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Stabilizing unstable periodic orbits with delayed feedback control in act-and-wait fashion

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ABSTRACT

A delayed feedback control framework for stabilizing unstable periodic orbits of linear periodic time-varying systems is proposed. In this framework, act-and-wait approach is utilized for switching a delayed feedback controller on and off alternately at every integer multiples of the period of the system. By analyzing the monodromy matrix of the closed-loop system, we obtain conditions under which the closed-loop system's state converges towards a periodic solution under our proposed control law. We discuss the application of our results in stabilization of unstable periodic orbits of nonlinear systems and present numerical examples to illustrate the efficacy of our approach.

1. Introduction

Stabilization of unstable periodic orbits of nonlinear systems using delayed feedback control was first explored in [1]. In the delayed feedback control scheme, the difference between the current state and the delayed state is utilized as a control input to stabilize an unstable orbit. The delay time is set to correspond to the period of the orbit to be stabilized so that the control input vanishes when the stabilization is achieved.

Delayed feedback controllers have been used in many studies for stabilization of the periodic orbits of both continuous- and discrete-time nonlinear systems (see, e.g., [2–4], and the references therein). More recently, [5] investigated delayed feedback control of nonlinear systems that are subject to noise, [6] explored delayed feedback control of a delay differential equation, and [7] utilized delayed feedback control for stabilizing quasi periodic orbits. The work [8] studied the relation between the delayed feedback control approach and the harmonic oscillator-based control methods for stabilizing periodic orbits in chaotic systems [9]. Furthermore, [10] and [11] explored the situation where the period of the orbit and the delay time in the delayed feedback controller do not match due to imperfect information about the periodic orbit or inaccuracies in the implementation of the controller.

The physical structure of delayed feedback control scheme is simple. However, the analysis of the closed-loop system is difficult. This is due to the fact that to investigate the system under delayed feedback control, one has to deal with delay-differential equations, the state space of which is infinite dimensional. To deal with the difficulties in the analysis of delay differential equations, an approach is to use approximation techniques (see, for instance, [12] and [13]). Another approach was taken in [14]. There, stabilization of a linear time-invariant system with a time-delay controller was considered, and "act-and-wait" concept was introduced. This concept is characterized by alternately applying and cutting off the controller in finite intervals. It is shown in [14] that by utilizing the act-and-wait concept, one may be able to derive a finite-sized monodromy matrix for the closed-loop system, which can then be used for stability analysis. Act-and-wait concept has been extended to discrete-time systems in [15], and tested through experiments in [16]. Furthermore, act-and-wait approach has been used together with delayed feedback control in [17] for stabilizing unstable fixed points of nonlinear systems, and more recently in [18] for stabilizing unstable periodic orbits of nonautonomous nonlinear systems.

In this paper, we explore the stabilization of periodic solutions to linear periodic systems with an act-and-wait-fashioned delayed feedback control framework. In this framework, a switching mechanism is utilized to turn the delayed feedback controller on and off alternately at every integer multiple of the period of a given linear periodic system. Act-and-wait scheme allows us to obtain the monodromy matrix associated with the closed-loop system under our proposed controller. We then use the obtained monodromy matrix for obtaining conditions under which the closed-loop system's state converges to a periodic solution. Our main motivation for studying a delayed feedback control problem for

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periodic systems stems from our desire to analyze the stability of a periodic orbit of a nonlinear system under delayed feedback control. In this paper we apply our results for linear periodic systems in analyzing periodic linear variational equations obtained after linearizing nonlinear systems (under delayed feedback control) around periodic trajectories corresponding to periodic orbits. The uncontrolled nonlinear systems that we consider are autonomous and as a result their stability assessment under the act-and-wait-fashioned delayed feedback controller differs from the nonautonomous case discussed in [18].

The paper is organized as follows. In Section 2, we introduce our act-and-wait-fashioned delayed feedback control framework for stabilizing periodic solutions of linear periodic systems; we present a method for assessing the asymptotic stability of a periodic solution of the closed-loop system under our proposed framework. Furthermore, in Section 3 we discuss an application of our results in stabilizing unstable periodic orbits of nonlinear systems. We present illustrative numerical examples in Section 4. Finally, we conclude our paper in Section 5.

We note that a preliminary version of this work was presented in [19]. In this paper, we provide additional discussions and examples.

2. Delayed feedback stabilization of periodic orbits

In this section, we provide the mathematical model for a linear periodic time-varying system and introduce a new delayed feedback control framework based on act-and-wait approach. We then characterize a method for evaluating convergence of state trajectories of a closed-loop linear time-varying periodic system towards a periodic solution.

