Systems & Control Letters 111 (2018) 18–26

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

# Relaxing the conditions for parameter estimation-based observers of nonlinear systems via signal injection

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ARTICLE INFO

*Article history:*
Received 1 May 2017
Received in revised form 13 October 2017
Accepted 23 October 2017
Available online 24 November 2017

*Keywords:*
Signal injection
Nonlinear systems
Observer
Parameter estimation

ABSTRACT

Parameter estimation-based observers are a new kind of state reconstruction methods where the state observation task is translated into an *on-line parameter estimation* problem. A key step for its application is the transformation of the system dynamics into a particular cascade form, which involves the solution of a partial differential equation that, moreover, should satisfy some injective requirement. In this note we use a recently proposed technique of signal injection to generate new outputs and simplify these tasks. In this way, we make this observer applicable to a wider class of nonlinear systems—even with indistinguishable states. The application of the proposed approach is illustrated with the design of a novel sensorless controller for magnetic levitation systems.



## 1. Introduction

In the last three decades we have witnessed rapid progress of nonlinear observer theory in the control literature. Recent reviews may be found in [1–3]. The first systematic observer theory for nonlinear systems was developed in [4]. In this seminal work, a nonlinear state transformation to obtain a linear system up to an output injection was proposed. The approach has some extremely restrictive conditions, stymying its applicability for most physical systems. Since then various approaches to design observers for nonlinear systems have been pursued, *e.g.*, based on nonlinearity domination via high-gain [5,6], on the generation of invariant manifolds [1,7], on passivity and dissipativity theory [8] and on optimization theory [9]. Particular attention has been given to the Kazantzis–Kravaris–Luenberger (KKL) observer [10], which is the extension of Luenberger's cornerstone work [11] for linear time-invariant (LTI) systems to the nonlinear case. For autonomous systems, KKL observers have been systematically studied from both theoretical and numerical aspects [12,13]. However, the existence of KKL observer and its design procedure for nonlinear systems with inputs are, to the best of our knowledge, still open problems.

In [14] a novel class of observers for nonlinear control systems, called parameter estimation-based observers (PEBOs), was proposed. The distinguishing feature of PEBOs is that it formulates the observer design problem as a parameter estimation problem. A key step for the application of PEBO is the transformation of the system dynamics into a particular cascaded form, a task that requires the solution of a partial differential equation (PDE). Moreover, this solution should define an injective map and an associated (consistent) parameter identification task has to be solved. In this note we propose to use the signal injection technique introduced in [15] to generate new "virtual outputs" that, included in the PEBO design, provide additional degrees of freedom to satisfy the three aforementioned requirements. The inclusion of probing signals for observer and controller design is standard practice in many practical applications like sensorless control of electrical machines [16], vibration control [17] and active islanding detection schemes [18]. It is also routinely used in parameter identification and adaptive control to achieve the persistency of excitation condition required for parameter convergence and control robustification [19].

To apply the signal injection technique in the PEBO scenario it is necessary to extend the results of [15] in three directions.

(i) Extend the theory for scalar systems to the case multi-input–multi-output (MIMO) systems.
(ii) Refine the averaging analysis considering the use in the control of the estimated virtual output instead of the virtual output itself.
(iii) Redesign the estimator of an alternative virtual output instead of the one obtained for the nominal system.

Another contribution of the paper is to propose a technique to solve the PDE using mappings pseudo inverses and Poincare's lemma.

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https://doi.org/10.1016/j.sysconle.2017.10.011
0167-6911/

This new procedure, which translates the problem of solution of the PDE into the solvability of a set of (nonlinear) algebraic equations and an, easily verifiable, integrability condition – both steps some free mappings – is more systematic and provides additional degrees of freedom to solve the problem.

The remaining of paper is organized as follows. In Section 2, a brief review of PEBO is given. The main idea and a motivating example are presented in Section 3. Sections 4 and 5 describe the use of signal injection in PEBO, while in Section 6 we discuss the new technique to solve the PDE. Examples that illustrate the main results of the paper are given in Section 7. Section 8 contains the conclusion and future work. To enhance readability the technically involved proofs are given in appendices.

**Notations.** $\mathbb{O} = \mathrm{col}(0,\ldots,0)$. $\mathbb{1} = \mathrm{col}(1,\ldots,1)$. $A^\dagger$ denotes the generalized inverse of the matrix $A$. All the functions are supposed smooth enough. Given a function $f : \mathbb{R}^n \to \mathbb{R}$ we define $\partial_i f := \frac{\partial f}{\partial x_i}$ and the differential operator $\nabla f := \left(\frac{\partial f}{\partial x}\right)^\top$. For a mapping $F : \mathbb{R}^n \to \mathbb{R}^m$ we define the $ij$th element of its $n \times m$ (transposed) Jacobian matrix as $(\nabla F)_{ij} := \frac{\partial F_j}{\partial x_i}$. $\mathcal{O}$ is the "uniform big O" symbol, that is, $f(z,\varepsilon) = \mathcal{O}(\varepsilon)$ if and only if $|f(z,\varepsilon)| \le C\varepsilon$ for a constant $C$ independent of $z$ and $\varepsilon$.

