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Cooperative control of high-order nonlinear systems with unknown control directions

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ABSTRACT

In this paper, we investigate output synchronization problem of nonlinear systems that can be transformed into strict feedback form with unknown control direction. To this end, an augmented Laplacian potential function is introduced to construct local level distributed adaptive controllers such that outputs of networked agents can be synchronized while all other states maintain bounded. Moreover, tuning functions are designed to reduce the order of the parameter updater. In contrast to conventional adaptive backstepping procedure, both the parameter updating law and tuning function of the proposed scheme take advantage of information flow among networked agents and thus distributive in nature. It is proved that output synchronization of the entire network can be achieved by properly choosing Nussbaum functions, provided the information graph is undirected and connected. Furthermore, we also proposed a way to define a proper Nussbaum function. Simulation results are presented to verify the effectiveness of the proposed schemes.

1. Introduction

Over the past decade, cooperative control of networked systems has attracted great attention in the control community [1]–[2]. Due to its simplicity, earlier effort in this venue mainly focuses on networked agents with linear dynamics, such as first-order integrator systems [3] and double-integrator systems [4,5]. However, uncertainties are inevitable in practical applications, mainly attributed to imprecise model, unmeasured state and external disturbance appear in agents' dynamics. Various controllers have been proposed for cooperative control of networked systems in the presence of uncertainty, such as leader-following control in the case of unmeasured velocity [6,7], synchronization controllers using neural network to approximate the uncertainty in the input channel [8], and consensus control of networked SISO (i.e., single input single output) nonlinear systems in strict feedback form [9,10]. Notice that when agents have heterogeneous or uncertain dynamics, synchronizing the full state becomes a challenge (e.g. the dynamics of agent i is $\dot{x}_{i1} = x_{i2} + \theta_i^T \varphi(x_{i1})$; $\dot{x}_{i2} = u_i$, and it is impossible to synchronize x_{i2} when x_{i1} is synchronized because of different θ_i), while output synchronization can be realized. In addition, an output synchronization control protocol was proposed in [11] for heterogeneous and non-introspective agents, and a sufficient and necessary condition for linear output synchronization basing on internal model principal has been introduced in [12] and output consensus for linear but uncertain multi-agent system can be found in [13].

In essence, control direction or the sign of high-frequency gain plays a key role in the control system development. In existing works, it is a common practice to assume the control direction be known and positive, in order to generalize and simplify the problem. However, this assumption becomes unrealistic in cases where control directions are inherently switching and unpredictable. For instance, uncalibrated visual servo control in [14] and autopilot design of time-varying ships [15]. While rare work has been done in networked nonlinear systems with unknown control directions, Nussbaum-type gain method is often used in dealing with the stabilization problem of systems with unknown control directions [16]. In addition, it is well knowledged that adaptive control has been proved to be effective and instrumental in overcoming parameter uncertainties [17], especially when the uncertainty is linearly parameterized in a semi(pure)-strict feedback form [18,19]. Hence, combining Nussbaum-type method with adaptive control is not only natural but imperative to unknown control direction problem, breakthroughs on this topic include adaptive control of high-order nonlinear systems [20,21], and stabilization or tracking of high-order systems [22].

It should be pointed out that application of Nussbaum-type method to a single system is rather straightforward. However, to the best of our knowledge, rare work has been done for networked high-order nonlinear systems with unknown control directions,

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particularly on how to ensure stability without sacrificing distributivity analysis becomes more difficult since the overall system may have multiple Nussbaum-type functions and furthermore these Nussbaum-type functions may be interacted. Until recently, authors in [23–25] have constructed a sub-Lyapunov function for each subsystem where only one Nussbaum-type function appears. In our previous work [26], consensus problem of multiple agents with unknown high-frequency gains has been investigated, where each agent has a first-order integrator dynamic. In [27], agent dynamic has been extended to second order, but uncertainty only appears in the input channel; hence, it is a particular case of this paper. Compared with the consensus controllers in [28] and [29], which are designed by internal model method, the dimensional advantage of our controller is obvious.

Motivated by the aforementioned discussion, this paper aims to solve synchronization problem of high-order nonlinear systems in the presence of uncertainties. The objective is to design a Nussbaum-type adaptive distributed controller for each agent whose dynamics can be transformed into strict feedback form in the presence of parameter uncertainty and unknown control coefficient, such that outputs of all agents in the network can be synchronized to the same while other states maintain bounded. The main contributions are twofold: (1) It solved output synchronization problem of networked nonlinear systems in strict feedback form with parameter uncertainty and unknown but identical control direction; (2) It proposed a novel strategy to combine backstepping procedure with graph theory to construct an augmented Laplacian potential function, such that local and adaptive parameter updaters can be designed for each agent to estimate the unknown parameters.

2. Preliminaries

2.1. Notions

Throughout this paper, $\mathbb{R}^{m\times n}$, Z^+ denote the family of $m \times n$ real matrices and the set of nonnegative integers, respectively, I_n is a $n \times n$ identity matrix, $M \geq (\leq) 0$ means that M is a semi-positive (semi-negative) definite matrix, $M > (<) 0$ means that M is a positive (negative) definite matrix, $Null(M)$ denotes the null space of matrix M, $sup(\cdot)$, $inf(\cdot)$ denote the least upper bound and the greatest lower bound, respectively, and $sign(\cdot)$ is the classical signum function. For a continuous differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, the row vector of $\partial f/\partial \boldsymbol{x}$ is $[\partial f/\partial x_1, \ldots, \partial f/\partial x_n]$.

