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Nonlinear discrete-time systems with delayed control: A reduction

Mattia Mattioni [a,b,*], Salvatore Monaco [a], Dorothée Normand-Cyrot [b]

[a] Dipartimento di Ingegneria Informatica, Automatica e Gestionale A. Ruberti (La Sapienza University of Rome), Via Ariosto 25, 00185 Rome, Italy
[b] Laboratoire des Signaux et Systèmes (L2S, CNRS), 3, Rue Joliot Curie, 91192, Gif-sur-Yvette, France

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ABSTRACT

In this work, the notion of reduction is introduced for discrete-time nonlinear input-delayed systems. The retarded dynamics is reduced to a new system which is free of delays and equivalent (in terms of stabilizability) to the original one. Different stabilizing strategies are proposed over the reduced model. Connections with existing predictor-based methods are discussed. The methodology is also worked out over particular classes of time-delay systems as sampled-data dynamics affected by an entire input delay.

1. Introduction

The seminal works by Smith [1] and Artstein [2] have inspired a research toward time-delay systems as an unavoidable paradigm in control theory because of their involvement in a lot of practical situations. Investigations have been addressed to the study of the effects of time delay in a control system emphasizing on drawback and also, unexpectedly, advantages. As an example, it has been shown that introducing a delay over the control system might make a non stabilizable (or not controllable) system stabilizable (or controllable) as shown, among others, in [3] or [4]. Furthermore, the huge developments in classical (non-delayed) nonlinear control motivated several important works devoted to extend those well-known results to time-delay systems (e.g., [3,5–8] and references therein). Nevertheless, a lot of questions still remain unanswered in the case of both continuous and discrete-time dynamics.

In this paper, the focus is set toward time-delay discrete-time systems which have proven themselves to be of extreme interest for several reasons [9–12]. Among them, a well-known motivation is provided by the fact that retarded discrete-time systems are finite dimensional so enabling one to restate the design problem over an extended and delay-free state–space model. That is even more interesting when the discrete-time retarded system is issued from the sampling of dynamics affected by input delays [13].

This paper addresses the stabilization of discrete-time nonlinear dynamics affected by input-delay. In this context, several works were carried out, especially in the linear context, by employing descriptor (mostly for linear systems, [3]) or prediction based feedback [14]. As this latter technique usually lacks in robustness, it was recently improved through Immersion and Invariance in [15]. Though, the aforementioned strategy is still hard to extend to larger classes of time-delay systems. Inspired by the work by Artstein [2], we aim at extending the reduction model approach to the discrete-time nonlinear context. Roughly speaking, given a nonlinear discrete-time dynamics affected by a N step input delay, we seek for a model which is delay-free and equivalent to the original retarded system at least as far as stabilizability is concerned. In doing so, we provide an explicit way of computing such a reduction and we prove that any feedback stabilizing its corresponding dynamics also achieves stabilization of the retarded dynamics. Then, we present several ways of designing control by exploiting the properties of the original delay-free system (i.e., the retarded system computed for $N = 0$) such as smooth stabilizability (in the Lyapunov sense) and u-average passivity (in the sense of [16]). Connections to predictor-based feedback laws are established and commented. The cases of Linear Time Invariant (LTI) and input-affine-like dynamics are illustrated as cases study as well as the case of sampled-data systems affected by the so-called entire delay [17,18].

The paper is organized as follows: the problem is formulated in Section 2 and general recalls on discrete-time delay-free systems are provided in Section 3; the definition of the reduction and its stabilizing properties with respect to the original retarded dynamics are in Section 4; the control design is addressed in Section 5 while some case studies are discussed in Section 6; conclusions and perspectives end the paper in Section 7.

* Corresponding author at: Dipartimento di Ingegneria Informatica, Automatica e Gestionale A. Ruberti (La Sapienza University of Rome), Via Ariosto 25, 00185 Rome, Italy.

E-mail addresses: mattia.mattioni@uniroma1.it, mattioni@l2s.centralesupelec.fr (M. Mattioni), salvatore.monaco@uniroma1.it (S. Monaco), cyrot@l2s.centralesupelec.fr (D. Normand-Cyrot).