## 2. Parameter estimation-based observers

To apply the signal injection technique it is necessary to develop a slight variation of the main result of [14], where the full state – instead of part of it as done in [14] – is observed. For the sake of completeness we give also the proof of this result.

**Proposition 1.** *Consider the nonlinear system*

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned} \tag{1}$$

*where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$ and $g(x)$ is full rank, together with the following assumptions.*

(**A1**) *(PDE solvability) There exist mappings*

$$\phi : \mathbb{R}^n \to \mathbb{R}^{n_z},\ \beta : \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}^{n_z},$$

*with $n_z \ge n - p$ satisfying the PDE*

$$\nabla^\top \phi(x)\big[f(x) + g(x)u\big] = \beta\big(h(x), u\big) \tag{2}$$

(**A2**) *(Left invertibility) There exists a mapping $\phi^L : \mathbb{R}^{n_z} \times \mathbb{R}^p \to \mathbb{R}^n$ such that*

$$\phi^L(\phi(x), h(x)) = x. \tag{3}$$

(**A3**) *(Consistent identification) There exist mappings*

$$\begin{aligned}&M : \mathbb{R}^{n_\zeta} \times \mathbb{R}^{n_z} \times \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}^{n_\zeta},\\ &N : \mathbb{R}^{n_\zeta} \times \mathbb{R}^{n_z} \times \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}^{n_z},\end{aligned}$$

*with $n_\zeta \in \mathbb{N}_+$ such that the parameter estimator*

$$\dot{\zeta} = M(\zeta, \mathfrak{z}, y, u) \tag{4}$$

$$\hat{\vartheta} = N(\zeta, \mathfrak{z}, y, u) \tag{5}$$

*generates a consistent estimate for the nonlinear regression model*

$$\dot{\mathfrak{z}} = \beta(y, u) \tag{6}$$

$$y = h(\phi^L(\mathfrak{z} + \vartheta, y)), \tag{7}$$

*where $\vartheta \in \mathbb{R}^{n_z}$ is a vector of constant, unknown parameters. That is, all signals are bounded and*

$$\lim_{t\to\infty} |\hat{\vartheta}(t) - \vartheta| = 0.$$

*Under these assumptions, the PEBO*

$$\hat{x} = \phi^L(\mathfrak{z} + \hat{\vartheta}, y)$$

*verifies* $\lim_{t\to\infty}|\hat{x}(t) - x(t)| = 0$.

**Proof.** Defining the function $z = \phi(x)$, it follows from (2) that

$$\dot{z} = \beta(h(x), u). \tag{8}$$

This, together with (6) and an integration, implies that

$$z(t) - \mathfrak{z}(t) = \vartheta,\ \ \forall t \ge 0, \tag{9}$$

where $\vartheta := z(0) - \mathfrak{z}(0)$ is the integration *constant*. Now, Assumption **(A2)** ensures that

$$x = \phi^L(z, y) = \phi^L(\mathfrak{z} + \vartheta, y),$$

where we have used (9) to obtain the second identity. Replacing the expression above in the output map yields (7). The proof is completed invoking Assumption **(A3)** and defining the state estimate $\hat{x}$ by replacing $\hat{\vartheta}$ in the place of $\vartheta$ in the equation above. □□□

## 3. Main idea and motivating example

Clearly, the design of the PEBO is simplified if additional signals are *available* for measurement. The main contribution of the paper is to propose a procedure – based on the signal injection technique of [15] – to generate an approximation of the signal $y_v = h_v(x) \in \mathbb{R}^p$, where we defined

$$h_v(x) := \nabla^\top h(x) g(x) b \tag{10}$$

with $b \in \mathbb{R}^m$ a *free* vector utilized in the scaling of the signal injections.

To motivate the addition of this virtual output signal in the context of PEBO we present in this section its application for a benchmark example. Namely, a magnetic levitation system consisting of an iron ball of mass $m$ in a vertical magnetic field as shown in Fig. 1. Assuming the flux, denoted $x_1$, and the current $y$ are related by

$$x_1 = \frac{k}{1 - x_2} y$$

where $k$ is a positive constant, $x_2 \in (-\infty, 1)$ is the distance between the centre of the ball and its nominal position and taking as state vector $x = \mathrm{col}(x_1, x_2, m\dot{x}_2)$, the dynamics of the system is described by (1) with

$$f(x) = \begin{pmatrix} -\frac{r}{k}(1 - x_2)x_1 \\ \frac{x_3}{m} \\ \frac{1}{2k}x_1^2 - mg_0 \end{pmatrix},\ g = \begin{pmatrix}1\\0\\0\end{pmatrix},\ h(x) = \frac{1}{k}(1 - x_2)x_1, \tag{11}$$

where $u$ is the voltage, $r$ is the inductors resistance and $g_0$ denotes the acceleration due to gravity—see [20] for additional details of the model.

It is clear that the observation of the state from the measurement of $y$ seems a daunting task. Our objective is to design a PEBO assuming measurable the additional signal

$$y_v := h_v(x) = \nabla^\top h(x) g = \frac{1}{k}(1 - x_2). \tag{12}$$

The first step is to solve the immersion PDE, which in this case becomes

$$\nabla^\top \phi(x)[f(x) + gu] = \beta\big(h(x), h_v(x), u\big).$$