Systems & Control Letters 111 (2018) 70–78

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Distributed formation tracking using local coordinate systems

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ARTICLE INFO

Article history:
Received 11 May 2017
Received in revised form 7 October 2017
Accepted 19 November 2017

Keywords:
Formation control
Cooperative control
Rigidity graph theory
Multi-agent systems

ABSTRACT

This paper studies the formation tracking problem for multi-agent systems, for which a distributed estimator–controller scheme is designed relying only on the agents' local coordinate systems such that the centroid of the controlled formation tracks a given trajectory. By introducing a gradient descent term into the estimator, the explicit knowledge of the bound of the agents' speed is not necessary in contrast to existing works, and each agent is able to compute the centroid of the whole formation in finite time. Then, based on the centroid estimation, a distributed control algorithm is proposed to render the formation tracking and stabilization errors to converge to zero, respectively. Finally, numerical simulations are carried to validate our proposed framework for solving the formation tracking problem.

1. Introduction

Formation control for multi-agent systems has attracted increasing attention from control scientists and engineers due to its broad applications [1–3]. A central problem is to drive the agents to realize some prescribed formation shape, and such a problem is usually referred to as the formation stabilization problem. In this line of research, formation stabilization for those with different shapes has been investigated, see, for example, circular formation [4,5], acyclic formation [6], and formations associated with tree graphs [7], minimally rigid graphs [8,9], and more general rigid graphs [10]. Time-varying formation control problems for linear multi-agent systems under switching directed topologies are also investigated in [11]. In addition, the effects of the measurement inconsistency between neighboring agents on the formation's stability are addressed in [12], where it is shown that the resulted distorted formation will move following a closed circular orbit in the plane for any rigid, undirected formation consisting of more than two agents. In [13], the steady-state rigid formation is achieved using an estimator-based gradient control law; in addition, both the static and time-varying mismatched compasses are studied in [14].

Another key problem concerned with formation control for multi-agent systems is formation tracking, which requires to stabilize the prescribed formation, and, additionally, requires that the whole formation follows a given reference trajectory. One commonly reported approach to deal with the formation tracking problem is to use the virtual structure strategy. This technique is built upon assigning a virtual leader to the centroid of the formation to be tracked while achieving the prescribed formation shape [15]. Under this framework, it is shown that the formation tracking can be achieved in finite time by employing the signum function if the virtual leader has directed paths to all the followers [16]. The virtual structure approach is also reported in [17], in which the control and estimation on a common virtual leader is addressed using a consensus algorithm. Integrating the techniques from nonsmooth analysis, collective potential functions and navigation feedback, a distributed algorithm for second-order systems is designed such that the velocity consensus to the virtual leader is achieved [18]. The formation tracking problem can also be solved using the distributed receding horizon control (RHC), for a group of nonholonomic multi-vehicle systems [19]. By applying RHC, some additional tasks, e.g., collision avoidance and consistency, can be realized through adding constraints on allowed uncertain deviation.

Akin to the virtual structure approach, the leader–follower strategy has also been widely employed to solve formation tracking problems (e.g., [20–24]). In [20], the formation tracking problem is solved based on formation stabilization with one designated leader among the group. To deal with the intrinsic unknown parameters for a class of nonlinear systems, an adaptive control law using the backstepping technique is proposed in [21], such that all the subsystems' outputs are regulated to achieve consensus tracking. In [22], to compensate the unknown slippage effect of mobile robots, a distributed recursive design strategy involving the adaptive function approximation technique is developed. More recently, the formation tracking problem for second-order multi-agent systems under switching topologies is studied in [23], where

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https://doi.org/10.1016/j.sysconle.2017.11.004
0167-6911/

one of the agents is set to be the leader to perform tracking tasks. The results therein are also feasible to the target enclosing problem for multi-quadrotor unmanned aerial vehicle systems. In [24], different from the one-leader tracking case, the formation tracking problem with multiple leaders is addressed. To drive the followers to the convex hull spanned by the leaders, a protocol is designed via solving an algebraic Riccati equation.

It should be noted that in the results discussed above, almost all the desired formations are specified by offset vectors with respect to the virtual/real leader or virtual centroid of the group. Those offset vectors are required to be set a priori in a common global coordinate system. In addition, each agent needs to know its corresponding desired offsets as well as its neighbors'. In particular, the agreement reached on the estimations of the virtual centroid is normally different from the real centroid of the group. However, it is sometimes meaningful to locate the real centroid when performing tasks like the transportation of objects. Furthermore, the approaches developed in these existing works are only applicable to the scenarios where the reference trajectory is an exogenous signal that is independent of the states of the system. To estimate the centroid of the formation, a consensus-based algorithm is proposed in [25], wherein the estimation of each agent is updated by averaging their projections and directions. However, the convergence can be ensured only when the underlying graph is complete. In [26], a tree-based algorithm is adopted to estimate the centroid, while each agent is required to maintain a list of trees with constant size. Recently, the weighted-centroid tracking problem has been considered in [27–29]. Unlike the leader–follower structures in which the dynamics of the followers and leaders can be separated, the control objective therein is to track some globally assigned function which is implicitly related to all agents' dynamics. In [27], a controller–observer scheme is designed for the single integrator dynamics such that the weighted centroid of the whole formation follows some given trajectory. As an extension, one additional task function for the formation is introduced in [28]. In [29], a finite-time centroid observer is constructed, and the distance-based control laws are developed by employing rigidity graph theory.

