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PDE based observer design for nonlinear systems with large output delay

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ABSTRACT

In this paper, we develop a new observer design method for nonlinear systems with large transport delays. The new observer design is a generalization of the PDE-based backstepping-like observer design approach. First developed for delayed linear systems, this approach relies on a modelling of the output time-delay by a 1st order hyperbolic equation, leading to an ODE–PDE representation of the system, and on coordinate transformations of the innovative system. The major technical challenge, that is faced in the generalization of the approach to nonlinear systems, consists in making it applicable in the case of an arbitrarily large time-delay D. This issue is presently coped with by redesigning the cascade observer method to fit ODE–PDE systems. A new class of observers is thus obtained involving a set of cascaded high-gain state observers and output predictors. The latter are defined by PDEs that provide estimates of the system future outputs $y(t+x)$ for all $x \in [0, D]$. The exponential stability of the observer is proved using a set of Lyapunov functionals and its performances are illustrated by simulation.

1. Introduction

In recent years, the problem of designing observers for continuous-time systems that contain time delay has been an active research topic, see e.g. [1,2]. Although time-delays are of fundamentally distributed parameter nature, observer design in the presence of these elements has proved to be possible using finite-dimensional design tools. Accordingly, one starts with exponentially convergent state observers of ODEs (without delay) and modify them by adding output and/or state predictors to compensate for the time delay effect. This approach has first been developed for linear systems in which case arbitrarily large delays can be accounted for, see e.g. [3,4]. In the more challenging case of nonlinear systems the maximum admissible delay (MAD) depends on the level of nonlinearity which, typically, is of globally Lipschitz nature. Roughly, the larger the Lipschitz constant, the smaller the MAD. This result has been illustrated using high-gain observers where the involved predictors proved to be useful in compensating the delay effect up to some upper limit. To get rid of this limitation, the concept of *chain observer* has been introduced [1,5–8]. Roughly, a cascade observer consists of a number of cascaded sub-observers, each one of them involves a predictor that compensates for a fraction of the total system time-delay. Therefore, for a time-delay of any size, there is a suitable finite number of sub-observers such that the resulting global observer is exponentially stable. So far, the idea of *cascade observer* has only been developed in the context of ODE based observer design. In the present work, we develop an infinite-dimensional version of the *cascade observer* concept.

In this paper, the problem of state observation for systems with output delay (Fig. 1) is addressed by letting the time delay be what it is: a distributed parameter phenomenon. Accordingly, time delay is captured through a first-order hyperbolic PDE connected in series with the ODE that describes the system finite-dimensional dynamics, leading to an ODE–PDE cascade representation of the system. Then, the observation problem consists in designing an observer that provides online estimates of both the (finite-dimensional) state of the ODE subsystem and the (infinite-dimensional) state of the PDE sensor. This formulation of the observer design problem has been introduced in [9] and [10] where the ability of the backstepping design approach, to yield full-order observers with feedback-predictors, has been demonstrated for linear systems. Then, arbitrary time-delay sizes can be compensated for, due to the system linearity. This paper aims at generalizing the PDE-based backstepping-like observer design approach of [9] and [10] to nonlinear systems. Specifically, the latter is described by an ODE of strict-feedback and globally Lipschitz nonlinearity. To cope with the system nonlinearity, we invoke the

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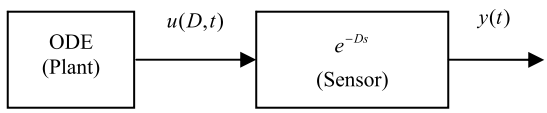

Fig. 1. System structure.

principle of high-gain observer design as in [11]. In the latter, we showed that high-gain observers, for cascade systems with parabolic PDEs, can be made exponentially convergent provided that the PDE domain length is sufficiently small. A similar result can be obtained in the case of (first-order) hyperbolic PDEs. A more challenging problem is one of designing exponentially convergent observers for ODE–PDE systems with nonlinear ODEs and PDEs of arbitrarily large domain length. This problem has yet to be solved both in the case of hyperbolic PDEs (of any order) and in the case of heat PDEs. In this paper, we develop a solution in the case of first-order hyperbolic PDEs using the PDE-based backstepping-like observer design approach. One key idea is to redesign within the ODE–PDE framework the cascade observer concept, so far only developed in an ODE framework. Accordingly, we re-express the initial ODE–PDE system representation in the form of $m \geq 2$ fictitious ODE–PDE subsystems involving first-order hyperbolic PDEs with domain length D/m (D being the arbitrarily-large time-delay) and appropriate boundary conditions defining the interaction between the subsystems. Then, we design an observer for each ODE–PDE subsystem using the high-gain observer principle backstepping-like design technique. It turns out that the global observer is composed of m (high-gain) observers connected in series. The interconnection is such that the first partial observer is directly driven by the physical system output. Then, the jth partial observer is driven by a virtual output generated by the $(j-1)$th observer. Each partial observer includes an output predictor which compensates for the effects of the fractional time-delay D/m. The predictors are defined by simple first-order hyperbolic PDEs that are much simpler compared to some previous works which involved output and state predictors. We then use a backstepping-like transformation of the estimation error system and construct an appropriate Lyapunov stability functional to analyse the transformed system. Doing so, we obtain sufficient conditions for the cascade observer to be exponentially convergent. The sufficient conditions involve the minimal number m of partial observers: the larger the system delay the larger the number m. Compared with ODE-based delay-compensating observers (e.g. [5–8]), the present observer is full-order in the sense that it estimates both the system (finite-dimensional) state and the sensor (infinite-dimensional) state. Also, the present output predictors feature a feedback structure, while those involved in ODE-based observers are open-loop.

