School of Automation, Nanjing University of Science and Technology, Nanjing 210094, PR China

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ABSTRACT

This paper deals with the admissibility analysis and stabilization of stochastic singular Markovian jump systems (SSMJSs) with time delays. More general conditions for the existence and uniqueness of the impulse-free solution to delayed SSMJSs are presented. Consequently, by constructing stochastic Lyapunov–Krasovskii functional and applying generalized Itô formula, new sufficient conditions for the stability of SSMJSs and delayed SSMJSs are obtained in terms of strict linear matrix inequalities. State feedback controller is designed to ensure stabilization of the delayed SSMJSs. Three illustrative examples, including an *RLC* circuit network and an oil catalytic cracking process, are employed to verify the effectiveness and usefulness of the obtained results.

1. Introduction

The study of singular systems has attracted much attention during the past decades [1–3]. It is well known that, besides the finite dynamic modes, singular systems also have infinite dynamic modes (which can generate undesired impulsive behavior) and infinite non-dynamic modes [4,5]. Therefore, singular systems are fundamentally different from normal state-space systems (also known as regular systems); more importantly, the use of singular system model can provide a more natural description of dynamic systems in comparison with normal systems [1–5]. A considerable number of fundamental concepts and results have been extended successfully from normal systems to singular systems. Pivotal problems for singular systems, such as stability, stabilization, normalization, passivity, dissipativity, controller and filter design, have been investigated in quick succession; see, for instance, [6–15], and the references therein.

On the other hand, as an important kind of hybrid systems, Markovian jump systems are known to be powerful when representing physical systems with abrupt structural variations, and thus they have been successfully employed in many practical regions such as networked control systems, economic systems, and manufacturing systems [5,16–18]. When singular systems are subjected to Markovian jump parameters, which leads to the well known singular Markovian jump systems (SMJSs) [5,9]. Recently, SMJSs have been widely studied by many researchers; lots of interesting results have been published such as [19–27].

Note that Itô stochastic systems have been encountered in many branches of engineering and science [28,29]. Very recently, stochastic singular systems (SSSs) and stochastic singular Markovian jump systems (SSMJSs) have been studied [30–38]. For example, the filtering problem for a class of stochastic descriptor systems was addressed in [30] while the problem of robust H_∞ output feedback control for uncertain stochastic singular systems was investigated in [31]; [32] considered passivity analysis and passification of T–S fuzzy descriptor systems with stochastic perturbation and time delay; [33] dealt with the problem of observer-based controller design for stochastic descriptor systems by applying a sequential design technique while the stabilization problem of stochastic singular nonlinear hybrid systems was studied in [34].

It is worth noting that there are some key challenges for the study of stochastic singular systems: how to derive the conditions for the existence and uniqueness of the solution to SSSs; how to define the non-impulsiveness; how to choose an appropriate Lyapunov function candidate and properly utilize the Itô formula and generalized Itô formula to show the stability of SSSs. It should be pointed out that, the aforementioned works either directly assume that the solution of the SSSs exists or completely ignore the crucial effect of the diffusion term. Recently, utilizing Kronecker product and column stacking operator technique, the mean square admissibility of singular stochastic systems with Markovian switching was studied in [35] by transforming a SSMJS into a deterministic singular system. Based on [35], the mean square admissibility

* Corresponding author.
E-mail address: syxu@njust.edu.cn (S. Xu).

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and optimal control for stochastic singular systems have been studied in [36], where the crucial effect of the diffusion term is ignored.

Very recently, [37] and [38] reported the new sufficient conditions for the existence and uniqueness of the solution to SSSs, and the SSSs were changed into deterministic singular systems by $\mathcal{H}$-representation method. However, the sufficient conditions in [35–38] do not relate to time-delays. Moreover, the Kronecker product and $\mathcal{H}$-representation techniques in [35,37,38] can be used to analyze the admissibility of SSSs directly, which have not been utilized to design controllers, observers or filters for SSSs. Therefore, finding more general sufficient conditions for the existence and uniqueness of the impulse-free solution to SSSs and SSMJSs with time delays and seeking more effective techniques for studying the corresponding synthesis problems are of considerable importance. This motivates the present investigation.

