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On design of interval observers with sampled measurement

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ABSTRACT

New design of interval observers for continuous-time systems with discrete-time measurements is proposed. For this purpose new conditions of positivity for linear systems with sampled feedback are obtained. A sampled-data stabilizing control is synthesized based on provided interval estimates. Efficiency of the obtained solution is demonstrated on examples.

1. Introduction

State estimation in dynamical systems is a rather complicated and practically important problem [1,2]. Sparse discrete measurement of the output for a continuous-time plant makes solution of this problem even more intricate [3–7]. An observer synthesis is especially problematical for the cases when the model of a nonlinear system is uncertain, and it contains parametric and/or signal uncertainties. An observer solution for these more complex situations is highly demanded in applications. Interval or set-membership estimation is a promising framework for observation in uncertain systems [8–13], when all uncertainty is included in the corresponding intervals or polytopes, and as a result the set of admissible values (an interval) for the state is provided at each instant of time. The size of that set is related with the level of uncertainty of the plant model.

In this work the problem of design of interval sampled-data observers is studied. A peculiarity of an interval observer is that in addition to stability conditions, some restrictions on positivity of estimation error dynamics have to be imposed (in order to envelop the system solutions). In the present work we are going to use the time-delay framework for modeling and analysis of sampled-data systems [14–17]. The first objective of this work is to recall the delay-dependent positivity conditions, which are based on the theory of non-oscillatory solutions for functional differential equations [18,19], and to develop them to the time-varying sampled-data case, *i.e.* new sampling dependent conditions of positivity are derived. Next, continuing the research direction of [20], where the pure time-delay case has been studied, design of interval observers is given for continuous-time linear systems with discrete measurements (with time-varying sampling). The existing solutions in the field [21,22] are based on delay-independent positivity conditions, and the interval observer constructed in [22] has a hybrid nature, which is more complicated than one proposed in the present work. Finally, following the ideas of [23] a sampled-data stabilizing control algorithm is synthesized based on interval estimates.

The paper is organized as follows. Some preliminaries are given in Section 2. The delay-dependent positivity conditions for continuous systems under sampled-data measurements are presented in Section 3. The interval observer design is performed for a class of linear systems (or a class of nonlinear systems in the output canonical form) with sampled measurements in Section 4. A dynamic output control design is carried out in Section 5. Examples of numerical simulation are presented in Section 6.

2. Notation and preliminaries

In the rest of the paper, the following notation will be used:

- $\mathbb{R}$ is the Euclidean space ($\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$);
- $|x|$ denotes the absolute value of $x \in \mathbb{R}$, $\|\cdot\|$ is the Euclidean norm of a vector or induced norm of a matrix;

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- for a Lebesgue measurable and locally essentially bounded input $u : \mathbb{R}_+ \to \mathbb{R}^p$ the symbol $\|u\|_{[t_0,t_1]}$ denotes its L_∞ norm $\|u\|_{[t_0,t_1)} = \text{ess sup}_{t\in[t_0,t_1)} \|u(t)\|$, or simply $\|u\|$ if $t_0 = 0$ and $t_1 = +\infty$, the set of all such inputs with the property $\|u\| < \infty$ will be denoted as $\mathcal{L}^p_\infty$;
- for a matrix $A \in \mathbb{R}^{n\times n}$ the vector of its eigenvalues is denoted as $\lambda(A)$;
- I_n and $0_{n\times m}$ denote the identity and zero matrices of dimensions $n \times n$ and $n \times m$ respectively;
- $a \mathcal{R} b$ corresponds to an elementwise relation $\mathcal{R}$ (a and b are vectors or matrices): for example $a < b$ (vectors) means $\forall i : a_i < b_i$;
- for a symmetric matrix Υ, the relation $\Upsilon \prec 0$ ($\Upsilon \preceq 0$) means that the matrix is negative (semi) definite.