The paper is organized as follows: first, the observation problem under study is formulated in Section 2; then, the observer design and analysis are respectively dealt with in Sections 3 and 4; simulation results are presented in Section 5; a conclusion and reference list end the paper.

2. Observation problem formulation

As depicted by Fig. 1, the system under study consists of a finite-dimensional nonlinear subsystem connected in series with a time delay. Analytically, the considered output-delayed system is described as follows:

$$\dot{X}(t) = AX(t) + f(X(t), v(t)), \quad t \geq 0, \tag{1a}$$

$$y(t) = CX(t - D) \quad \text{(output)} \tag{1b}$$

with the known matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbf{R}^{n\times n}, \; c = [10\ldots 0] \in \mathbf{R}^{1\times n} \tag{1c}$$

where $X(t) \in \mathbf{R}^n$ is the system state vector, $y(t) \in \mathbf{R}$ is the system output, $v \in C^0([0, \infty) : \Omega_v)$ is an external signal (control input) taking values in some known subset $\Omega_v \subset \mathbf{R}$, $f \in C^2([0, \infty) : \mathbf{R}^{n\times n})$ is a known bounded vector field with the triangular form:

$$f(X, v) = \begin{pmatrix} f_1(X_1, v) \\ f_2(X_1, X_2, v) \\ \vdots \\ f_n(X_1, \ldots, X_n, v) \end{pmatrix}; \tag{1d}$$

where $f_i : \mathbf{R}^i \to \mathbf{R}$. It is supposed that $f(.)$ is globally Lipschitz with respect to X, uniformly in $v \in \Omega_v$. That is, the following property holds:

$$\exists \beta_0 > 0, \forall X \in \mathbf{R}^n, \forall v \in \Omega_v : \|f_X(X)\| \leq \beta_0. \tag{1e}$$

The positive real constant D denotes a time-delay that is arbitrarily large but known. Both the input v and output y are accessible to measurements, but the state vector $X(t)$ is not. Following the approach developed in [9] and [10], the output Eq. (1b) is given an equivalent representation in the form of a first-order hyperbolic equation. Accordingly, the system model (1a)–(1c) rewrites as follows:

$$\dot{X}(t) = AX(t) + f(X(t), v(t)), \quad t \geq 0 \tag{2a}$$

$$u(D, t) = CX(t) \tag{2b}$$

$$u_t(x, t) = u_x(x, t), \quad 0 \leq x < D, \quad t \geq 0 \tag{2c}$$

$$y(t) = u(0, t). \tag{2d}$$

The solution of (2b)–(2c) is well known to be:

$$u(x, t) = CX(t + x - D), \quad 0 \leq x < D, \quad t \geq 0. \tag{3}$$

That is, the boundary measurement (2d) gives the delayed output $y(t) = CX(t - D)$, which is identical to (1b).

We seek the design of an observer that provides accurate online estimates of the finite-dimensional state $X(t)$ and the distributed state $u(x, t)$ $(0 \leq x \leq D)$. The observer must only make use of the measurements of $y(t)$, $v(t)$.

Remark 1. (1) In the case of a zero function $f(.)$ (i.e. case of linear systems), exponentially convergent observers have been designed in [9] and [10] using the (infinite-dimensional) backstepping transformation approach. In such a linear context, there is no limitation on the delay size.

(2) In the case of nonlinear systems (nonzero function $f(.)$) and a parabolic PDE (instead of (2c)), an exponentially convergent observer has been designed in [11] combining the backstepping transformation and the high-gain observer design technique (this motivated the triangular structure (1d) of the nonlinear function $f(.)$). The exponential convergence of the observer in [11] was established under the condition that the PDE domain length (presently, equivalent to the delay D) is sufficiently small.

(3) The present class of systems, described by (2a)–(2d), differs from that [11] in that the PDE is hyperbolic type and the domain length D is of arbitrary size which, together with the nonlinear