In this paper, we address the problems of admissibility and stabilization of SSMJSs with time delays. The first aim is to find more complete sufficient conditions for the existence and uniqueness of the impulse-free solution to delayed SSMJSs. To the best of our knowledge, up to now, there are few other conditions for the existence and uniqueness of the impulse-free solution to SSSs with time-varying delays. At the same time, the systems equivalence method is employed to overcome the computational complexity. The second aim is, based on stochastic Lyapunov–Krasovskii functionals and generalized Itô formula, to derive new sufficient conditions for the stability of delayed SSMJSs, the obtained conditions are given in terms of strict linear matrix inequalities (LMIs). The third aim is to design state feedback controller so as to insure stabilization of the delayed SSMJSs. Finally, examples including an *RLC* circuit network and an oil catalytic cracking process (OCCP) are applied to show the effectiveness and usefulness of the obtained results.

The main contributions of this paper are summarized as follows: (i) more general conditions for the existence and uniqueness of the impulse-free solution to delayed SSMJSs are proposed; (ii) new conditions for the stability of delayed SSMJSs are presented in terms of strict LMIs; (iii) stochastic Lyapunov–Krasovskii functionals are employed to reflect the state delays and Markovian jump modes information; (iv) state feedback controllers are designed to stabilize the delayed SSMJSs.

Notation. The notation used throughout this paper is fairly standard. $sym(A)$ means $A + A^T$, $deg(\cdot)$ stands for the degree of a polynomial, $diag(\cdot)$ represents a block diagonal matrix. For symmetric matrices X and Y, $X > Y$ implies that matrix $X - Y$ is positive definite. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations. I and 0 denote the identity matrix and a zero matrix with appropriate dimensions, respectively. R^n and $R^{n\times m}$ refer to the n-dimensional Euclidean space and set of all $n \times m$ real matrices, respectively. R_+ means the set of all nonnegative real numbers, $C^{2,1}(R^n \times R_+; R)$ stands for the family of all real-valued functions $V(x,t)$ defined on $R^n \times R_+$, which are continuously twice differentiable in $x \in R^n$ and once differentiable in $t \in R_+$. $(\Omega, \mathcal{F}, \mathcal{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $\mathcal{E}\{\cdot\}$ represents expectation operator with respect to the given probability measure $\mathcal{P}$. $\mathcal{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$, $\|\cdot\|$ refers to the spectral norm for matrices, $\|\cdot\|_2$ stands for the usual $\mathcal{L}_2[0,\infty)$ norm. $\|\cdot\|_{E_2}$ denotes the norm in $\mathcal{L}_2((\Omega,\mathcal{F},\mathcal{P}), [0,\infty))$, $|\cdot|$ is the Euclidean vector norm in R^n, $\|\phi(t)\|_{\bar{\tau}}$ means $Sup_{-\bar{\tau}\leq t\leq 0}\|\phi(t)\|$.

2. Problem formulation and preliminaries

For a given complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, we consider the SSMJS with time delays represented by

$$\begin{cases} E(r_t)\,dx(t) = [A(r_t)x(t) + A_d(r_t)x(t-\tau(t)) \\ \qquad\qquad + C(r_t)u(t)]dt + B(r_t)x(t)\,d\varpi(t), \\ x(t) = \phi(t), \quad \forall\, t \in [-\bar{\tau},\ 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input; $\phi(t)$ is a vector-valued initial continuous function. $\tau(t)$ is the unknown time-varying delay satisfying

$$0 \leq \tau(t) \leq \bar{\tau} < \infty, \quad \dot{\tau}(t) \leq \mu < 1, \tag{2}$$

where $\bar{\tau}$ and μ are constant scalars.

