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Observer design based on immersion technics and canonical form

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ABSTRACT

A crucial problem in immersion based observer design is the case where the dimension of the immersed system is greater than that of the original system, and the analytical expression of the inverse of the immersion is unknown. In this paper, we have proposed a method for constructing observers for such autonomous nonlinear systems.

1. Introduction

An observer is a dynamical system used for accurate reconstruction of another process or system's state variables through its measurable variables (inputs and outputs). Observer theory for linear systems is well-understood and many practical observers have been designed for both single and multi-output cases, most prominent being Luenberger observer and Kalman filter. Observers for nonlinear systems however are much more challenging to design, and nonlinear observer research has received considerable attention since early 1980s [1]. Many approaches can be found in the existing literature: (i) Filtering approach based on extending the Kalman filter to deterministic nonlinear systems (Extended Kalman Filter: EKF), its convergence is guaranteed under some hypotheses for example, [2] for local asymptotic convergence in the case of control affine nonlinear systems, and [3,4] for the global stability limited to a particular class of nonlinear systems. (ii) Geometric approach based on characterization of nonlinear systems for which an observer can be designed. In this approach, the most widespread method is the error linearization method [5–11], which consists of characterizing nonlinear systems that can be transformed via change of coordinates and output variables into a linear system plus a nonlinear term depending only on the inputs and outputs measurement. These systems can be observed using Luenberger observer. Another approach for the error linearization problem has been proposed in [12–14]. Under some local observability hypothesis, the authors propose a change of variables resulting from the resolution of a linear first order PDE. From observability point of view, all the above-mentioned classes of systems are similar to the class of linear systems in the sense that the observability is not affected by inputs. An extension of these works is to characterize nonlinear systems which are observable independently on the inputs. For single output systems [15,16], this characterization is completely determined by a normal form of observability (in the local generic sense), and a high gain observer can be designed. Some extensions to multi-output systems have been proposed in the literature [17–26]. All the observers proposed for these systems are generally based on a normal form of observability and the design of such an observer requires an inverse transformation.

It can be noted that the diffeomorphisms required to obtain normal forms are generally local and consequently less applicable for state estimation (only the trajectories that are in the domain of the transformation can be estimated). In the presence of singularities, injective immersions can be considered to obtain such a normal form. Initiated in [27], this problem has been extensively addressed in the control context in [19,28], and the details are available in [29]. Contrary to normal forms based on diffeomorphisms transformations, the construction of an observer based on immersion technics is a difficult task. The reasons are many and varied: the nonlinear elements of the dynamics of normal forms are generally unknown, the observer requires an inversion procedure necessitating an optimization algorithm, thereby increasing calculation times.

In this paper, we propose a constructive observer based on immersion of an autonomous system into a normal form. The immersion is achieved by embedding a system of dimension n in a system of dimension N (generally $N > n$). Under some observability hypotheses, this embedding is well-defined on sufficiently large domains of the state space. Motivated by this immersion, we propose an algorithm for state estimation of the original system, using an observer that does not require the knowledge of the inverse transformation (which is usually incalculable). This observer is defined by a system of differential equations of dimension $n+N$.

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The remainder of the paper is organized as follows: Section 2 contains the preliminary results required to establish the main result and Section 3 addresses the theorem stating the main result, i.e. the observer, along with its proof. A simulation example is presented in Section 4, using a simple mechanical system which illustrates the practical applicability of the observer. Some concluding remarks are given in Section 5.

2. Preliminary results and problem statement

Consider the following nonlinear systems:

$$\begin{cases} \dot{x} = f(x), & x \in \mathbb{R}^n \\ y = h(x), & y \in \mathbb{R} \end{cases} \tag{1}$$

where f and h are assumed to be of class $\mathcal{C}^\infty$.

$$\begin{cases} \dot{z} = F(z), & z \in \mathbb{R}^N \\ y = H(z), & y \in \mathbb{R} \end{cases} \tag{2}$$

where F, H are of class $\mathcal{C}^\infty$.

Definition 1. We say that system (1) is immersible into system (2), if there exists a $\mathcal{C}^\infty$ map Φ such that for every $x(0) \in \mathbb{R}^n$, $\Phi(x(.))$ is the trajectory of system (2) starting from $\Phi(x(0))$ at $t = 0$ and $H(\Phi(x(.))) = h(x(.))$ in the interval in which both $x(.)$ and $\Phi(x(.))$ are well defined. In order to distinguish between a geometric immersion and immersion of system (1) into (2), the map Φ will henceforth be referred to as an S-immersion.

2.1. Normal forms and high gain observer

Under some observability hypotheses, it can be shown that system (1) can be immersed into system (2) of the following normal form:

$$\begin{cases} \dot{z} = Az + F_N(z)b \\ y = z_1 = Cz \end{cases} \tag{3}$$

where, F_N is a $\mathcal{C}^\infty$ function defined on $\mathbb{R}^N$, $A = \begin{pmatrix} 0 & 1 & & 0 \\ 0 & & \ddots & 0 \\ 0 & & 0 & 1 \\ 0 & 0 & \dots & 0 \end{pmatrix}$; $b = \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & & 0 \end{pmatrix}$.

