Systems & Control Letters 113 (2018) 65–70

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Distributed optimal control and L_2 gain performance for the multi-agent system with impulsive effects

Lijuan Chen [a,b], Jitao Sun [b,c,*]

[a] College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian, 350108, China
[b] School of Mathematical Sciences, Tongji University, Shanghai, 200092, China
[c] Institute for Intelligent Systems, University of Johannesburg, South Africa

ARTICLE INFO

Article history:
Received 15 May 2017
Received in revised form 6 December 2017
Accepted 9 January 2018

Keywords:
Multi-agent system
Impulsive effect
Distributed optimal control
L_2 gain performance
Cross weighting term

ABSTRACT

In this paper, we investigate the distributed optimal control and L_2 gain performance for the multi-agent system with impulsive effects. First, we obtain sufficient conditions to ensure that the distributed protocols can minimize the desired performance index with state-control cross weighting terms. Second, we propose and solve the bounded L_2 gain synchronization problem for the impulsive system with hybrid disturbance inputs. Finally, an example is presented to illustrate the efficiency of the obtained results.

© 2018 Elsevier B.V. All rights reserved.

1. Introduction

Recently people have witnessed an increasing interest in networks of systems [1–10], especially, the distributed control for multi-agent system due to its wide applications in various areas, such as air traffic control, automated highway systems, and sensor networks [11–13]. Also, optimality of the protocol is an important topic for multi-agent system (MAS). Discussing the cost function or the performance index aims at getting a balance between the state convergence and the control effort because of the limited resources constraint. Compared with the centralized control, the distributed controller will save more efforts and be more feasible to implement in practice. Ma et al. [14] investigated the optimal consensus of the first-order and the second-order multi-agent systems, respectively, and mentioned that the optimal topology is a star topology under the special performance index. Ma et al. [15] studied the competition phenomena of multi-agent systems consisting of three groups of agents. Hengster-Movric and Lewis [16] and Lewis et al. [17] obtained the optimal distributed consensus protocols by the inverse optimality theory. The main idea of the inverse optimality theory is to analyze the optimality of a stabilizing control with respect to some prescribed performance index [18]. Zhang et al. [19] provided the conditions for globally optimal cooperative control problems of MAS on directed graphs which contain a spanning tree. Liao et al. [20] and Wu et al. [21] investigated the multi-agent systems with error integral and preview action and designed the controller which can achieve the cooperative optimal preview tracking. Xie and Lin [22] studied the global optimal consensus problem for a multi-agent system with bounded controls.

As we know, the actual system is often deeply disturbed by human exploring activities such as planting and harvesting. Impulsive differential equation has a wider application when analyzing many real world phenomena such as the frequency-modulated signal processing systems and flying object motions. Systems with impulse and the potential engineering applications have received extensive attention in the past few years, see Chen et al. [23], Han et al. [24], Wang and Shen [25], Guan et al. [26], Suo and Sun [27], Li and Wu [28] and references therein. However, to the best of our knowledge, to this day, for the MAS with impulse, except for [29], there exist no results which focus on discussing the problem that the distributed control law can make the desired performance, especially with cross weighting terms, reach minimum.

Meanwhile, it is well known that the external disturbance is quite common in real world and we hope that the influence of the external disturbance is as small as possible. In order to find the control law such that the closed-loop networked system reaches the synchronization in absence of disturbances, and that synchronization L_2 gain is bounded for all L_2 disturbance inputs, it is indispensable to discuss the distributed performance synthesizing problem [30–33]. For example, Wang et al. [30] considered the consensus problem of MAS with external disturbances using the

* Corresponding author at: School of Mathematical Sciences, Tongji University, Shanghai, 200092, China.
E-mail address: sunjt@tongji.edu.cn (J. Sun).

https://doi.org/10.1016/j.sysconle.2018.01.007
0167-6911/© 2018 Elsevier B.V. All rights reserved.

H_∞ control theory. Wang et al. [31] discussed the distributed robust control of uncertain linear multi-agent systems. Amini et al. [32] investigated a new method for consensus in nonlinear multi-agent systems using fixed-order non-fragile dynamic output feedback controller, via an LMI approach. However, to the best of our knowledge, there exists no research on the bounded L_2 gain synchronization problem for the impulsive MAS with hybrid disturbance inputs. The study on this issue is meaningful and challenging.