2.1. Linear periodic time-varying system

Consider the linear periodic time-varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0, \quad t \geq t_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $A(t) \in \mathbb{R}^{n\times n}$ and $B(t) \in \mathbb{R}^{n\times m}$ are periodic matrices with period $T > 0$, that is, $A(t+T) = A(t)$ and $B(t+T) = B(t)$, $t \geq t_0$. For simplicity of exposition, we assume $t_0 = 0$ for the rest of the discussion because the case where $t_0 \neq 0$ can be similarly handled. Furthermore, we assume that the uncontrolled ($u(t) \equiv 0$) dynamics possess a periodic solution $x(t) \equiv x^*(t)$ with period T satisfying $x^*(t+T) = x^*(t)$, $t \geq 0$. It follows from Floquet's theorem that there exists such a periodic solution to the uncontrolled system (1) of period T if and only if there exists a nonsingular matrix $C \in \mathbb{R}^{n\times n}$ possessing 1 in its spectrum such that $V(t+T) = V(t)C$, where $V(t)$ denotes a fundamental matrix of the uncontrolled system (1). Moreover, note that since $x(t) \equiv x^*(t)$ is a T-periodic solution of the uncontrolled system (1), $x(t) \equiv \alpha x^*(t)$ is also a T-periodic solution for all $\alpha \in \mathbb{R}$, that is, $x(t) \equiv \alpha x^*(t)$ satisfies (1) with $u(t) \equiv 0$.

We investigate the asymptotic stability of periodic solutions of the closed-loop system (1) under the delayed feedback control input

$$u(t) = -g(t)F(x(t) - x(t-T)), \tag{2}$$

where $F \in \mathbb{R}^{m\times n}$ is a constant gain matrix and

$$g(t) \triangleq \begin{cases} 0, & 2kT \leq t < (2k+1)T, \\ 1, & (2k+1)T \leq t < 2(k+1)T, \end{cases} \quad k \in \mathbb{N}_0 \tag{3}$$

is a time-varying function that switches the controller on and off alternately at every integer multiples of the period T.

Note that the feedback term characterized in (2) vanishes after the periodic solution is stabilized. Specifically, for $x(t) \equiv x^*(t)$, we have $u(t) = 0$, $t \geq 0$, since $x(t) = x(t-T)$.

We remark that our control approach is a specific case of the act-and-wait approach introduced in [14]. In particular, in our control law (2), both the acting and the waiting durations have length T. Specifically, in every $2T$ period, the controller first waits for a duration of length T, and then acts for a duration of length T. Note that the controllers in [14] are more general in the sense that acting and waiting times need not be equal. In Section 4, we also consider different switching functions $g(t)$ that lead to different acting and waiting times.

The reason why $g(t)$ is set to be a time-varying function can be understood if we compare it to the case where $g(t)$ is constant. For instance, if $g(t) \equiv 1$ in (2), then (1) becomes

$$\dot{x}(t) = (A(t) - B(t)F)x(t) + B(t)Fx(t-T), \tag{4}$$

which is a delay-differential equation. Analysis of the solution of (4) is difficult, as the state space associated with (4) is infinite-dimensional.

On the other hand, for the linear periodic system

$$\dot{x}(t) = A(t)x(t), \quad A(t) = A(t+T), \tag{5}$$

where there are no delay terms, stability of an equilibrium solution can be assessed by analyzing the corresponding monodromy matrix. Let $\Phi(\cdot,\cdot)$ denote the state-transition matrix of (5). The monodromy matrix associated with the T-periodic system (5) is given by $\Phi(T,0) \in \mathbb{R}^{n\times n}$. The eigenvalues of the monodromy matrix, known as the Floquet multipliers, are essential in the analysis of the long-term behavior of the state-transition matrix of (5), because

$$\Phi(t+kT, 0) = \Phi(t,0)\Phi^k(T,0), \quad k \in \mathbb{N}_0. \tag{6}$$

Moreover, the state of the periodic system (5) satisfies

$$x((k+1)T) = \Phi x(kT), \quad k \in \mathbb{N}_0.$$

Observe that if $g(t) \equiv 1$ in (2), we would not be able to find a homogeneous expression in the form of (5), let alone find a corresponding "monodromy matrix", because of the existence of the delay term.

However, in our case, following the act-and-wait approach, we define $g(t)$ as in (3) as a switching function. Consequently, we are able to construct a monodromy matrix $\Lambda \in \mathbb{R}^{n\times n}$ for the closed-loop system (1), (2) with the *doubled period* $2T$ such that

$$x(2(k+1)T) = \Lambda x(2kT), \quad k \in \mathbb{N}_0. \tag{7}$$

Note that the spectrum of the monodromy matrix Λ characterizes long-term behavior of the state trajectory.

In the following sections, we first derive the monodromy matrix, and then we present conditions for the convergence of the state trajectory towards a periodic solution of the closed-loop system (1), (2).

2.2. Monodromy matrix

In this section, we obtain the monodromy matrix associated with the closed-loop system given by (1), (2). In our derivations, we use $\Phi(\cdot,\cdot)$ to denote the state-transition matrix associated with (5). Furthermore, let $\Upsilon(\cdot,\cdot)$ denote the state-transition matrix for the linear T-periodic system

$$\dot{x}(t) = (A(t) - B(t)F)x(t). \tag{8}$$

Now, let $\mathcal{T}_0(k) \triangleq [2kT, (2k+1)T)$, $\mathcal{T}_1(k) \triangleq [(2k+1)T, 2(k+1)T)$, $k \in \mathbb{N}_0$. Note that when $t \in \mathcal{T}_1(k)$, the controller is on, that is, $g(t) = 1$. Hence, it follows from (1) and (2) that for $t \in \mathcal{T}_1(k)$,

$$\dot{x}(t) = (A(t) - B(t)F)x(t) + B(t)Fx(t-T). \tag{9}$$