2.1. Interval bounds

Given a matrix $A \in \mathbb{R}^{m\times n}$ define $A^+ = \max\{0, A\}$, $A^- = A^+ - A$ and $|A| = A^+ + A^-$. Let $x \in \mathbb{R}^n$ be a vector variable, $\underline{x} \le x \le \overline{x}$ for some $\underline{x}$, $\overline{x} \in \mathbb{R}^n$, and $A \in \mathbb{R}^{m\times n}$ be a constant matrix, then [24]:

$$A^+\underline{x} - A^-\overline{x} \le Ax \le A^+\overline{x} - A^-\underline{x}. \quad (1)$$

A matrix $A \in \mathbb{R}^{n\times n}$ is called Metzler if $A_{i,j} \ge 0$ for all $1 \le i \ne j \le n$.

2.2. Delay-dependent positivity

Consider a scalar time-varying linear system with time-varying delays [18]:

$$\dot{x}(t) = a_0(t)x[g(t)] - a_1(t)x[h(t)] + b(t), \quad (2)$$

$$x(\theta) = 0 \quad \text{for } \theta < 0,\ x(0) \in \mathbb{R}, \quad (3)$$

where $a_0 \in \mathcal{L}_\infty$, $a_1 \in \mathcal{L}_\infty$, $b \in \mathcal{L}_\infty$, $h(t) - t \in \mathcal{L}_\infty$, $g(t) - t \in \mathcal{L}_\infty$ and $h(t) \le g(t) \le t$ for all $t \ge 0$. For the system (2) the initial condition in (3) is, in general, not a continuous function (if $x(0) \ne 0$).

The following result proposes delay-independent positivity conditions.

Lemma 1 *([18, Corollary 15.7]). Let $0 \le a_1(t) \le a_0(t)$ for all $t \ge 0$. If $x(0) \ge 0$ and $b(t) \ge 0$ for all $t \ge 0$, then the corresponding solution of (2), (3) $x(t) \ge 0$ for all $t \ge 0$.*

Recall that in this case positivity is guaranteed for "discontinuous" initial conditions. The peculiarity of the condition $0 \le a_1(t) \le a_0(t)$ is that it may correspond to an unstable system (2). In order to overcome this issue, delay-dependent conditions can be introduced.

Lemma 2 *([18, Corollary 15.9]). Let $0 \le \frac{1}{e}a_0(t) \le a_1(t)$ for all $t \ge 0$ and*

$$\sup_{t\in\mathbb{R}_+} \int_{h(t)}^{t} \left[a_1(\xi) - \frac{1}{e}a_0(\xi)\right] d\xi < \frac{1}{e},$$

where $e = \exp(1)$. If $x(0) \ge 0$ and $b(t) \ge 0$ for all $t \ge 0$, then $x(t) \ge 0$ for all $t \ge 0$ in (2), (3).

These lemmas describe positivity conditions for a scalar system, they can also be extended to a n-dimensional system.

Corollary 1 *([20]). The system*

$$\dot{x}(t) = A_0x(t) - A_1x(t - \tau(t)) + b(t), \quad t \ge 0,$$

where $x(t) \in \mathbb{R}^n$, $\tau : \mathbb{R}_+ \to [-\overline{\tau}, 0]$ and $b : \mathbb{R}_+ \to \mathbb{R}^n_+$ are Lebesgue measurable functions of time, $\overline{\tau} \in \mathbb{R}_+$, with initial conditions

$$x(\theta) = 0 \quad \text{for } -\overline{\tau} \le \theta < 0,\ x(0) \in \mathbb{R}^n_+,$$

is positive (i.e. $x(t) \ge 0$ for all $t \ge 0$) if $-A_1$ is Metzler, $A_0 \ge 0$, and

$$0 \le (A_0)_{i,i} \le e(A_1)_{i,i} < (A_0)_{i,i} + \overline{\tau}^{-1}$$

for all $1 \le i \le n$.

