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Disturbance observer-based control for consensus tracking of multi-agent systems with input delays from a frequency domain perspective

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ABSTRACT

The consensus tracking controller design of multi-agent systems with diverse input delays is studied in this paper. A universal block diagram is established to describe the linear multi-agent systems based on transfer functions. The stabilization of the whole system is decomposed into the zero steady-state error control problem of each independent agent. A sufficient and necessary condition is accordingly deduced to impose on each controller. Based on the H_2 performance index of each subsystem, both the optimal consensus controller and disturbance observer (DOB) are derived analytically. The distributed H_2 DOB-based consensus controller can not only achieve consensus tracking for the reference input, but also reject the effects of external disturbance and model uncertainty. Some simulations are performed to illustrate the validity of the proposed design approach.

1. Introduction

Motivated by applications in various fields, such as cooperative control of mobile robots [1,2] and unmanned aerial vehicles [3,4], and formation control of marine surface crafts [5,6], the consensus tracking problem of multi-agent systems has attracted more and more attention in the last decade. The objective of consensus tracking is to make all the agent outputs achieve asymptotic tracking for a common reference input while rejecting the effects of external disturbance and model uncertainty.

The consensus algorithms for homogeneous single-integrator [7], double-integrator [8], high-order [9], general linear [10], nonlinear agents [11], and heterogeneous agents [12] are under consideration in state-space successively. Among all the prime tools for stability analysis [13], the method of frequency domain is a powerful tool for the description of complex multi-agent systems and time delay issues [14,15]. Initially, [16] introduced a general frequency-domain framework for homogeneous interconnected systems and developed an efficient Nyquist-like method for stability check. A more general multi-input multi-output (MIMO) model is considered in [17], which can solve the consensus tracking problem of complex multi-agent systems in a uniform framework. However, the consensus protocol designs in [17] are insufficient to guarantee the overall system stability. With that, [18] revised the condition and adopted a PI controller as an example to verify. And in the frequency domain, the time delay $e^{-\tau s}$ is a non-minimum phase term whose amplitude is 1. This characteristic plays an important role in the performance optimization. Both input and communication delays are studied based on the frequency-domain analysis in [19], whose conclusion is that the consensus condition is dependent on input delays but independent of communication delays. Throughout most of the previous studies [20–22], the way to deal with the input delay issue in the multi-agent agents is finding the consensus conditions to restrict the delay. In other words, there usually exist appropriate protocols to achieve consensus as long as the delay is in the limited range. As far as we know, how to analytically design a fully distributed controller to reach consensus tracking of the multi-agent systems with given input delays has not been well solved.

In addition, the control performance for input disturbances is seldom considered in the protocol design. The existence of unexpected disturbance will easily destroy the balance status of large scale systems that has been appropriately set up. The two widely-used performance criterions are H_∞ and H_2 indices [23,24]. The basic design objective of H_2 control is to search for a controller such

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that the integral square error of the system is minimized for the particular input, such as impulse or the white noise with zero mean and unit variance. It can quantify the tracking performance and robustness of the system. [25] introduced the notions of H_∞ and H_2 performance regions and showed that H_2 performance limit of the whole system scaled with the number of agents in the network. In order to enhance the system performance with evident model mismatch and strong external disturbance, an excellent way is to introduce DOB into the consensus controller besides the conventional feedback part. The DOB, which is originally presented in [26], uses the inverse of the nominal model to observe the disturbances and then to directly cancel the effect of the disturbances in the input channel. As a result, the closed-loop is forced to act as the nominal agent. DOB-based control designs have been proposed for handling disturbances in various industrial sectors [27], and also are available in the connected autonomous systems. Asymptotical consensus tracking of the multi-agent systems is achieved under a DOB-based pinning control strategy in [28]. How to analytically design a DOB aiming to attenuate the effect of input disturbances as per the prescribed performance index is a meaningful issue.

On account of the former conclusions, the intention of this work is to design an analytical DOB-based control scheme to optimize the tracking and disturbance suppression performance for consensus of multi-agent systems with input delays. First, an MIMO framework with transfer function is established to uniformly describe all kinds of linear multi-agent systems. The consensus tracking control is transformed into the zero steady-state error control problem of some independent subsystems. Then a sufficient and necessary condition is given to guarantee the stability of the whole system. For each separate agent, both an optimal controller and DOB are developed based on the H_2 performance criterion. Compared with the previous DOB-based consensus controllers, the main merits of the proposed control scheme are the following: (1) The MIMO block diagram is suitable for all kinds of linear systems, including homogeneous, heterogeneous and high-order agents. (2) The fully distributed controllers can be equivalent to low-order PID controllers. (3) Both the derived H_2 controller and observer are given in analytical forms.

This paper is organized as follows. In Section 2 some basic concepts about graph theory are introduced. In Section 3, the MIMO model of linear multi-agent systems is established, and a sufficient and necessary condition is developed to guarantee the consensus tracking of the whole system. For each independent agent, both the optimal consensus controller and DOB with H_2 performance index are analytically derived in Section 4. Section 5 gives two numerical examples to comprehensively illustrate the validity of this novel DOB-based controller. Section 6 concludes this paper.

