Systems & Control Letters 111 (2018) 58–63

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Global stabilization for feedforward nonlinear systems with unknown control direction and unknown growth rate ☆

Fang Shang [a,*], Yungang Liu [b], Chengdong Li [a]

[a] School of Information and Electrical Engineering, Shandong Jianzhu University, Jinan 250101, PR China
[b] School of Control Science and Engineering, Shandong University, Jinan 250061, PR China

ARTICLE INFO

Article history:
Received 22 February 2016
Received in revised form 12 June 2017
Accepted 17 November 2017

Keywords:
Feedforward nonlinear systems
Unknown control direction
Unknown growth rate
Global stabilization
Adaptive control

ABSTRACT

This paper investigates the global adaptive state feedback controller design for a class of feedforward nonlinear systems with completely unknown control direction and unknown growth rate. Since the control direction, i.e., the sign of the control coefficient, is unknown, the control problem becomes much more challenging, to which a Nussbaum-type function is exploited. Moreover, the systems heavily depend on the unmeasured states with unknown growth rate, and hence a dynamic gain, rather than a constant one, is adopted to compensate the large system unknowns. For control design, a suitable state transformation is first introduced for the original system. Then, the state feedback controller is proposed based on an appropriate Nussbaum-type function and a dynamic high gain. It is shown that the state of the original system converges to zero, while the other states of the closed-loop system are globally bounded. Finally, a simulation example is provided to illustrate the effectiveness of the theoretical results.

1. Introduction

Since the pioneering work of Teel [1], many important results have been obtained for feedforward nonlinear systems, see e.g., [2–12] and references therein. Most of these works on global stabilization are for feedforward nonlinear systems with exactly known control coefficients. However, when the control coefficients are uncertain or unknown, the problem becomes much more challenging and difficult, and few results are proposed [13–16]. In [13], the problem of input disturbance suppression was studied for a class of feedforward nonlinear systems. In [14], global stabilization problem was investigated for feedforward systems with nonlinearities allowed to be lower-order growing. The systems considered in [13,14] have uncertain control coefficients with known lower and upper bounds. In [15], the considered feedforward systems have unknown control coefficient with known sign. Recently, [16] investigated state feedback controller design for feedforward nonlinear systems with unknown control direction. Later on, [17] studied the problem of adaptive state feedback stabilization a class of feedforward nonlinear systems, whose control coefficient is unknown but with known sign.

In this paper, we generalize the systems studied in our earlier work to a more general ones:

$$\begin{cases} \dot{\zeta}_i = f_i(\zeta, x),\ i = 1, \ldots, m, \\ \dot{x}_i = x_{i+1} + \phi_i(\zeta, x, u),\ i = 1, \ldots, n-1, \\ \dot{x}_n = gu + \phi_n(\zeta, x, u), \end{cases} \tag{1}$$

where $\zeta = [\zeta_1^{\mathrm{T}}, \ldots, \zeta_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{n_1+\cdots+n_m} = \mathbf{R}^{n_\zeta}$, $1 \le m \le n-1$, and $x = [x_1, \ldots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$, $n \ge 3$, are the system states with the initial states $\zeta(0) = \zeta_0$, and $x(0) = x_0$; $u \in \mathbf{R}$ is the control input. The control coefficient g is an unknown nonzero constant. The sign of g, i.e., the control direction, is also unknown. Functions $f_i : \mathbf{R}^{n_\zeta} \times \mathbf{R}^n \to \mathbf{R}^{n_i}$, $i = 1, \ldots, m$, and $\phi_i : \mathbf{R}^{n_\zeta} \times \mathbf{R}^n \times \mathbf{R} \to \mathbf{R}$, $i = 1, \ldots, n$, are locally Lipschitz continuous. Suppose system (1) satisfies the following assumptions:

Assumption 1. The subsystems ζ_i, $i = 1, \ldots, m$ are Input-to-State Stable (ISS) with ISS Lyapunov functions $U_i(\zeta_i)$ satisfying

$$\underline{U}_i\|\zeta_i\|^2 \le U_i(\zeta_i) \le \overline{U}_i\|\zeta_i\|^2, \quad \forall \zeta_i \in \mathbf{R}^{n_i},$$

$$\dot{U}_i \le -\alpha_i\|\zeta_i\|^2 + c_0\sum_{j=i+2}^{n} x_j^2, \quad i = 1, \ldots, m,$$

where $\underline{U}_i, \overline{U}_i, \alpha_i, i = 1, \ldots, m$ are known positive constants, and c_0 is an unknown positive constant.