2.2. Graph theory

We first introduce some graph terminologies that can also be found in [30]. A weighted graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is a nonempty finite set of N nodes, an edge set $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is used to model the communications among agents. The neighbor set of node i is denoted by $\mathcal{N}_i = \{j | j \in \mathcal{V}, (i, j) \in \mathcal{E}\}$. $j \notin \mathcal{N}_i$ means that there is no information flow from node j to node i. A sequence of successive edges in the form $\{(i, k), (k, l), \ldots, (m, j)\}$ is defined as a path from node i to node j. For undirected graph, it is said to be connected if there is a path from node i to node j, for all the distinct nodes $i, j \in \mathcal{V}$.

A weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ where $a_{ii} = 0\,(\forall i)$ and $a_{ij} > 0\,(i \neq j)$ if $(i, j) \in \mathcal{E}$ and 0 otherwise. In undirected graph, $a_{ij} = a_{ji}$, that is information exchange is uniformly balanced. In what follows, we set $a_{ij} = 1$ when $a_{ij} > 0$ without loss of any generality. In addition, we define the in-degree of node i as $d_i = \sum_j a_{ij}$ and $D = diag\{d_i\} \in \mathbb{R}^{N\times N}$ is thus the in-degree matrix. Then, the Laplacian matrix of graph is $L = \mathcal{D} - \mathcal{A}$. Let $\mathbf{1}_N = [1, 1, \ldots, 1]^T \in \mathbb{R}^N$, it is well-known that 0 is one of the eigenvalues of the Laplacian matrix L associated with the right eigenvector $\mathbf{1}_N$.

Lemma 1 (*[31]*). *Let the undirected graph be connected, then* $L \in \mathbb{R}^{N\times N}$, $L = L^T \geq 0$ *and* $Null(L) = span\{\mathbf{1}_N\}$.

2.3. Nussbaum-type function

A Nussbaum-type function $N(\cdot)$ is the one with the following properties [16]:

$$\begin{aligned} &\lim_{k\to\infty} \sup \left(\frac{1}{k} \int_0^k N(\tau) d\tau \right) = +\infty \\ &\lim_{k\to\infty} \inf \left(\frac{1}{k} \int_0^k N(\tau) d\tau \right) = -\infty \end{aligned} \tag{1}$$

commonly used Nussbaum-type functions include $e^{k^2}\cos(k)$, $k^2\sin(k)$ and $k^2\cos(k)$ [23].

3. Problem formulation

Consider a network of N agents with the dynamic of agent i formulated as, $i = 1, \ldots, N$

$$\begin{cases} \dot{x}_{i1} = x_{i2} + \boldsymbol{\theta}_i^T \boldsymbol{\varphi}_1(x_{i1}) \\ \dot{x}_{i2} = x_{i3} + \boldsymbol{\theta}_i^T \boldsymbol{\varphi}_2(\bar{x}_{i2}) \\ \quad\vdots \\ \dot{x}_{i,n-1} = x_{in} + \boldsymbol{\theta}_i^T \boldsymbol{\varphi}_{n-1}(\bar{x}_{i,n-1}) \\ \dot{x}_{in} = b_i u_i + \boldsymbol{\theta}_i^T \boldsymbol{\varphi}_n(\bar{x}_{in}) + h_i \end{cases} \tag{2}$$
$$y_i = x_{i1}$$

where $\boldsymbol{x}_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \in \mathbb{R}^n$, $u_i \in \mathbb{R}$, $y_i \in \mathbb{R}$ are the state, input and output of agent i, respectively. Unknown constant h_i is the disturbance in the input channel. $\boldsymbol{\theta}_i \in \mathbb{R}^p$ is a vector of unknown parameters, $\boldsymbol{\varphi}_1(x_{i1}), \ldots, \boldsymbol{\varphi}_m(\bar{x}_{im}), \ldots, \boldsymbol{\varphi}_n(\bar{x}_{in}) \in \mathbb{R}^p$ are sufficiently smooth known functions bounded for all x_{i1}, where $\bar{x}_{im} = [x_{i1}, \ldots, x_{im}]^T (m = 1, \ldots, n)$, and b_i is the unknown control coefficient which satisfies the following assumption:

Assumption 1. Unknown constants $b_i\,(i = 1, \ldots, N)$ have the same sign and $0 < b_{min} \leq |b_i| \leq b_{max}$.

In this paper, we choose a Nussbaum function $N_0(\cdot)$ such that its integration on the interval $[0, k]$, i.e., $M(k) = \int_0^k N_0(\sigma)d\sigma$ meets the following assumption:

Assumption 2. $M(k) = \int_0^k N_0(\sigma)d\sigma$ possesses the following properties:

1 : $M(k)$ is an even function, i.e., $M(k) = M(-k)$.
2 : there exists $0 < \kappa_1 < \kappa_2 \cdots < \kappa_{2n}$, $n \in Z^+$ such that

$$\begin{aligned} &\max_{k\in[0,\kappa_{2n}]} \{M(k)\} = M(\kappa_{2n-1}) > 0 \\ &\min_{k\in[0,\kappa_{2n}]} \{M(k)\} = M(\kappa_{2n}) < 0 \end{aligned} \tag{3}$$

and

$$\begin{aligned} &\max_{k\in[0,\kappa_{2n-1}]} \{M(k)\} = M(\kappa_{2n-1}) > 0 \\ &\min_{k\in[0,\kappa_{2n-1}]} \{M(k)\} = M(\kappa_{2n-2}) < 0 \end{aligned} \tag{4}$$

3 : the growth rates of local maximums and minimums satisfy

$$\left| \frac{M(\kappa_{2n})}{M(\kappa_{2n-1})} \right| > \kappa^* > 0 \tag{5}$$

and

$$\left| \frac{M(\kappa_{2n-1})}{M(\kappa_{2n-2})} \right| > \kappa^* > 0 \tag{6}$$

where κ^* is a positive constant.