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New formulation of predictors for finite-dimensional linear control systems with input delay

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ABSTRACT

This paper focuses on a prediction-based control for linear time invariant systems subject to a constant input delay, also known as the Artstein reduction approach. Standardly, this method consists in considering a predicted delay-free system, on which one can design straightforwardly a stabilizing controller. The resulting controller is then defined through an implicit integral equation, involving both the original system state and past values of the input. We propose here an alternative formulation which allows to write explicitly the Artstein transformation, and thus the corresponding controller, in terms of past values of the state only. This formal explicit formulation is the main contribution of the paper.

1. Introduction

Even if voluntary delay introduction can sometimes benefit to the control action [1], most of the time, the appearance of delay in control loops is a source of substantial performance degradation, and even of instability if the controller has been designed neglecting this delay (see [2–4] for introductions to time-delay systems). Interestingly, these undesirable effects can be circumvented using a predictor-based approach [5–7] which enables to recover closed-loop performance similar to the delay-free case. The basic idea of this technique grounds on the use of system state prediction instead of the current state in the control loop, thus compensating for the input delay.

This method has been first introduced for linear time-invariant dynamics subject to a constant input-delay. This is also the framework considered in this paper. It is worth mentioning that numerous improvements and extension of this technique have been proposed in the last decades, such as for nonlinear plants [8–10], for various classes of non-constant delays [11,12], for uncertain [13] or multiple delays [14,15] or the design of alternative predictions to counteract the effect of integral discretization in the prediction (see the works of [16] on the addition of a low-pass filter or the ones of [17] on truncated predictors or again the ones of [18] on alternative recursive differential predictions).

In this paper, we aim at presenting an alternative formulation of the standard prediction-based technique for constant input delay, the so-called Artstein approach. Standardly, this prediction-based control law is obtained by solving an implicit integral equation involving past values of the input, namely, a Volterra equation of the second kind [19]. Here, we propose to inverse this transformation and obtain an expression of both the Artstein transform and the corresponding controller in terms of the state history only. This is the main contribution of the paper. We wish to emphasize that the novelty of this paper does not relate to implementation aspects, but rather to providing a new tool to study, e.g., implementation issues or robustness properties of standard prediction-based controllers, using the original Artstein transformation.

The paper is organized as follows. In Section 2, we briefly recall the Artstein approach before stating our main results, namely, the inversion of the Artstein reduction (see Theorems 1 and 2). Then, we illustrate the interest of this result in different technical applications in Section 3. Finally, Section 4 collects the proofs of all results, whereas some concluding remarks are given in Section 5.

2. Main results

2.1. Standard prediction — Artstein approach

In this section, we briefly recall the standard Artstein approach.

Consider the following input-delay finite-dimensional linear system

$$\dot{x}(t) = Ax(t) + Bu(t - D), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, A is a real matrix of size $n \times n$, B is a real matrix of size $n \times m$ and D is a constant input-delay. In order

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to stabilize the control system (1), introduce the so-called Artstein model reduction (see [5], see also [4,6,7]), i.e., define, for $t \in \mathbb{R}$,

$$z(t) = x(t) + \int_{t-D}^{t} e^{(t-s-D)A} Bu(s)\, ds \tag{2}$$

which gives, from an easy computation,

$$\dot{z}(t) = Az(t) + e^{-DA}Bu(t), \tag{3}$$

that is, a delay-free linear system. Therefore, assuming controllability of the pair $(A, e^{-DA}B)$, this leads to the natural control choice[1]

$$u(t) = K_D z(t) = K_D \left(x(t) + \int_{t-D}^{t} e^{(t-s-D)A} Bu(s)\, ds \right), \quad t \geqslant 0 \tag{5}$$

in which the gain matrix K_D is chosen such that $A + e^{-DA}BK_D$ is Hurwitz. Then, by construction, $t \mapsto z(t)$ converges exponentially to the origin, and hence both $t \mapsto u(t)$ and $t \mapsto \int_{t-D}^{t} e^{(t-D-s)A} Bu(s)\, ds$ converge exponentially to the origin as well. Then the equality (2) implies that $t \mapsto x(t)$ converges exponentially to the origin.

Theoretically, the predictor-based control (5) stabilizes exponentially the delay control system (1), whatever the value of the delay D may be, and without any restriction on the matrices of the system. This should be put in contrast with the use of a standard proportional controller $u(t) = Kx(t)$ which achieves closed-loop stabilization if sufficient conditions bearing on the feedback gain and involving both delay and dynamics matrices are satisfied. Yet, the prediction-based controller (5) is now infinite-dimensional as it involves an integral term depending on past values of the input, the implementation of which can generate serious computational issues [21].

2.2. Inversion of the Artstein transform

As emphasized previously, the Artstein transformation and, thus, the corresponding prediction-based control law depend on past values of the control input over a time-horizon $[t - D, t]$. In order to provide an alternative theoretical tool, we propose in this section to invert the Artstein transform (2), that is, to obtain an expression of it depending only on $x(\cdot)$ (and potentially the input over a fixed time-horizon). By expressing both the stabilization feedback law and a Lyapunov functional in terms of the state, we aim at potentially improving robustness margin but also provide new tools to study, e.g., the impact of the discretization of the integral in (5) in an implementation context.

