Systems & Control Letters 112 (2018) 42–50

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Stability of two-dimensional descriptor systems with generalized directional delays

Le Van Hien [a,*], Le Huy Vu [b], Hieu Trinh [c]

[a] Department of Mathematics, Hanoi National University of Education, Hanoi, Vietnam
[b] Faculty of Natural Science, Hong Duc University, Thanhhoa, Vietnam
[c] School of Engineering, Deakin University, Geelong, VIC 3217, Australia

ARTICLE INFO

Article history:
Received 7 June 2016
Received in revised form 27 June 2017
Accepted 14 December 2017

Keywords:
2-D descriptor systems
Roesser model
Interval delays
2-D summation inequality

ABSTRACT

This paper is concerned with the problem of stability analysis of two-dimensional (2-D) descriptor systems described by a Roesser-type model with generalized directional time-varying delays. By constructing an improved 2-D Lyapunov–Krasovskii functional candidate and utilizing zero-type free matrix equations, new delay-dependent conditions are derived in terms of linear matrix inequalities (LMIs) to ensure that the system under consideration is regular, causal and internally stable. The obtained results are shown to extend the existing literature by numerical examples.

1. Introduction

Descriptor model is commonly used to describe dynamics of various practical phenomena such as electrical circuit networks, power systems, multibody mechanics, aerospace engineering and chemical and physical processes [1,2]. The systems of this type are also known as singular systems [3], implicit systems [4,5] or differential/difference-algebraic equations [6,7]. In such a system, the state variables are subject to both dynamical equations and algebraic constraints which result a number of different features from classical systems such as impulsive behaviors in the state response, non-properness of transfer matrix or non-causality between input/output and states. These characteristic properties make the study of descriptor systems much more complicated and challenging than classical systems. On the other hand, as an inherent characteristic, time-delay is ubiquitously encountered in engineering systems which has various effects on the system performance [8]. Thus, the study of qualitative behavior of time-delay systems plays an important role in applied models which has received significant research attention in the last two decades (see, e.g. [9–13]). In particular, a great deal of effort from researchers has been devoted to the problems of stability analysis and control of singular systems with delays and many results have been reported in the literature. To mention a few, we refer the reader to [14–18] for the problem of stability analysis and [19–24] for some other control issues related to singular delayed systems.

Two-dimensional systems can be used to describe dynamics of many practical models where the information propagation occurs in each of the two independent directions [25,26]. Recently, due to their widespread applications in circuit analysis, image processing, seismographic data transmission or multi-dimensional digital filtering, the theory of 2-D systems has attracted considerable research attention (see, e.g. [27–30] and the references therein). There have been a few papers concerning the problems of stability and stabilization of 2-D descriptor systems. For example, in [31], the stability problem was studied for 2-D linear singular systems in general model. Sufficient conditions were derived in a type of Lyapunov matrix inequalities to ensure that a 2-D system is acceptable (see Definition 1) and asymptotically stable. In [32], the problems of stability and stabilization via state feedback controllers were investigated for a class of delay-free 2-D singular Roesser systems. By decomposing the system into slow- and fast-subsystems, and based on the Lyapunov function method, sufficient conditions in terms of linear matrix inequalities (LMIs) were derived to design a stabilizing state feedback controller. The problem of H_∞ control was also considered in [33,34] for 2-D singular Roesser models with constant delays. By using the bounded real lemma approach, delay-independent LMI-based conditions were derived for the design of state feedback controllers that make the closed-loop system to be acceptable and stable with a prescribed H_∞ performance level. However, the proposed method of [33,34] cannot be extended to 2-D singular systems with time-varying delays which are encountered in many practical systems, for instance, in 2-D models of networked control systems. Looking at the literature so far, apart from [35], it is clear that there has been no reported

* Corresponding author.
E-mail address: hienlv@hnue.edu.vn (L.V. Hien).

https://doi.org/10.1016/j.sysconle.2017.12.003
0167-6911/

work on stability of 2-D singular systems with time-varying delays. Based on the Lyapunov–Krasovskii functional (LKF) method and by employing a Jensen-type discrete inequality to manipulate the difference of a LKF candidate, delay-dependent stability conditions were first derived in [35] for a class of 2-D singular Roesser systems with state-varying delays.