To the best of the authors' knowledge, this paper is the first work that addresses the problem about the distributed optimal control and L_2 gain performance for the multi-agent system with impulsive effects. The detailed contribution is in two aspects. One is to present the distributed protocol to optimize some specified performance index with state-control cross weighting terms, which has wider applications in engineering [34]. The other is to investigate the bounded L_2 gain synchronization problem for the impulsive MAS with hybrid disturbance inputs. We transform the hybrid disturbance rejection problem into an optimal control problem, design the distributed control and obtain the worst disturbance case.

This paper is organized as follows. In Section 2, we present the preliminaries. In Section 3, for the impulsive MAS, we discuss the distributed optimal control with state-control cross weighting terms in the performance index. In Section 4, we propose and solve the bounded L_2 gain synchronization problem for the impulsive MAS with hybrid disturbance inputs. In Section 5, numerical simulations are presented. The paper ends up with a brief discussion.

2. Preliminaries

In this section, we present several notations and graph theory preliminaries which shall be used throughout this paper.

Let $Z_+ = \{1, 2, \dots\}$ and $N_+ = \{1, 2, \dots, N\}$. R^n denotes the Euclidean space of n-dimension, $R^{n\times m}$ is the set of all $n \times m$ real matrices and I is defined as an identity matrix with compatible dimension. Moreover, $A \otimes B$ denotes the Kronecker product of matrix A and B. A S.P.D. matrix A or $A > 0$ means that A is symmetric and positive definite. Also, a S.P.S-D. matrix A or $A \geq 0$ means that A is symmetric and positive semi-definite. $d(x, S) = \inf_{y\in S} d(x, y)$ is the distance of a point from the manifold S as given by the distance function d of the embedding space. A function φ defined in $[0, +\infty)$ is a $\mathcal{K}$-class function which means that φ is continuous, strictly monotone increasing, $\varphi \geq 0$ and $\varphi(0) = 0$. In the whole paper, we suppose that $0 = t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$.

A graph $G = (N_+, E, A)$ consists of a vertex set N_+, an edge set $E = \{(j, i) : i, j \in N_+\} \subset N_+ \times N_+$ and an adjacency matrix $A = (a_{ij}) \in R^{N\times N}$. An edge $(i, j) \in E$ implies that the agent i can receive the information of agent j. The adjacency matrix $A = (a_{ij}) \in R^{n\times n}$ is defined where $a_{ij} > 0$, if $(i, j) \in E$ and $a_{ij} = 0$, otherwise. The set of neighbors of vertex i is denoted by $N_i = \{j \in N_+ : (i, j) \in E, j \neq i\}$. The Laplacian matrix $L = (l_{ij}) \in R^{n\times n}$ is denoted as $l_{ii} = \sum_{j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij},\ i \neq j$. If there is a path between any two vertices of G, then G is connected, otherwise disconnected. The graph is said to contain a directed spanning tree if there exists a vertex, υ_0, such that every other vertex can be connected to υ_0 by a directed path starting from υ_0. In this paper, we consider the leader-following case. And we assume that the graph G contains a directed spanning tree with at least one non-zero pinning gain connecting to a root node. $\tilde{D} = \mathrm{diag}\{d_1, d_2, \dots, d_N\}$ and $d_i \geq 0,\ i \in N_+$. When the ith agent is directed connected to the leader, $d_i > 0$ and $d_i = 0$ otherwise. For simplicity, denote $\hat{L} = L + \tilde{D}$.

3. Performance optimization with cross weighting terms

In this section, we consider the leader-following multi-agent system with impulse effects as follows.

$$\begin{cases} \dot{x}_i(t) = Fx_i(t) + Gu_i(t),\ t \neq t_k, \\ \Delta x_i(t_k) = (D_k - I)(x_i(t_k) - x_0(t_k)),\ k \in Z_+ \end{cases} \tag{1}$$

and

$$\begin{cases} \dot{x}_0(t) = Fx_0(t),\ t \neq t_k, \\ \Delta x_0(t_k) = 0,\ k \in Z_+ \end{cases} \tag{2}$$

where $x_i(t) \in \mathbb{R}^n,\ i \in N_+$ and $x_0(t) \in \mathbb{R}^n$ mean the position of the ith follower and the leader at time t, respectively. $F,\ G$ and impulse matrix $D_k, k \in Z_+$ are matrices with compatible dimension. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k)$. Also, $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$ and $x_i(t_k) = \lim_{t\to t_k^-} x_i(t)$.