In detail, by inverting the Artstein transform, we mean to solve the fixed point implicit equality (5) or, equivalently, to invert the definition of the variable z, which, through (2) and (5), satisfies

$$\begin{aligned} z(t) = x(t) &+ \int_{\max(t-D,0)}^{t} e^{(t-s-D)A} BK_D z(s) ds \\ &+ \int_{t-D}^{\max(t-D,0)} e^{(t-s-D)A} Bu_0(s) ds \end{aligned} \tag{6}$$

[1] It is interesting to note that the approach (2)–(5) is formally equivalent to ones considering a pole placement in terms of the original dynamics matrices A, B as done, e.g., in [20], introducing

$$\begin{aligned} u(t) &= K \left[e^{DA} x(t) + \int_{t}^{t+D} e^{(t+D-s)A} Bu(s-D) ds \right] \\ &= Ke^{DA} z(t). \end{aligned} \tag{4}$$

Indeed, one formally obtains that the two control laws are similar with $K_D = Ke^{DA}$. Moreover, noting that

$$A + e^{-DA}BK_D = e^{-DA}(A + BK)e^{DA},$$

it follows that the closed-loop matrices $A + e^{-DA}BK_D$ and $A + BK$ (with $K = K_D e^{-DA}$) have the same eigenvalues and thus the same stability properties.

in which u_0 denotes the control values for time $t < 0$, i.e., $u(t) = u_0(t)$ for $t \in [-D, 0)$.

With this aim in view, for every function f defined on $\mathbb{R}$ and locally integrable, we define

$$(T_D f)(t) = K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} Bf(s)\, ds\,, \tag{7}$$

$$(T_0 f)(t) = K_D \int_{t-D}^{\max(t-D,0)} e^{(t-D-s)A} Bf(s)\, ds. \tag{8}$$

It follows that (6) can be rewritten as $u(t) = K_D x(t) + (T_D u)(t) + (T_0 u_0)(t)$, for every $t \geqslant 0$. An explicit manual iteration leads to the following expression of the feedback u at time t,

$$\begin{aligned} u(t) = K_D x(t) &+ K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} BK_D x(s)\, ds \\ &+ K_D \int_{t-D}^{\max(t-D,0)} e^{(t-D-s)A} Bu_0(s)\, ds \\ &+ K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} BK_D \int_{\max(s-D,0)}^{s} e^{(s-D-\tau)A} BK_D x(\tau)\, d\tau\, ds \\ &+ K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} BK_D \\ &\times \int_{s-D}^{\max(s-D,0)} e^{(s-D-\tau)A} Bu_0(\tau)\, d\tau\, ds + \cdots \end{aligned} \tag{9}$$

We summarize more formally this relation in the following theorem (proved in Section 4.1).

Theorem 1. *There holds*

$$u(t) = \begin{cases} u_0(t) & \text{if } t \in [-D, 0)\,, \\ \sum_{j=0}^{+\infty} (T_D^j K_D x)(t) + \sum_{j=0}^{+\infty} (T_D^j T_0 u_0)(t) & \text{if } t \geqslant 0, \end{cases} \tag{10}$$

and the series is convergent, whatever the value of the delay $D \geqslant 0$ may be.

Note that, according to this result, the control law at time t depends on past values of x over the time interval $(0, t)$ and on the initial control values over the interval $(-D, 0)$. We reformulate this fact explicitly in the following result (proved in Section 4.2).

Theorem 2. *For every $t \in \mathbb{R}_+$, there holds*

$$x(t) = z(t) - \int_{0}^{t} \Phi_D(t, s) x(s)\, ds - \int_{-D}^{0} \Phi_0(t, s) u_0(s)\, ds \tag{11}$$

where $\Phi_D = 0$ if $D = 0$ and, otherwise, is defined as, for $(t, s) \in \mathbb{R}_+^2$,

$$\Phi_D(t, s) = f_{\lfloor \frac{t-s}{D} \rfloor} \left(t - s - \lfloor \frac{t-s}{D} \rfloor D \right), \tag{12}$$

in which $\lfloor \cdot \rfloor$ denotes the integer part of a real number and the sequence of functions $f_i : [0, D] \to \mathcal{M}_n(\mathbb{R})$ is defined as follows:

- *f_0 is the solution of the fixed-point equation*

$$f_0(r) = \tilde{f}(r) + (\tilde{T}_0 f_0)(r)\,, \; r > 0 \tag{13}$$

with, for $r > 0$,

$$\tilde{f}(r) = e^{(r-D)A} BK_D \tag{14}$$

$$(\tilde{T}_0 f_0)(r) = \int_{0}^{r} e^{(r-\tau-D)A} BK_D f_0(\tau) d\tau \tag{15}$$