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Notations and definitions: $\mathbb{N}$ and $\mathbb{R}$ denote, respectively, the set of natural and real numbers including the 0. For any $u^j \in \mathbb{R}$ and $j = 1, \dots, m$ and $w^i \in \mathbb{R}$ for a fixed $i \leq m$, we denote $\mathbf{w}^i = (u^1, \dots, u^{i-1}, w^i, 0, \dots, 0) \in \mathbb{R}^m$. $u_{[k-N,k[}$ denotes the story of u over time window $[k-N, k[$ (i.e., $u_{[k-N,k[} = \{u_{k-N}, \dots, u_{k-1}\}$). All the functions and vector fields defining the dynamics are assumed smooth over the respective definition spaces. Id and I denote the identity function and matrix respectively. Given a vector field f, L_f denotes the Lie derivative operator, $L_f = \sum_{i=1}^n f_i(\cdot)\nabla_{x_i}$ with $\nabla_{x_i} := \frac{\partial}{\partial x_i}$. Given two vector fields f and g, $ad_f g = L_f \circ L_g Id - L_g \circ L_f Id = [f, g]$ and iteratively $ad_f^i g = [f, ad_f^{i-1} g]$. $e^{L_f} Id$ (or $e^f Id$, when no confusion arises) denotes the associated Lie series operator, $e^{L_f} := I + \sum_{i\geq 1} \frac{L_f^i}{i!}$. Given any smooth function $h : \mathbb{R}^n \to \mathbb{R}$ then $e^{L_f} h(x) = h(e^{L_f} Id\big|_x)$.

2. Problem statement

In this paper, we address the problem of stabilizing via reduction *discrete-time dynamics with discrete input delays* of the form

$$x_{k+1} = F(x_k, u_{k-N}) \tag{1}$$

with $N \in \mathbb{N}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $F(\cdot) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and the origin as equilibrium to be stabilized. The approach consists in defining a *reduction variable* (or simply reduction) whose dynamics (the *reduced dynamics*) is delay-free and of the same dimension as the original retarded system. Moreover, the stabilizability properties of the reduced model are equivalent to those of the original system; namely, any feedback stabilizing the reduce model ensures stabilization of the retarded dynamics as well.

3. Recalls on discrete-time systems

In the following, we refer to

$$\Sigma_d : x_{k+1} = F(x_k, u_k) \tag{2}$$

as the *delay-free dynamics* associated to (1) when $N = 0$.

3.1. The differential-difference (or generically (F_0, G)) representation

As proposed in [19], (2) can be equivalently represented by two coupled difference and differential equations whenever the drift term dynamics $F(\cdot, 0) := F_0(\cdot)$ admits an inverse.[1] More in detail, assuming $m = 1$, Σ_d described as a map by (2) can be equivalently represented in the (F_0, G)-form below

$$x^+ = F_0(x), \quad x^+ := x^+(0) \tag{3a}$$

$$\frac{\partial x^+(u)}{\partial u} = G(x^+(u), u) \tag{3b}$$

where $x^+(u)$ denotes a curve parametrized by u over $\mathbb{R}^n$ and $G(\cdot, u) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ satisfies[2] $G(x, u) := \nabla_u F(x, u)\big|_{x=F^{-1}(x,u)}$. It is a matter of computations to verify that for any pair (x, u), the map $F(\cdot, u)$ can be recovered by integrating (3b) over $[0, u[$ with initial condition fixed by (3a) as $x^+ = F_0(x)$. One gets

$$F(x, u) = x^+(u) = F_0(x) + \int_0^u G(x^+(v), v)\mathrm{d}v \tag{4}$$

and thus $x^+(u_k) = x_{k+1} = F(x_k, u_k)$ for any pair (x_k, u_k).

Remark 3.1. Invertibility of $F_0(\cdot)$ guarantees the existence of $G(\cdot, u)$ and integrability of (3b) with well defined solution (4) for u sufficiently close to zero. Invertibility of $F_0(\cdot)$ can be relaxed to require the existence of a nominal control value $\bar{u} \in \mathbb{R}$ for which $F(\cdot, \bar{u})$ admits an inverse. In such a case, integrability of (3b) between $\bar{u}$ and u is still guaranteed for u in a neighborhood of $\bar{u}$.