In the present paper, we consider the formation tracking problem, in which the centroid of the formation moves as the agents move and is unknown to all of the agents. Under this case, the problem becomes more challenging due to the inner coupling and conflict between centroid estimation, formation stabilization and reference tracking. By adopting the feedback term from the gradient descent control, we design a new class of finite-time centroid estimator that is continuously differentiable. Based on the output of the estimator, the proposed distance-based control laws render the convergence to the prescribed formation shape while keeping its centroid following the reference. Compared with the previous work of using virtual/real leader structure, the proposed estimator–controller framework can be implemented in agents' local coordinate systems, which not only increases the robustness to the noises in the sensing signals but also reduces the equipment cost of the overall system. Moreover, the control law in this paper is more scalable and distributed in the sense that some constraints are removed, including the a priori knowledge of the position information of the reference trajectory [27,28] and the agents' maximum speed [29]. In addition, the precise knowledge of the time-varying centroid can be obtained in finite time via the proposed smooth centroid estimator, which renders a faster convergence speed than that in [25,26]. In addition, the centroid estimator in [25] is only valid under complete graphs whereas the one in this paper can be directly applied to any general undirected graphs.

The paper is organized as follows. Section 2 introduces the formation tracking problem and basic concepts of graph rigidity. In Section 3, the main results are presented including the estimator–controller scheme and the theoretical analysis. Section 4 extends the results to a more general case. The numerical simulations are presented in Section 5. Finally, we give the conclusions in Section 6.

2. Problem formulation

A team of $n > 1$ agents is considered, each of which is characterized by the single integrator dynamics

$$\dot{q}_i^g = u_i^g, \quad i = 1, \dots, n, \tag{1}$$

where $q_i^g \in \mathbb{R}^d$ and $u_i^g \in \mathbb{R}^d$ are, respectively, the position and the control input of mobile agent i with respect to the global coordinate system ${}^g\Sigma$. Each agent i is also assigned with the local coordinate system ${}^i\Sigma$, whose origin is exactly the point q_i^g. In this paper, the local coordinate systems are assumed to share the same orientations. We use q_j^i to denote agent j's position with respect to ${}^i\Sigma$. This definition also applies to other variables. Note that the local variable q_j^i and the global one q_j^g have the following relationship:

$$q_j^g = q_j^i + q_i^g.$$

Here, q_i^g is actually unknown to the agents, since the global coordinate system is introduced only for analysis purposes.

The neighboring relationships between the agents are defined by an undirected graph $\mathcal{G}$ with the vertex set $\mathcal{V} = \{1, 2, \dots, n\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ where there is an edge (i, j) if and only if agents i and j are neighbors of each other. We use $\mathcal{N}_i$ to denote the set of neighbors of agent i. The graph $\mathcal{G}$ is embedded in $\mathbb{R}^d$ when $q = [q_1^T, q_2^T, \dots, q_n^T]^T$ is realizable and the pair $(\mathcal{G}, q)$ is called a *framework*. The *adjacency matrix* $A = a_{ij} \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ is defined as $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The interaction relationships among the agents and the reference signal is denoted by matrix $B = \text{diag}\ \{b_1, \dots, b_n\}$, where $b_i = 1$ if agent i has access to the reference signal directly, and $b_i = 0$ otherwise. By assigning an arbitrary orientation to $\mathcal{G}$, the *incidence matrix* $H = [h_{ij}] \in \mathbb{R}^{|\mathcal{E}|\times n}$ is defined by

$$h_{ij} = \begin{cases} 1, & i\text{th edge enters node } j, \\ -1, & i\text{th edge leaves node } j, \\ 0, & \text{otherwise}, \end{cases}$$

where $|\mathcal{E}|$ represents the cardinality of the edge set $\mathcal{E}$, and it is taken to be m throughout the paper. The *Laplacian matrix* is then given by $L = H^T H \in \mathbb{R}^{n\times n}$.

Now, we formulate the problem to be investigated in this paper. On one hand, to achieve a desired shape of the formation, each agent i is required to keep some prescribed distance $d_{ij}, j \in \mathcal{N}_i$, namely, the agents are driven to the following target set:

$$\mathcal{T}_d = \{q^g \in \mathbb{R}^{nd} |\ \|q_i^g - q_j^g\| = d_{ij},\ \forall (i, j) \in \mathcal{E}\}. \tag{2}$$

On the other hand, at the same time, the stabilized formation is guided through the control law such that its centroid q_c^g tracks some smooth reference signal $q_d^g(t) : t \to \mathbb{R}^d$, where the centroid of the formation is defined by

$$q_c^g = \frac{1}{n}\sum_{i=1}^{n} q_i^g. \tag{3}$$

Equivalently, the tracking task can be written as

$$\lim_{t\to\infty}(q_c^g - q_d^g(t)) = \mathbf{0}. \tag{4}$$

To introduce the notion of graph rigidity, we first define a function

$$f_{\mathcal{G}}(q_1^g, \dots, q_n^g) = [\cdots, \|q_i^g - q_j^g\|^2, \dots]^T, \tag{5}$$