It is noted also that practical models in real world applications produce different types of delays because of variable networks transmission conditions. Since the properties of these delays may not be identical, it is not reasonable to lump all the delays into one type [18]. Thus, it is interesting and important to study 2-D singular models with different types of delays. Besides, the use of Jensen-type inequalities usually produces undesired conservatism in the derived stability conditions. Therefore, reducing the conservativeness of stability conditions is always an important issue in applications of control engineering which needs further investigation.

Motivated from the above discussion, in this paper, we study the problem of stability analysis of a general class of 2-D descriptor systems described by the Roesser model with delays. The novelties of this paper are three points.

- The stability problem is studied for 2-D descriptor systems with interval discrete and distributed time-varying delays in both horizontal and vertical directions which encompass the 2-D descriptor models considered in the existing literature as some special cases.
- An improved 2-D LKF candidate which comprises of some quadratic terms and summation terms in single, double and triple forms, is constructed.
- The technique of zero-type free matrix equations is utilized to further reduce conservativeness of the proposed stability conditions.

On the basis of these features, new delay-dependent conditions are formulated in terms of LMIs to ensure that the system under consideration is regular, causal and internally stable. The obtained results are shown to extend the existing results in the literature by numerical examples.

Notation. $\mathbb{Z}$ denotes the set of integers, $\mathbb{Z}[a,b] \triangleq \{a, a+1, \dots, b\}$ for $a, b \in \mathbb{Z}$, $a \le b$. $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices and $\mathrm{diag}(A,B) \triangleq \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ for two matrices A, B of appropriate dimensions. $\mathrm{Sym}(A) \triangleq A + A^\top$ for $A \in \mathbb{R}^{n\times n}$. A matrix $M \in \mathbb{R}^{n\times n}$ is semi-positive definite, $M \ge 0$, if $x^\top Mx \ge 0$, $\forall x \in \mathbb{R}^n$; M is positive definite, $M > 0$, if $x^\top Mx > 0$, $\forall x \in \mathbb{R}^n$, $x \ne 0$.

2. Preliminaries

Consider a class of 2-D descriptor systems with mixed directional time-varying delays described by the following Roesser model (2-D DRM)

$$E\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + A_\tau \begin{bmatrix} x^h(i-\tau_h(i),j) \\ x^v(i,j-\tau_v(j)) \end{bmatrix} + A_d \begin{bmatrix} \sum_{l=1}^{d_h(i)} x^h(i-l,j) \\ \sum_{l=1}^{d_v(j)} x^v(i,j-l) \end{bmatrix}, \quad i,j \in \mathbb{Z}^+, \tag{1}$$

where $x^h(i,j) \in \mathbb{R}^{n_h}$ and $x^v(i,j) \in \mathbb{R}^{n_v}$ are the horizontal state vector and the vertical state vector, respectively, $E, A, A_\tau, A_d \in \mathbb{R}^{n\times n}$ $(n = n_h + n_v)$ are given real matrices, where E is singular with $\mathrm{rank}(E) = r \le n$. $\tau_h(i)$, $d_h(i)$ and $\tau_v(j)$, $d_v(j)$ are respectively the directional time-varying delays along the horizontal and vertical directions satisfying

$$\tau_{hm} \le \tau_h(i) \le \tau_{hM}, \quad \tau_{vm} \le \tau_v(j) \le \tau_{vM}, \tag{2a}$$
$$d_{hm} \le d_h(i) \le d_{hM}, \quad d_{vm} \le d_v(j) \le d_{vM}, \tag{2b}$$

where $\tau_{hm}, \tau_{hM}, \tau_{vm}, \tau_{vM}, d_{hm}, d_{hM}, d_{vm}$ and d_{vM} are known nonnegative integers involving the upper and the lower bounds of delays. Denote $\sigma_h = \max(\tau_{hM}, d_{hM})$ and $\sigma_v = \max(\tau_{vM}, d_{vM})$.