☆ This work is supported by National Natural Science Foundations (NNSF) of China (61104069, 61325016 and 61473176).
* Corresponding author.
E-mail address: shangfang@sdjzu.edu.cn (F. Shang).

https://doi.org/10.1016/j.sysconle.2017.11.005
0167-6911/

Assumption 2. There exists an unknown constant $c > 0$ such that

$$|\phi_i(\zeta, x, u)| \le c\left(\sum_{j=1}^{m}\|\zeta_j\| + \sum_{j=i+2}^{n}|x_j|\right), \quad i = 1, \dots, n.$$

By Assumption 1, the zero-dynamics of the considered systems are input-to-state stable, and it is similar to those in [17,18]. Assumption 2 shows that system (1) is in a feedforward form, similar or more general assumptions are given in [6,19,20]. Besides, the considered system has unmeasured states dependent growth with unknown constant growth rate c. Therefore, the problem of global stabilization is investigated for feedforward nonlinear systems with unknown growth rate. For nonlinear systems with unknown growth rate, some results of global stabilization have been obtained, see e.g., [19,21]. However, the systems considered in [19,21] have exactly known control coefficient and do not contain zero-dynamics. As we know from the description of system (1), the control coefficient g is completely unknown, including its sign. Therefore, we have to deal with the difficulties caused by both the unknown growth rate and the control coefficient. On the one hand, since the control direction is unknown, it is hard to decide the direction along which the control operates [22]. On the other hand, the existence of unknown g makes the control design of the feedforward system (1) more difficult and challenging [14,17]. As a result, it is difficult to relax the restrictions on system nonlinearities.

Comparing with local stabilization of nonlinear systems, the result of global stabilization is more delicate. The initial state is not restricted to belong to some subset of the state space [7,10,14,17,23–25]. In this paper, we consider the global state feedback stabilization problem for system (1) with unknown control direction. This paper generalizes the systems studied in [16] to those with unknown growth rate c, and [17] to those with unknown control direction $\text{sign}(g)$. Accordingly, the method of construct a dynamic high gain and a Nussbaum-type function is flexibly adopted to deal with the difficulties caused by c and $\text{sign}(g)$. First, a state transformation is introduced to obtain a new system, such that it is more convenient to design the state feedback controller. Then, the state feedback controller is proposed with an appropriate Nussbaum-type function and a dynamic high gain. By suitable choice of the design parameter and the Lyapunov function, the state of the closed-loop system is proved to be globally bounded and the original state of system (1) converges to zero.

Notations. Throughout this paper, I denotes the identity matrix with appropriate dimension; $\mathbf{R}$ denotes the set of all real numbers, $\mathbf{R}^n$ denotes the real n-dimensional space. For a vector or matrix X, X^{T} denotes its transpose, and $\|X\|$ (i.e., $\|X\|_2$) and $\|X\|_1$ denote the Euclidean norm (or 2-norm), and the 1-norm for vectors, and the corresponding induced norms for matrices, respectively.

2. Adaptive state feedback controller

Considering system (1), it is not easy to propose a controller directly. Hence, similar to [17], we first introduce the following state transformation for the subsystem x of system (1):

$$z_i = \frac{1}{r^{n-i+1}}x_i, \quad i = 1, \dots, n, \tag{2}$$

where r is a dynamic gain and satisfies

$$\dot{r} = \frac{1}{r^2}\sum_{i=1}^{n} z_i^2, \quad r(0) = 1. \tag{3}$$

In the maximal interval of existence for the solution of system (3), r is monotonically nondecreasing and hence $r(t) \ge 1$ for $t \ge 0$.