In system (1), $\varpi(t)$ is one-dimensional standard Brownian motion defined on probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $x(t)$ is $\mathcal{F}_t$-measurable and the $\sigma-$field $\mathcal{F}_t = \sigma(\varpi(s) : 0 \leq s \leq t)$. Let $d\varpi(t)$ and $x(t)$ are linear independent and $d\varpi(t)$ has the following property:

$$\mathcal{E}\{d\varpi(t)\} = 0, \quad \mathcal{E}\left\{d\varpi(t)^2\right\} = dt. \tag{3}$$

$\{r_t\}$ is a right continuous Markovian process independent of $\varpi(t)$ and taking values in a finite set $S = \{1, 2, \cdots, N\}$. Transition probability is given by

$$P\{r_{t+h} = j\,|r_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ii}h + o(h), & i = j \end{cases} \tag{4}$$

where $h > 0$, $\lim_{h\to 0}\frac{o(h)}{h} = 0$, and $\pi_{ij} \geq 0$, for $i \neq j$, is the transition rate from mode i at time t to mode j at time $t+h$, satisfying

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}. \tag{5}$$

$\Pi = \left[\pi_{ij}\right]_{N\times N}$ is the so-called transition rate matrix. The matrix $E(r_t) \in \mathbb{R}^{n\times n}$ may be singular and it is assumed that $rank(E(r_t)) = r \leq n$. For notation concision, in the sequel, for each $r_t = i \in S$, a matrix $L(r_t)$ will be represented by L_i, for instance, the aforesaid matrix $E(r_t)$ is depicted by E_i.

System (1) is the well-known time-delay stochastic singular Markovian jump systems (SSMJS). When $u(t) = 0$, we get the following unforced system:

$$\begin{cases} E(r_t)\,dx(t) = [A(r_t)x(t) + A_d(r_t)x(t-\tau(t))]dt \\ \qquad\qquad + B(r_t)x(t)\,d\varpi(t), \\ x(t) = \phi(t), \quad \forall\, t \in [-\bar{\tau},\ 0]. \end{cases} \tag{6}$$

In order to guarantee the well-posedness of the solution to (6), we first give the following assumption:

Assumption 1. For every $i = r_t \in S$, there are a pair of invertible matrices $M_i \in \mathbb{R}^{n\times n}$, $N \in \mathbb{R}^{n\times n}$ such that one of the following conditions is satisfied:

(1.1) $M_iE_iN = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \mathcal{J}_{in_2} \end{bmatrix}$, $M_iA_iN = \begin{bmatrix} \tilde{A}_{i1} & 0 \\ 0 & I_{n_2} \end{bmatrix}$, $M_iA_{di}N = \begin{bmatrix} \tilde{A}_{di1} & \tilde{A}_{di2} \\ 0 & 0 \end{bmatrix}$, $M_iB_iN = \begin{bmatrix} \tilde{B}_{i1} & \tilde{B}_{i2} \\ 0 & 0 \end{bmatrix}$,

(1.2) $M_iE_iN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, $M_iA_iN = \begin{bmatrix} \hat{A}_{i1} & \hat{A}_{i2} \\ \hat{A}_3 & \hat{A}_4 \end{bmatrix}$, $M_iA_{di}N = \begin{bmatrix} \hat{A}_{di1} & \hat{A}_{di2} \\ 0 & 0 \end{bmatrix}$, $M_iB_iN = \begin{bmatrix} \hat{B}_{i1} & \hat{B}_{i2} \\ 0 & 0 \end{bmatrix}$,

(1.3) $M_iE_iN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, $M_iA_iN = \begin{bmatrix} \check{A}_{i1} & \check{A}_{i2} \\ 0 & \check{A}_{i3} \end{bmatrix}$, $M_iA_{di}N = \begin{bmatrix} \check{A}_{di1} & \check{A}_{di2} \\ 0 & 0 \end{bmatrix}$, $M_iB_iN = \begin{bmatrix} \check{B}_{i1} & \check{B}_{i2} \\ 0 & \check{B}_{i3} \end{bmatrix}$,