In the *multi-input case* ($m > 1$), one defines analogously the (F_0, G)-form with $G(x, u) = (G^1(x, u), \dots, G^m(x, u))$ and $G^i(\cdot, u) := \nabla_{u^i} F(x, u)\big|_{x=F^{-1}(x,u)}$ for $i = (1, \dots, m)$ by setting

$$x^+ = F_0(x), \quad x^+ := x^+(0) \tag{5a}$$

$$\frac{\partial x^+(u)}{\partial u^1} = G^1(x^+(u), u) \tag{5b}$$

$$\dots \tag{5c}$$

$$\frac{\partial x^+(u)}{\partial u^m} = G^m(x^+(u), u). \tag{5d}$$

The family of controlled vector fields $(G^j(\cdot, u))_{j=1,\dots,m}$ verifies by definition the so-called *compatibility conditions* that guarantee integrability of the so built system of partial derivatives (see [19]). In the multi-input case, (4) generalizes as

$$F(x, u) = F_0(x) + \sum_{i=1}^m \int_0^{u^i} G^i(x^+(\mathbf{w}^i), \mathbf{w}^i)\mathrm{d}w^i \tag{6}$$

with $\mathbf{w}^i = (u^1, \dots, u^{i-1}, w^i, 0, \dots, 0)$.

As discussed through several contributions (e.g., [20,21]), the $(G^j(\cdot, u))_{j=1,\dots,m}$ provide a differential geometric apparatus to analyze and formulate in an elegant way the properties of nonlinear discrete-time dynamics and their associated flows. Some of the aspects that are instrumental in the present context are recalled below when $m = 1$ with intuitive extension to $m > 1$.

At first, given $G(\cdot, u)$, one defines $Ad_{F_0} G(\cdot, u)$ as its *transport along the drift term* $F_0(\cdot)$ as (see [19,21])

$$Ad_{F_0} G(x, u) := [\nabla_x F_0(x)]_{F_0^{-1}(x)} G(F_0^{-1}(x), u). \tag{7}$$

Iteratively, one sets $Ad_{F_0}^i G(x, u) := Ad_{F_0} \circ Ad_{F_0}^{i-1} G(x, u)$ with $Ad_{F_0}^0 G(x, u) := G(x, u)$.
Given any smooth mapping $S(\cdot) : \mathbb{R}^n \to \mathbb{R}$, a useful outcome of the (F_0, G)-representation is to split the evolution of $S(\cdot)$ along the dynamics (2) into the free (or uncontrolled) and forced contributions; namely, one writes

$$S(F(x, u)) = S(F_0(x)) + \int_0^u L_{G(\cdot,v)} S(x^+(v))\mathrm{d}v. \tag{8}$$

This is useful in the definition of u-average passivity that is recalled below [16].

3.2. u-average passivity and stabilization

The notion of *u-average passivity* has been introduced in discrete time in [16]. First, consider the case of a single-input system (i.e., when $m = 1$).

Definition 3.1. Σ_d with $u \in \mathbb{R}$ and output $H(\cdot)$ is *u-average passive* (or *average passive*) if there exists a positive semi definite function $S(\cdot) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, the storage function, such that for any pair (x_k, u_k), $k \geq 0$, one verifies the inequality

$$S(F(x_k, u_k)) - S(x_k) \leq H_{av}(x_k, u_k)u_k \tag{9}$$

where $H_{av}(x, u)$ denotes the *u-average output mapping* associated with $H(x)$; i.e.

$$H_{av}(x, u) := \frac{1}{u} \int_0^u H(x^+(v))\mathrm{d}v$$

with $H_{av}(x, 0) = H(x^+(0)) = H(F_0(x))$.

[1] There exists $F_0^{-1} : \mathbb{R}^n \to \mathbb{R}^n$ such that $F_0^{-1}(\cdot) \circ F_0(x) = F_0(\cdot) \circ F_0^{-1}(x) = x$.

[2] Given a smooth mapping $F(x, u) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $F^{-1}(x, u)$ denotes the inverse of F with respect to x; i.e., $F(F^{-1}(x, u), u) = F^{-1}(F(x, u), u) = x$.