Remark 1. The form of the dynamics of r is the same as that in [17]. It is simple and effective. By introducing the dynamic high gain r, the negative effect caused by the unknown c in Assumption 2 can be successfully handled, as will be seen later.

After state transformation, the following equations are obtained

$$\begin{cases} \dot{z}_i = \dfrac{1}{r}z_{i+1} + \dfrac{1}{r^{n-i+1}}\phi_i - \dfrac{(n-i+1)\dot{r}}{r}z_i, \ i = 1, \dots, n-1, \\ \dot{z}_n = \dfrac{1}{r}gu + \dfrac{1}{r}\phi_n - \dfrac{\dot{r}}{r}z_n. \end{cases} \tag{4}$$

For system (4), we propose the controller in the form of

$$u = N(k)(a_1 z_1 + \cdots + a_n z_n) = N(k)a^{\mathrm{T}}z, \tag{5}$$

$$\dot{k} = \frac{2}{r}z^{\mathrm{T}}Pe_n a^{\mathrm{T}}z, \tag{6}$$

where $z = [z_1, \dots, z_n]^{\mathrm{T}}$, $a = [a_1, \dots, a_n]^{\mathrm{T}}$ with constants a_i, $i = 1, \dots, n$ to be determined later, and $N(\cdot)$ is a smooth function with Nussbaum property:

$$\lim_{s\to\infty}\sup \frac{1}{s}\int_0^s N(k)\mathrm{d}k = +\infty,$$

$$\lim_{s\to\infty}\inf \frac{1}{s}\int_0^s N(k)\mathrm{d}k = -\infty.$$

When the sign of the control coefficient is unknown, it becomes much more difficult to solve the control problem, because we cannot decide the direction along which the control operates [22]. Nussbaum function is successfully developed to deal with this situation. In light of [22,26–30], the difficulty caused by the unknown control direction can be overcome using the Nussbaum function. In fact, the Nussbaum-type function is not restricted to be even or odd [22]. However, in this paper, we restrict the Nussbaum-type function to be even for convenience, such as $N(k) = \mathrm{e}^{k^2}\cos(\frac{\pi}{2}k)$, $N(k) = k^2\cos(\frac{\pi}{2}k)$, and so on. Then, the following lemma is given:

Lemma 1 ([26]). *Let $V(\cdot)$ and $k(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0, \forall t \in [0, t_f)$, $N(\cdot)$ be an even smooth Nussbaum-type function, and g be a nonzero constant. If the following inequality holds:*

$$V(t) \le \int_0^t (N(k(\tau))g + 1)\dot{k}(\tau)d\tau + c', \quad \forall t \in [0, t_f), \tag{7}$$

where c' represents some suitable constant, then $V(t)$, $k(t)$ and $\int_0^t (N(k(\tau))g + 1)\dot{k}(\tau)d\tau$ must be bounded on $[0, t_f)$.

Substituting (5) into (4), system (1) is converted into

$$\begin{cases} \dot{\zeta}_i = f_i, \ i = 1, \dots, m, \\ \dot{z} = \dfrac{1}{r}Az + \psi + \dfrac{1}{r}(N(k)g + 1)e_n a^{\mathrm{T}}z - \dfrac{\dot{r}}{r}Dz, \end{cases} \tag{8}$$

where $\psi = [\psi_1, \dots, \psi_n]^{\mathrm{T}}$ with $\psi_i = \frac{1}{r^{n-i+1}}\phi_i$, $e_n = [0, \dots, 0, 1]^{\mathrm{T}} \in \mathbf{R}^n$, $D = \mathbf{diag}[n, \dots, 1]$, and

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix}.$$

The constants a_i, $i = 1, \dots, n$ are chosen to make matrix A Hurwitz, and such that there exists a symmetric matrix $P > 0$ satisfying

$$A^{\mathrm{T}}P + PA \le -I, \quad DP + PD \ge 0. \tag{9}$$

The choice can always be carried out according to [23,31,32].

Up to now, the main result of the paper is summarized as follows: