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Necessary and sufficient conditions of observer-based stabilization for a class of fractional-order descriptor systems[☆]

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This paper is concerned with observer-based stabilization for fractional-order descriptor systems with order $1 \leq \alpha < 2$. A necessary and sufficient condition is proposed for the control design by using appropriate matrix variable decoupling technique. The developed results are more general and useful than some existing works, and cover them as special cases, in which only sufficient conditions were presented, in particular, for normal fractional-order systems. Illustrative examples are provided to verify and demonstrate the effectiveness and potential of the theoretic results obtained.

1. Introduction

Fractional-order systems represent a useful class of systems which are more complicated than integer-order systems. They often appear in various practical applications such as viscoelastic systems, electrochemistry, economy and biology systems, see for example, [1,2] and the references therein. Due to its importance in both theory and practical applications, such class of systems continues receiving much attention, especially with respect to stability analysis and control design. Many efficient stability conditions have been established as reported in [3–6]. In particular, recent developments witness the effectiveness of stability results in terms of linear matrix inequalities (LMIs) for two types of fractional orders: $0 < \alpha \leq 1$ and $1 \leq \alpha < 2$ [4–7]. Accordingly, stabilization via state feedback control is studied based on these developments and parallel design conditions are obtained in terms of LMIs [4–6,8]. It should be noted that although necessary and sufficient stability conditions are available in terms of LMIs for both cases of $0 < \alpha \leq 1$ and $1 \leq \alpha < 2$, the stabilization problem is hard to solve. For stabilization via state feedback, the case of $1 \leq \alpha < 2$ could be dealt with [5] while the case of $0 < \alpha \leq 1$ is difficult and it has been solved in terms of complex LMIs [4]. In this regard, an alternative method other than LMI approach is presented in [9] where the authors have introduced a unified and concise stability test for fractional order systems with $0 < \alpha < 2$. The exact stability test therein is developed using results on regular chains for semi-algebraic sets and accordingly stabilization method is provided.

In view of unavailable pseudo-states of a fractional-order system, an observer has to be designed so as to fulfil estimation and control. This issue has also been investigated, see for example, [1] and the references therein. As for the study on observer-based stabilization control, much effort has been made and corresponding results have been reported for both cases of $0 < \alpha \leq 1$ and $1 \leq \alpha < 2$ [10–13]. However, to the best of our knowledge, all the existing conditions on observer-based stabilization control design are sufficient only, even for the simple case of $1 \leq \alpha < 2$. These sufficient conditions limit the practical applications.

Recently, extensions to study on fractional-order descriptor systems have been made [2,14,15]. It is well known that descriptor systems represent a wider class of systems which are mathematically described by both differential equations and algebraic equations [16–18]. Fruitful results have been extended from normal systems to descriptor systems, coving most of research issues with respect to stability analysis, control design, estimation and filtering [18–22]. But extension of results from fractional-order systems to fractional-order descriptor systems has not been fully investigated due to its difficulty. Existence of solutions and stability problem are studied in [2,23,24]. A necessary and sufficient LMI

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condition is given in [2] for keeping admissibility of fractional-order descriptor systems with order $0 < \alpha \leq 1$. The stabilization via state feedback is considered in [14] where the design is indeed not directly with respect to descriptor systems as the considered system is normalized through derivative type state feedback prior to control design. As for observer-based control, there is no result reported for fractional-order descriptor systems, which deserves a full investigation in both theoretic and practical point of view, thus motives us for this study.

In view of the above observation, this paper focuses on the observer-based stabilization control for fractional-order descriptor systems with order $1 \leq \alpha < 2$. Firstly, a necessary and sufficient condition is presented for the admissibility of open-loop systems by using a procedure similar to that for the case of $0 < \alpha \leq 1$ in [2]. Then, a necessary and sufficient condition is obtained for observer-based stabilization control design by using appropriate matrix variable decoupling technique. When reduced to normal fractional-order systems, the present method is straightforward necessary and sufficient one which improves existing sufficient condition based results. Finally, illustrative examples are given to verify the usefulness of the proposed new design techniques.

The rest of the paper is organized as follows. The problem formulation is described in Section 2. The main results are established in Section 3, while verification examples are given in Section 4. The paper is concluded in Section 5.

Notation: Notation used in this paper is fairly standard. $X < 0$ (respectively, $X \leq 0, X > 0$ and $X \geq 0$) means that matrix X is symmetric and negative definite (respectively, negative semi-definite, positive definite and positive semi-definite). I_n is the n dimensional identity matrix and especially I is identity matrix with compatible dimension. The superscript 'T' represents the transpose. $\|\cdot\|$ denotes the spectral norm for matrices. $spec(\cdot)$ is the spectrum (set of all eigenvalues) of a matrix. $\otimes$ is the Kronecker product. The symbol $*$ denotes a block matrix inferred by symmetry, and $Sym\{M\}$ denotes $M + M^T$.