3. Positivity of sampled systems

Consider a time-invariant version of (2):

$$a_0(t) = a_0, \quad a_1(t) = a_1, \quad g(t) = t, \quad (4)$$

$$h(t) = t_k \quad \forall t \in [t_k, t_{k+1}),$$
$$t_{k+1} = t_k + T_k, \quad k \ge 0,\ t_0 = 0, \quad (5)$$

where $0 < T_k \le \overline{T}$ is a time-varying sampling rate. Then Lemma 2 admits the following corollary.

Corollary 2. *For (4), (5) let $0 \le a_0 \le ea_1 < a_0 + \overline{T}^{-1}$. If $x(0) \ge 0$ and $b(t) \ge 0$ for all $t \ge 0$, then the corresponding solution of (2)–(5) $x(t) \ge 0$ for all $t \ge 0$.*

However, as we can see from the result given below, the conditions of Corollary 2 are very conservative:

Lemma 3. *Consider the system (2), (4), (5) with $x(0) \ge 0$ and $b(t) \ge 0$ for all $t \ge 0$, then $x(t) \ge 0$ for all $t \ge 0$ provided that one of the following conditions is satisfied:*

(i) $a_1 \le 0$;
(ii) $a_0 \ge a_1 > 0$;
(iii) $a_1 > 0,\ a_1 > a_0,\ \overline{T} \le \frac{1}{a_0} \ln \frac{a_1}{a_1 - a_0}$.

Note that

$$\lim_{a_0\to 0} \frac{1}{a_0} \ln \frac{a_1}{a_1 - a_0} = \frac{1}{a_1},$$

then condition (iii) for $a_0 = 0$ reads: $a_1 > 0$ and $\overline{T} \le a_1^{-1}$.

Proof. Such a system for $t \in [t_k, t_{k+1})$ for any $k \ge 0$ has solution:

$$x(t) = e^{a_0(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{a_0(t-s)}[b(s) - a_1x(t_k)]ds$$

and for any $b(t) \ge 0$ the integral $\int_{t_k}^{t} e^{a_0(t-s)}b(s)ds$ is always positive, then in order to identify the conditions of positivity of the solutions the worst case $b(t) = 0$ for $t \ge 0$ has to be analyzed. For this case and for $t \in [t_k, t_{k+1})$, if $a_0 \ne 0$ we obtain

$$x(t) = \left[\left(1 - \frac{a_1}{a_0}\right) e^{a_0(t-t_k)} + \frac{a_1}{a_0}\right] x(t_k),$$

and for $a_0 = 0$,

$$x(t) = [1 - a_1(t - t_k)]x(t_k).$$

Therefore, for $x(t_k) \ge 0$ the solutions are positive if $\left(1 - \frac{a_1}{a_0}\right)e^{a_0(t-t_k)} + \frac{a_1}{a_0} \ge 0$, which is true for ($a_0 \ne 0$)

$$a_1 \le 0 \quad \text{or} \quad a_0 \ge a_1 > 0 \quad \text{or} \quad a_1 > 0,\ a_1 > a_0,$$
$$\overline{T} \le \frac{1}{a_0} \ln \frac{a_1}{a_1 - a_0},$$

or $1 - a_1(t - t_k) \ge 0$ that is satisfied for ($a_0 = 0$)

$$a_1 \le 0 \quad \text{or} \quad a_1 > 0,\ \overline{T} \le a_1^{-1}.$$

Using L'Hôpital's rule we derive

$$\lim_{a_0\to 0} \frac{1}{a_0} \ln \frac{a_1}{a_1 - a_0} = \lim_{a_0\to 0} \frac{\ln \frac{a_1}{a_1-a_0}}{a_0} = \lim_{a_0\to 0} \frac{\frac{1}{a_1-a_0}}{1} = \frac{1}{a_1},$$

then the stated delay-dependent positivity conditions follow (the case for $a_0 \ne 0$ includes $a_0 = 0$). □

Note that the result of Lemma 3 deals only with positivity of the solutions, but not with stability, and the case of Lemma 1 is completely covered. Lemma 2 deals (implicitly through non oscillatory solution behavior) with stable positive systems, then the following extension of Lemma 3 can be proposed.