Let $\delta(t) = (\delta_1(t), \delta_2(t), \dots, \delta_N(t))^T$ where $\delta_i(t) = x_i(t) - x_0(t),\ i \in N_+$. We can obtain the following error system:

$$\begin{cases} \dot{\delta}(t) = (I \otimes F)\delta(t) + (I \otimes G)u(t),\ t \neq t_k, \\ \delta(t_k^+) = (I \otimes D_k)\delta(t_k),\ k \in Z_+. \end{cases} \tag{3}$$

As we know, for the leader-following multi-agent system with impulse effects, it is important to design the distributed protocol which not only guarantees the consensus but also optimizes the specified performance index. Here we pay attention to discuss the following performance index with cross weighting terms:

$$J(\delta_0, u) = \int_0^{+\infty} [\delta^T Q_1 \delta + 2\delta^T Q_2 u + u^T R u] dt + \sum_{k\in Z_+} \delta^T Q_{3k} \delta \tag{4}$$

subject to the system (3). Following we will consider the above optimization problem by using the inverse optimal approach for the impulsive system. First we present a useful lemma [29] as follows.

Lemma 1. *Consider the impulsive system with control*

$$\begin{cases} \dot{x}(t) = f_c(x(t), u_c(t)),\ x(t_0) = x_0,\ t \neq t_k, \\ \Delta x(t_k) = f_d(x(t_k), u_d(t_k)),\ k \in Z_+ \end{cases} \tag{5}$$

subject to quadratic hybrid performance functional

$$\begin{aligned} J(x_0, u_c(\cdot), u_d(\cdot)) = &\int_0^{+\infty} \bar{L}_c(x(t), u_c(t)) dt \\ &+ \sum_{k\in Z_+} \bar{L}_d(x(t_k), u_d(t_k)), \end{aligned} \tag{6}$$

where $x(t) \in R^n$ is the state vector, $(u_c(t), u_d(t_k)) \in R^{mc} \times R^{md}$ is the hybrid control input. $f_c,\ f_d,\ \bar{L}_c$ and $\bar{L}_d$ are Lipschitz continuous functions. $\Delta x(t_k) = x(t_k^+) - x(t_k)$. Also, $x(t_k^+) = \lim_{t\to t_k^+} x(t)$ and $x(t_k) = \lim_{t\to t_k^-} x(t)$. Denote by S a target manifold. If there exists a continuous differentiable function $V(x)$, $(\phi_c(x), \phi_d(x))$ and $\mathcal{K}$-class functions $\upsilon,\ \chi$ satisfying the following conditions:

$$\begin{aligned} &V(x) = 0 \Leftrightarrow x \in S;\ \phi_c(x) = 0, x \in S;\ V(x) \geq \upsilon(d(x, S)); \\ &V'(x) f_c(x, \phi_c) \leq -\chi(d(x, S)), t \neq t_k; \\ &V(x + f_d(x, \phi_d)) - V(x) \leq 0, t = t_k; \\ &H_c(x, \phi_c(x)) = 0, t \neq t_k;\ H_d(x, \phi_d(x)) = 0, t = t_k; \\ &H_c(x, u_c(x)) \geq 0, t \neq t_k;\ H_d(x, u_d(x)) \geq 0, t = t_k, \end{aligned}$$

where

$$\begin{aligned} &H_c(x, u_c) = \bar{L}_c(x, u_c) + V'(x) f_c(x, u_c), t \neq t_k, \\ &H_d(x, u_d) = \bar{L}_d(x, u_d) + V(x + f_d(x, u_d)) - V(x), t = t_k. \end{aligned}$$

Download English Version:

https://daneshyari.com/en/article/7151551

Download Persian Version:

https://daneshyari.com/article/7151551

Daneshyari.com