2. Problem formulation

Consider the following fractional-order descriptor system with order $1 \leq \alpha < 2$:

$$\begin{cases} ED^{\alpha}x(t) = Ax(t) + Bu(t) \\ \quad y(t) = Cx(t) \end{cases} \tag{1}$$

where D^{α} denotes the fractional order operator, $x \in \mathbb{R}^n$ is the pseudo-state vector, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^p$ is the measured output. A, B and C are constant matrices with appropriate dimensions and $rank(E) = r \leq n$. Here, the fractional-order derivative can, as mentioned in [1,4], be any type, such as the Caputo type, the Grunwald–Letnikov type and the Riemann–Liouville type, whereas definition is applicable for resultant stability criterion. Note that different type of fractional-order derivatives may affect its dependence on initialization requirements and subsequent trajectories but does not affect the stability results.

The following definitions are made analogous to those for integer-order descriptor systems [16,18] and fractional-order descriptor systems [2].

Definition 2.1. The autonomous system (1) with $u = 0$ is called regular if the pseudo-polynomial $|s^{\alpha}E - A|$ is not identically zero for some complex number $s \in \mathbb{C}$.

The regularity ensures the existence and uniqueness of solutions to the autonomous system (1) with $u = 0$. Under the regularity property, it is seen from [2] that there exist two invertible matrices N_1 and N_2 such that

$$N_1EN_2 = diag\{I_{n_1},\ J\},\quad N_1AN_2 = diag\{A_1,\ I_{n_2}\}, \tag{2}$$

where $A_1 \in \mathbb{R}^{n_1}$, $J \in \mathbb{R}^{n_2}$ is nilpotent, and $n_1 + n_2 = n$. The set of finite eigenvalues of the pair (E, A) is $spec(A_1)$. The infinite eigenvalues are associated with vectors v satisfying $Ev = 0$. Note that impulsive modes exist if there are vectors $v_1 \neq 0$ and $v_2 \neq 0$, such that $Ev_1 = 0$ and $Ev_2 = Av_1$ [2,16,25]. If the nilpotent matrix J is of index one, say $J = 0$, the autonomous system (1) with $u = 0$ is impulse-free as in the integer-order case [16,18].

Definition 2.2. The autonomous system (1) with $u = 0$ is called admissible if it is regular, impulse-free and $|arg(spec(A_1))| > \alpha\pi/2$.

It is obvious that, in the special case of $E = I$, the admissibility concept reduces to the asymptotical stability for normal fractional-order systems (see, e.g., [5]).

The purpose of this work is to find an observer-based controller

$$\begin{cases} ED^{\alpha}\hat{x}(t) = A\hat{x}(t) + Bu(t) + L(C\hat{x}(t) - y(t)) \\ \quad u(t) = K\hat{x}(t) \end{cases} \tag{3}$$

where $\hat{x} \in \mathbb{R}^n$ is the estimation pseudo-state, $L \in \mathbb{R}^{n\times p}$ and $K \in \mathbb{R}^{m\times n}$ are, respectively, observer gain and controller gain to be designed, such that the closed-loop system

$$\bar{E}D^{\alpha}\bar{x}(t) = \bar{A}\bar{x}(t) \tag{4}$$

is admissible. Here,

$$\bar{x} = [x^T,\ (x - \hat{x})^T]^T,$$
$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix},\quad \bar{A} = \begin{bmatrix} A + BK & -BK \\ 0 & A - LC \end{bmatrix}. \tag{5}$$

3. Main results

The following lemma provides a necessary and sufficient condition for the admissibility of system (4).

Lemma 3.1. *The system of the form (4) is admissible if and only if there exists a matrix $\bar{P}$ such that*

$$\bar{P}^T\bar{E} = \bar{E}^T\bar{P} \geq 0, \tag{6}$$

$$Sym\{\Theta \otimes (\bar{P}^T\bar{A})\} < 0, \tag{7}$$

where

$$\Theta = \begin{bmatrix} \sin(\pi - \alpha\frac{\pi}{2}) & \cos(\pi - \alpha\frac{\pi}{2}) \\ -\cos(\pi - \alpha\frac{\pi}{2}) & \sin(\pi - \alpha\frac{\pi}{2}) \end{bmatrix},\quad 1 \leq \alpha < 2. \tag{8}$$

Proof. Let us first assume that $(\bar{E}, \bar{A})$ is regular and impulse-free. Then, we have the decomposition form (2). Noticing the well-established result (see, e.g., [5]) that the normal fractional-order system $D^{\alpha}x_1(t) = A_1x(t)$ is asymptotically stable if and only if $Sym\{\Theta \otimes (PA_1)\} < 0$ holds for a positive definite matrix $P > 0$, the rest of proof follows similarly to that of [2] which is for the case of fractional-order $0 < \alpha < 1$. This proves that, under the regular and impulse-free property of the pair $(\bar{E}, \bar{A})$, system (4) is admissible if and only if there exists a matrix $\bar{P}$ such that (6) and (7) hold.

Now, note that the conditions in (6) and (7) imply that $\bar{P}^T\bar{E} = \bar{E}^T\bar{P} \geq 0$ and $Sym\{\bar{P}^T\bar{A}\} < 0$, which assure that $\bar{P}$ is invertible and $(\bar{E}, \bar{A})$ is regular and impulse-free. Hence, this completes the proof of the lemma.

Remark 3.1. Note that Lemma 3.1 is true for any pair (E, A) of the form (4) but not merely for the specific system appeared in (4) with (5).