Initial condition of (1) is defined by

$$\begin{aligned} x^h(k,j) &= \phi(k,j), \ k \in \mathbb{Z}[-\sigma_h, 0], \ j \in \mathbb{Z}^+, \\ x^v(i,l) &= \psi(i,l), \ l \in \mathbb{Z}[-\sigma_v, 0], \ i \in \mathbb{Z}^+, \end{aligned} \tag{3}$$

where $\phi(k,.) \in l_2(\mathbb{Z}^+)$, $\forall k \in \mathbb{Z}[-\sigma_h, 0]$ and $\psi(.,l) \in l_2(\mathbb{Z}^+)$, $\forall l \in \mathbb{Z}[-\sigma_v, 0]$.

By adopting some concepts from 1-D singular systems [3,25], we give the following definitions.

Definition 1.

(i) The pair (E, A) is said to be regular if $\det[E\mathbb{I}(z,w) - A]$ is not identically zero, where $\mathbb{I}(z,w) = \mathrm{diag}(zI_{n_h}, wI_{n_v})$ and I_n is the identity matrix of $n \times n$ dimension.
(ii) The pair (E, A) is said to be causal if $\deg(\det(sE - A)) = \mathrm{rank}(E)$.
(iii) The 2-D DRM (1) is said to be acceptable if the pair (E, A) is regular and causal.

Definition 2 (*[33]*). The acceptable 2-D DRM (1) is said to be internally stable if for any initial condition (3) it holds that

$$\lim_{q\to\infty} \sup \left\{ \left\| \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \right\| : \ i+j = q \right\} = 0.$$

System (1) is admissible if it is acceptable and internally stable.

It is noted that regularity and causality of the pair (E, A) guarantee system (1) to have a decomposition into slow- and fast-subsystems which deduce the global existence and uniqueness of solution of system (1) [3]. Besides that if an acceptable 2-D DRM in the form of (1) is causal then it can be equivalently transformed into a standard form where $E = \mathrm{diag}(E_h, E_v)$ via linear transformations. On the basis of this observation, our main goal in this paper is to establish new delay-dependent conditions in terms of LMIs ensuring the regularity, causality and internal stability of system (1), where for convenience we assume that $E = \mathrm{diag}(E_h, E_v)$, $E_h \in \mathbb{R}^{n_h\times n_h}$ and $E_v \in \mathbb{R}^{n_v\times n_v}$.

Since $\mathrm{rank}(E) = r \le n$, there always exist two nonsingular matrices M, N such that

$$\bar{E} = MEN = \left[\begin{array}{c|c} I_r & 0_{r\times(n-r)} \\ \hline 0_{(n-r)\times r} & 0_{(n-r)\times(n-r)} \end{array}\right]. \tag{4}$$

Let

$$\begin{aligned} \bar{A} &= MAN = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right], \\ \bar{A}_\tau &= MA_\tau N = \left[\begin{array}{c|c} A_{\tau 11} & A_{\tau 12} \\ \hline A_{\tau 21} & A_{\tau 22} \end{array}\right], \\ \bar{A}_d &= MA_d N = \left[\begin{array}{c|c} A_{d11} & A_{d12} \\ \hline A_{d21} & A_{d22} \end{array}\right]. \end{aligned} \tag{5}$$

Lemma 1. *With the decompositions (4) and (5), system (1) is acceptable if A_{22} is nonsingular.*

Proof. Denote

$$\tilde{M} = \begin{bmatrix} I_r & -A_{12}A_{22}^{-1} \\ 0_{(n-r)\times r} & I_{n-r} \end{bmatrix} M, \quad \tilde{N} = N \begin{bmatrix} I_r & 0_{r\times(n-r)} \\ -A_{22}^{-1}A_{21} & A_{22}^{-1} \end{bmatrix},$$