Systems & Control Letters 112 (2018) 24–30

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Prediction-based control of LTI systems with input and output time-varying delays

V. Léchappé [a,*], E. Moulay [b], F. Plestan [c]

[a] University of South Pacific, School of Engineering and Physics, Laucala campus, Suva, Fiji
[b] XLIM, UMR CNRS 7252, Université de Poitiers, Poitiers, France
[c] Ecole Centrale de Nantes - LS2N, UMR CNRS 6004, Nantes, France

ARTICLE INFO

Article history:
Received 9 November 2016
Received in revised form 14 July 2017
Accepted 14 December 2017

Keywords:
Prediction-based control
Input delay
Output delay
Time-varying delay
Reduction method

ABSTRACT

The stability of a prediction-based controller is studied in presence of time-varying delays both in the input and in the output. Thanks to the reduction method and a Lyapunov–Krasovskii analysis, stability conditions are derived. A comparison is also made between the single input delay and single output delay cases. It is shown that this method can be applied to stabilize output delay systems without restriction on the delay rate. The results are illustrated numerically on a double integrator.

1. Introduction

Input delay and output delay systems are a subclass of time delay systems (TDS). The reader can refer to survey papers [1,2] and books [3,4] for a general review on TDS. Input and output delays can arise because of data acquisition or because of latencies during communications between the controller, the plant and the sensors. The latter example is particularly common for remote controlled devices such as UAVs, satellites or in Networked Control Systems (NCS). Usually input and output delays are treated similarly because they have similar effects on the system.

There exist two different approaches to control such systems: memoryless (or memory free) and memory controllers. The advantage of memory free controllers is that they do not require the computation of an integral. The reader can refer to the following articles: [5] for bounded control, [6] for adaptive control, [7] and [8] for a truncated predictor, [9] for continuous pole placement, [10] for Partial Spectrum Assignment (PSA) and [11] for sliding mode techniques. The drawback of this approach is that they usually cannot guarantee a good level of performance for unstable systems with large delays. In this case, memory controllers can be designed. For systems with a single delay (in the input or in the output), a memory controller is often a controller based on the computation of a prediction. It has been highlighted in [12] that *state prediction is a fundamental concept for delay systems, much like state observation is for systems with incomplete state measurements*. The most well-known method is the Smith predictor. This frequency approach was introduced by Smith at the end of the 1950s in [13]. At the end of the 1970s and the beginning of the 1980s, the result of Smith has been extended to state–space representation and unstable systems in [14] and [15]. In [16], this approach has been extended for input, output and state delays. In [17], the standard prediction is modified to get more robustness against external disturbances. All these methods are designed for constant delays.

When the delay is time-varying, it has been shown in [18] that it is possible to perfectly compensate it but it requires the knowledge of the delay in advance. This result has been extended to nonlinear systems with both input and state delays in [19]. In practice, this is generally not possible to know the delay value in advance that is why alternative predictive techniques have been developed. In [20], the sub predictor method developed in [21] has been modified and extended to time-varying output and input delays. The advantage of this method is that it is finite dimensional. However it cannot deal with arbitrarily large input or output delays. In [22], an approximate predictor (based on the constant delay predictor) is computed for time-varying delays in the input. It is shown that the stability is preserved if the delay rate is sufficiently small.

In this article, the method presented in [22] (for time-varying input delay) is extended to LTI systems with time-varying delays both in the input and the output. It is considered that the full state is known but that the measurement is delayed as well as the input.

* Corresponding author.
E-mail addresses: vincent.lechappe@insa-lyon.fr (V. Léchappé), emmanuel.moulay@univ-poitiers.fr (E. Moulay), franck.plestan@ec-nantes.fr (F. Plestan).

https://doi.org/10.1016/j.sysconle.2017.12.006
0167-6911/

In the constant delay case, input and output delays have similar effects on the stability of the systems. As a consequence, for the time-varying delay case, it is expected that the stability conditions that hold for the input delay case will also hold for the output delay case. However, it is shown that for a time-varying output delay, no bound on the delay time-derivative is required.

The paper is organized as follows. The problem, the assumptions and the stability analysis when both input and output delays affect the system are presented in Section 2. The special cases of a single input delay and a single output delay are given in Section 3. Simulations illustrate previous theoretical results in Section 4. Finally, some perspectives are given in Section 5.

2. Main result

2.1. System presentation and assumptions

The systems considered in this work have the following form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t - h_I(t)) \\ y(t) = x(t - h_O(t)) \\ x(\theta) = \phi_x(\theta) for \theta \in [-h_{max}, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $u(t) \in \mathbb{R}^p$ and ϕ_x is continuous.[1] The delays $h_I(t)$ and $h_O(t)$ are known, time-varying and verify the assumptions below.