2. Mathematical preliminaries

In this section, we shall review some useful results of algebraic graph theory. A communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a finite node set $\mathcal{V} = \{1, 2, \ldots, N\}$ and a finite edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. The pair $(i, j) \in \mathcal{E}$ with distinct nodes $i, j \in \mathcal{V}$ represents an edge from vertex i to vertex j. The set of neighbors of agent i is denoted by $\mathcal{N}_i = \{j \mid j \in \mathcal{V}, (j, i) \in \mathcal{E}\}$. The adjacency matrix of the graph is defined as $A = (a_{ij})_{N\times N}$ where $a_{ii} = 0, a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix is defined as $L = (l_{ij})_{N\times N}$, where $l_{ii} = \sum_{j\neq i} a_{ij}, l_{ij} = -a_{ij}, i \neq j$. If $a_{ij} = a_{ji}$, the graph $\mathcal{G}$ is undirected, otherwise the graph is directed. An undirected graph is said to be strongly connected if, for any pair of distinct nodes, there exists a path between them. L is a positive semi-definite real symmetric matrix and all eigenvalues of L are non-negative real in light of the Gershgorin disc theorem [29]. In general, the Laplacian matrix of a directed graph is asymmetric. If there exists a directed path from node i to any other nodes, the graph is said to contain a directed spanning tree with node i as the root.

Lemma 1 ([7]). *Both for the undirected and directed topology, zero is always a simple and the smallest eigenvalue of the Laplacian matrix with* $\mathbf{1}$ *as the corresponding right eigenvector if and only if* $\mathcal{G}$ *is strongly connected or it has a directed spanning tree.*

Notation: $\mathcal{R}^n$ is the set of $n \times 1$ real vectors. The symbol T denotes the transpose of a matrix. Denote by $\mathbf{1}_n$ and $\mathbf{0}_n$ column vectors with all entries equal to one and zero of dimension n. $\mathbf{1}_n^m = \left(\mathbf{1}_m{}^T, \mathbf{0}_{n-m}{}^T\right)^T$. For a set of N matrices $\{A_1, \ldots, A_N\}$, we define the direct sum as $\hat{A} = \oplus\sum_{i=1}^{N} A_i = diag\{A_1, \ldots, A_N\}$. The 2-norm of transfer function $F(s)$ is defined by

$$\|F(s)\|_2 = \left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{tr}\left[F^*(j\omega)F(j\omega)\right] \mathrm{d}\omega\right\}^{\frac{1}{2}}.$$

3. MIMO framework for consensus tracking of linear multi-agent systems

The traditional topology structure of multi-agent systems can be represented by Fig. 1. The systems consist of N single-input single-output (SISO) linear autonomous agents. All the agents are divided into two parts: the leading M agents have access to the same reference signal, $r_1(s) = r_2(s) = \cdots = r_M(s) = r(s) \neq 0$. The following $N - M$ agents are only accessible to the relative output errors with respect to their neighboring agents, $r_{M+1}(s) = r_{M+2}(s) = \cdots = r_N(s) = 0$. The transfer function of agent i is

$$G_i(s) = G_{ip}(s)e^{-\tau_i s}, i = 1, 2, \ldots, N, \tag{1}$$

where $G_{ip}(s)$ is the dynamic free of delay. τ_i is a positive real constant denoting the pure input delay, that is the response time to the control effect. Disturbance and uncertainty are, in general, lumped together. $d_{i-in}(s)$ is the lumped input disturbance. $e_i(s)$, $u_i(s)$ and $y_i(s)$ are, respectively, the system error, output of the corresponding controller $C_i(s)$, and output state of agent i.

$$deg(i) = \begin{cases} 1 + \sum\limits_{j\in\mathcal{N}_i} a_{ij}, i = 1, 2, \ldots, M \\ \sum\limits_{j\in\mathcal{N}_i} a_{ij}, i = M + 1, M + 2, \ldots, N \end{cases}$$

represents the in-degree of agent i. Thus,

$$y_i(s) = G_i(s)[u_i(s) + d_{i-in}(s)] = G_i(s)[C_i(s)e_i(s) + d_{i-in}(s)],$$

with

$$deg(i)e_i(s) = \begin{cases} r(s) - y_i(s) + \sum\limits_{j\in\mathcal{N}_i} a_{ij}[y_j(s) - y_i(s)], \\ \quad i = 1, 2, \ldots, M \\ \sum\limits_{j\in\mathcal{N}_i} a_{ij}[y_j(s) - y_i(s)], \\ \quad i = M + 1, M + 2, \ldots, N. \end{cases} \tag{2}$$

Now the N input N output block diagram of linear multi-agent systems can be established as Fig. 2. $D_{in}(s)$, $E(s)$ and $Y(s)$ denote the column stack vector of $d_{i-in}(s)$, $e_i(s)$ and $y_i(s)$. $R(s) = r(s)\mathbf{1}_N^M$ is the expected consensus value. Introduce an auxiliary matrix $\Lambda = \oplus\sum_{i=1}^{N} 1/\deg(i)$. Let $\tilde{R} = \Lambda \cdot R(s)$. $\hat{G}(s) = \oplus\sum_{i=1}^{N} G_i(s)$ and $\hat{C}(s) = \oplus\sum_{i=1}^{N} C_i(s)$. Define $z_i(s)$ as the output error of agent i relative to its neighbors and $Z(s) = [z_1(s), \ldots, z_N(s)]^T$. The information exchange among the agents is represented as the matrix $\tilde{L}$, where $\tilde{L} = \Lambda \cdot \left(L + I_N^M\right)$.