An observer for system (3) can be constructed as:

$$\dot{\hat{z}} = A\hat{z} + F_N(\hat{z})b + \Delta_\theta K(\hat{z}_1 - y) \tag{4}$$

where Δ_θ is the diagonal matrix $\Delta_\theta = \begin{pmatrix} \theta & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & \theta^N \end{pmatrix}$, and K is a constant column vector that renders $A + KC$ Hurwitz.

If the nonlinear term F_N is a global Lipschitz function, i.e. $|F_N(z) - F_N(z')| \le c\|z - z'\|$ for some constant $c > 0$, then system (5) is an exponential observer, and its convergence rate may be chosen arbitrarily ([15–17,21,22] and [23]). In the case where the trajectories of system (3) lie in a compact set $\mathcal{Z} \subset \mathbb{R}^N$, then replacing the nonlinear dynamics $F_N(z)$ by $\chi(z)F_N(z)$ (where χ is any $\mathcal{C}^\infty$ function which takes 1 on $\mathcal{Z}$ and 0 outside a bounded open set containing $\mathcal{Z}$) permits to obtain a global Lipschitz dynamics. Moreover trajectories of the system are not changed, as they lie in $\mathcal{Z}$.

Now let us assume that there exists an S-immersion Φ which immerses system (1) into the normal form (3). In order to estimate the unknown state $x(t)$ of system (1), we can encounter the following three situations:

(1) $N = n$ and the map Φ is a diffeomorphism. In this case an observer for system (1) can be constructed as follows:

$$\dot{\hat{x}} = f(\hat{x}) + \left[\frac{\partial \Phi}{\partial x}(\hat{x})\right]^{-1} \Delta_\theta K(h(\hat{x}) - y). \tag{5}$$

Notice that this observer does not require the knowledge of the nonlinear term F_N.

(2) $N > n$ and the analytic expression of the nonlinear term F_N is known. In this case, $x(t)$ can be estimated using

$$\begin{cases} \dot{\hat{z}} = A\hat{z} + F_N(\hat{z})b + \Delta_\theta K(\hat{z}_1 - y) \\ \hat{x}(t) = Arg \min_{x \in \Omega} \|\hat{z}(t) - \Phi(x)\| \end{cases} \tag{6}$$

(3) $N > n$ and the expression of F_N is unknown. In this case, the observer in Eq. (6) cannot be used to estimate $x(t)$.

The third case is the subject of this paper. Let us consider the following motivating example in order to illustrate its importance:

Example 1. Consider the following system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 \\ y = h(x_1) \end{cases} \tag{7}$$

where h is an analytic bijective function from $\mathbb{R}$ onto itself. The injectivity of h implies that system (7) is observable on $\mathbb{R}^2$.

Now consider the Lie derivatives of h with respect to the vector field $f(x) = (x_2, -x_1)^T$, we obtain $L_f^0(h)(x) = h(x_1)$, $L_f(h)(x) = x_2h'(x_1)$, $L_f^2(h)(x) = -x_1h'(x_1) + h''(x_1)x_2^2$. More generally, $L_f^k(h)(x) = \varphi_1(x_1) + \dots + \varphi_k(x_1)x_2^k$ is polynomial function in x_2 with coefficients in a ring of an analytic function of x_1. Assuming that there exists an integer N such that the rank of the jacobian of the map $\Phi_N = (h, \dots, L_f^{N-1}(h))$ is equal to 2, for every x. From Proposition 1, Φ_N is an embedding map from any bounded open set $\Omega \subset \mathbb{R}^2$ into $\mathbb{R}^N$, and the restriction of system (7) to Ω can be immersed into a normal form (3). However the calculation of the explicit expression of F_N is another matter. Calculating F_N such that $L_f^N(h)(x) = F_N(h)(x), \dots, L_f^{N-1}(h)(x)$ is a difficult task (generally impossible), as shown in the following example.

Example 2. The following simple example illustrates the practical difficulty in the calculation of F_N. Taking $h(x) = x_1 + sin(x_1)$ as the output of the dynamical system (7) gives us

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 \\ y = x_1 + sin(x_1). \end{cases} \tag{8}$$

Clearly h is bijective, and the explicit expressions of the Lie derivatives $L_f^k(h)$ are as follows:

$L_f(h)(x) = x_2(1 + \cos(x_1))$, $L_f^2(h)(x) = -x_2^2 \sin(x_1) - x_1(1 + \cos(x_1))$,

$L_f^3(h)(x) = -x_2^3 \cos(x_1) + 3x_1x_2 \sin(x_1) - x_2 \cos(x_1) - x_2$,

$L_f^4(h)(x) = x_2^4 \sin(x_1) + 4x_1x_2^2 \cos(x_1) - 3x_1^2 \sin(x_1) + 4x_2^2 \sin(x_1) + x_1 \cos(x_1) + x_1$.

A simple calculation shows that the rank of the jacobian of $\Phi_4 = (h, \dots, L_f^3(h))$ is 2 at every point of $\mathbb{R}^2$. However the calculation of the explicit expression of a function F_4 such that $L_f^4(h)(x) = F_4(h(x), \dots, L_f^3(h)(x))$ is not easy to obtain.

In this paper we have addressed this third situation, i.e. estimation of $x(t)$ in the case where $N > n$ and that the explicit expression