Assumption 1. The delays $h_I(t)$ and $h_O(t)$ are bounded, i.e. there exist $h_{min} \geq 0$ and $h_{max} > 0$ such that

$$h_{min} \leq h_I(t) \leq h_{max} \tag{2}$$

and

$$h_{min} \leq h_O(t) \leq h_{max}. \tag{3}$$

Assumption 2. The delays $h_I(t)$ and $h_O(t)$ are differentiable and their time derivatives are bounded, i.e. there exist $\delta_I > 0$ and $\delta_O > 0$ such that

$$|\dot{h}_I(t)| \leq \delta_I \tag{4}$$

and

$$|\dot{h}_O(t)| \leq \delta_O. \tag{5}$$

In addition, it is assumed that the following assumption holds.

Assumption 3. The pair (A, B) is stabilizable, so there exists a matrix K such that $A+BK$ is Hurwitz and this ensures the existence of a symmetric positive matrix P, solution of the Lyapunov equation

$$(A + BK)^T P + P(A + BK) = -c_u I_n \tag{6}$$

with $c_u > 0$ and I_n the identity matrix of order n.

The objective is to design a prediction-based controller inspired by the constant delay case prediction and to study the influence of the time-varying delay on the closed-loop stability. In the case of a time-varying delay, it is very difficult to compute the exact prediction since it would require to know future values of the delay [18,19]. However, this case is not very common in practice. Therefore, an approximate prediction $z(t)$ is computed thanks to the current value $h(t) = h_I(t) + h_O(t)$ ($h(t) \in [2h_{min}, 2h_{max}]$) as follows

$$z(t) = e^{Ah(t)}x(t - h_O(t)) + \int_{t-h(t)}^{t} e^{A(t-s)}Bu(s)ds \tag{7}$$

for all $t \geq 0$.

Remark 2.1. On top of the initial condition on $x(t)$ for $t \in [-h_{max}, 0]$, an initial condition $u(s) = \phi_u(s)$ for $s \in [-2h_{max}, 0]$ is needed to compute $z(t)$. Note that ϕ_u has to be bounded on $[-2h_{max}, 0]$ to ensure that $\sup_{s\in[-2h_{max},0]} \|u(s)\|^2$ is well defined. In addition, it is also required that ϕ_u is differentiable and that $\dot{\phi}_u(0) = K\dot{z}(0)$ in order to have u differentiable for all $t \geq -2h_{max}$.

Note that $z(t)$ is an approximate prediction so the delay is not be perfectly compensated. This approximate prediction can then be used to compute the controller

$$u(t) = Kz(t) \tag{8}$$

for all $t \geq 0$. The convergence analysis of the closed-loop system (1)–(8) is given in the next section.

2.2. Convergence result

The condition for the stability of the closed-loop system (1)–(8) is given in the following theorem.

Theorem 1. *Consider system (1) which satisfies Assumptions 1, 2, 3. Suppose that system (1) is controlled by (8) with z defined by (7) and define*

$$\begin{aligned} \Upsilon_{IO}(t) = & \sup_{s\in[t-h_{max},t]} \|x(s)\|^2 + \sup_{s\in[t-2h_{max},t]} \|u(s)\|^2 \\ & + \sup_{s\in[t-2h_{max},t]} \|\dot{u}(s)\|^2. \end{aligned} \tag{9}$$

Then, there exist $\varsigma_{IO}, \varrho_{IO}, \delta_I^* > 0$ such that, provided

$$\delta_I < \delta_I^*, \tag{10}$$

one has

$$\Upsilon_{IO}(t) \leq \varsigma_{IO}\Upsilon_{IO}(0)e^{-\varrho_{IO}t}, \quad \forall t \geq 0 \tag{11}$$

and therefore $\lim_{t\to+\infty}\|x(t)\| = 0$.

Proof. By differentiating (7) thanks to Leibniz's rule and using (1), it can be verified that the prediction $z(t)$ is solution of the following equation

$$\begin{aligned} \dot{z}(t) = & Az(t) + Bu(t) + \dot{h}_I Az(t) + \dot{h}_I e^{Ah}Bu(t - h(t)) \\ & - \dot{h}_I A \int_{t-h(t)}^{t} e^{A(t-s)}Bu(s)ds \\ & + (1 - \dot{h}_O)e^{Ah}B \int_{t-h(t)}^{\phi(t)} \dot{u}(s)ds \end{aligned} \tag{12}$$

for all $t \geq 0$ and where $\phi(t) = t-h_O(t)-h_I(t-h_O(t))$. The following Lyapunov–Krasovskii functional candidate is chosen

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{13}$$

where

$$V_1(t) = z^T(t)Pz(t), \tag{14}$$

$$V_2(t) = \int_{t-2h_{max}}^{t} (2h_{max} + s - t)\|u(s)\|^2 ds, \tag{15}$$

$$V_3(t) = \int_{t-2h_{max}}^{t} (2h_{max} + s - t)\|\dot{u}(s)\|^2 ds. \tag{16}$$

Note that P is defined in (6). The term V_1 is similar to the one used in the delay free case. The terms V_2 and V_3 are required to deal

[1] This guarantees that $\sup_{s\in[-h_{max},0]} \|x(s)\|^2$ is well defined.